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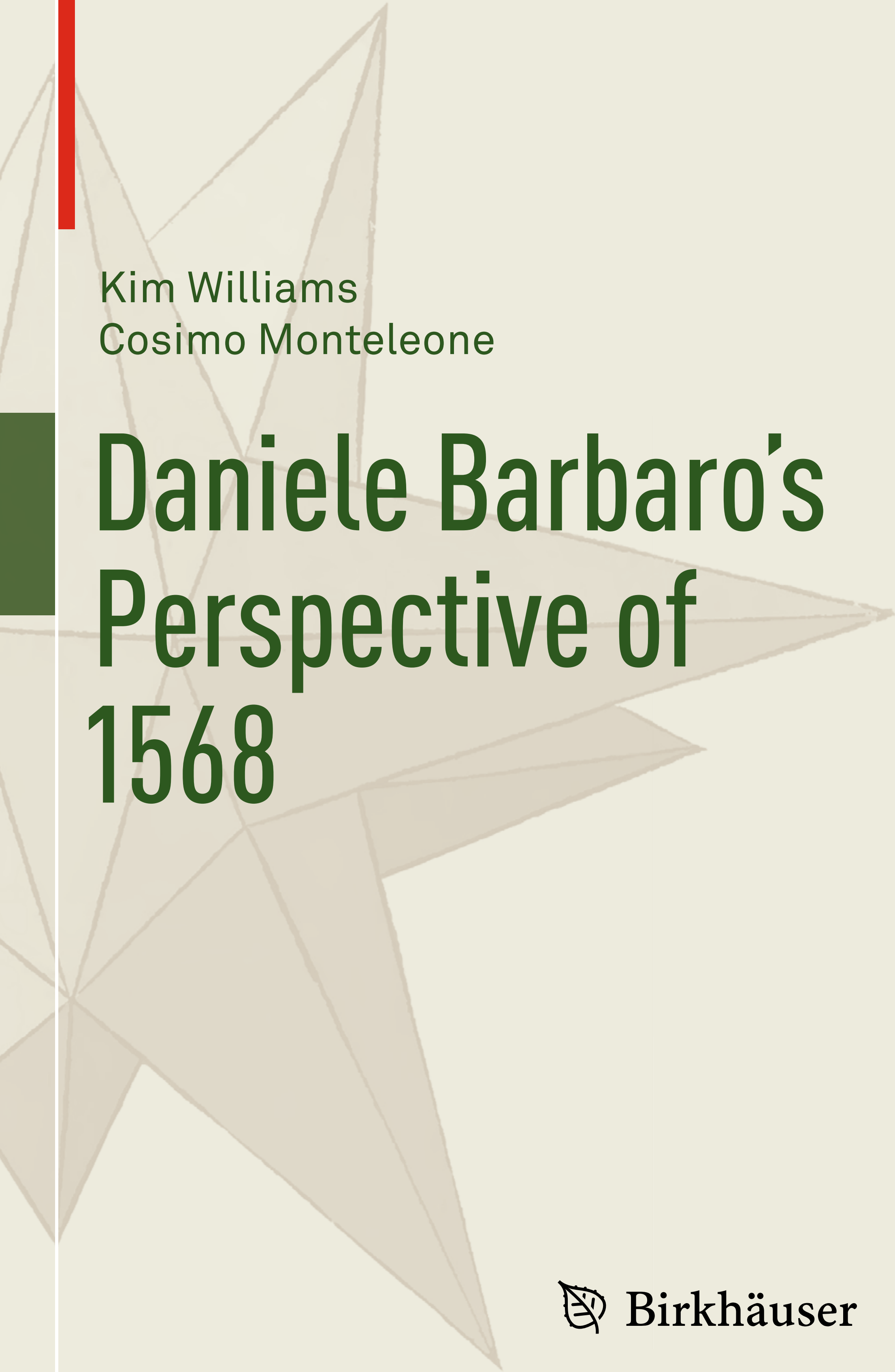
Daniele Barbaro's Perspective of 1568



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Daniele Barbaro's Perspective of 1568

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FOREWORD

Philip Steadman

This is the first translation into English of Daniele Barbaro's *The Practice of Perspective* of 1568. Kim Williams and Cosimo Monteleone have done a splendid job with a text that is unclear in many places. Several other important Renaissance treatises still lack English translations, including Piero della Francesca's crucial *De prospectiva pingendi*, written in the late fifteenth century, and Albrecht Dürer's highly influential *Underweysung der Messung* of 1525. It will do a great service to scholars and amateurs who lack the languages in question if this edition of Barbaro stimulates further translations, offering insights into obscure passages and provoking new research.

Barbaro's book is an enthusiastic compendium of contemporary knowledge, not just of conventional perspective methods, but other more specialised topics in projective geometry: anamorphic pictures, the construction of sundials, map projection, a qualitative account of cast shadows, the design of stage sets in perspective, and mechanical picture-making devices. Only parts of the material are original to Barbaro. For techniques of perspective construction, he draws heavily on Piero and Dürer. From Dürer he also takes a method of drawing human heads, Dürer's famous machines for perspective drawing, and material on the geometry of regular polyhedra.

The book starts in a conventional way with some Euclidean optics – a subject that greatly interested Barbaro – followed by the basic elements of perspective method, and techniques for 'degrading' plane figures into perspective views. The subjects of the exercises become more elaborate, leading up to architraves, vaults, and the bases and capitals of classical columns. Barbaro explains methods derived from Piero, using marked rulers to produce views of complex objects. (His attempt at drawing a spiral staircase however – something that has defeated many perspectivists – is a disaster.)

The longest section is devoted to the Platonic and Archimedean solids, following in the tradition of Luca Pacioli's *Divina proportione* – with its spectacular illustrations by Leonardo – which in turn owed large debts to

Piero. Barbaro provides 'nets' for all the polyhedra, to be cut out, folded and glued to create solid models, and gives detailed instructions for making perspective views.

This is the moment when the flow of ideas reverses its previous one-way south-to-north direction, and Germany starts to speak to Italy. It is worth remarking that Wenzel Jamnitzer's *Perspectiva corporum regularium* was published in Nuremberg in the same year as Barbaro's treatise. The drawings of solid bodies in the two books are very comparable, although Barbaro's polyhedra remain coolly classical, while Jamnitzer gets carried away with more elaborate stellations and fanciful encrustations.

Barbaro's chapters on stage scenery are a disappointment. He draws material from Vitruvius, whose *Ten Books on Architecture* he had edited, on *scenographia* in the ancient drama. He reproduces Sebastiano Serlio's versions of Vitruvius's Tragic, Comic and Satiric Scenes. But he is muddled about the use of *periaktoi*, the triangular scenic prisms of the classical stage. He says that the three types of scene were painted on the three sides of a single *periaktos*, and that the images on multiple *periaktoi* were drawn in perspective to share a common eyepoint. There is no authority from Vitruvius for this, and the three classes of drama would not have been combined in a single play.

There follows a section on anamorphic or stretched perspectives – drawings that only look right and become comprehensible when viewed at extremely shallow angles. Barbaro describes a method by which a design is drawn on paper, pricked with holes, and projected obliquely onto a wall or screen with sunlight or a lantern. This is the first printed account of how to make anamorphoses. The architect Vignola had written on the subject previously, but his work was only edited and published by Egnazio Danti in 1583, and Vignola/Danti's method is less sophisticated than Barbaro's.

Chapters on the construction of terrestrial globes and sundials move away from the central theme of perspective drawing to look at other applications of projective geometry. Barbaro explains how to make globes from tapered paper 'gores', and how to project the great circles of the earth onto the flat planes of maps. Several chapters are devoted to a 'universal clock' invented by Barbaro himself, taking the form of a spherical framework, with which the lines of sundials of different types could be positioned and marked by the use of silk threads.

A section on lighting, shadows and colours moves back to the interests of painters. It covers sources of natural and artificial light, the shading of solid bodies, and an example due to Dürer of the shadow projected by a cube (imperfectly illustrated). The measurement and proportions of the human body are treated with material taken from Vitruvius and Dürer, including orthographic plan and side views of heads in different orientations.

In his discussion of optical and mechanical aids to perspective, Barbaro gives one of the very earliest accounts of a camera obscura with a lens, which made the instrument useful to artists. Barbaro grows lyrical in his description of this 'very beautiful experiment in perspective':

> Here you will see the forms on the paper as they are, with the degradations and colours, and shadows and motions, and clouds, the shimmering of water, the flight of birds, and all else that can be seen.

What an experience this must have been for the Renaissance viewer, a kind of proto-cinema in motion and colour. Barbaro also describes Dürer's machine for drawing a lute using stretched strings, and says that he has made many drawings with the device 'in the chambers of the most reverend Cardinal Turnone, to his great delight.' He also describes – but does not illustrate – another interesting device invented by Baldassarre Lanci for use in surveying and making cylindrical panoramas.

Barbaro's style is unavoidably dry when he is taking the reader point by point and line by line through his demonstrations – a characteristic shared with Piero, whose style J. V. Field criticises as 'grimly repetitious'. The charm of the book is in the author's evident pleasure and excitement in his subject, especially those topics where he is covering unfamiliar ground. Barbaro repeatedly expresses his delight at the ingenuity and subtlety of his fellow geometers and their beautiful discoveries.

London, March 2021

Notes on the Translation

Kim Williams and Cosimo Monteleone

The subject of perspective, treated in the way that Barbaro treats it – as an obligatory subject of study by those who aim at perfecting their artistic skills – was actually already a little out of style when Barbaro was writing. In fact, in his Foreword we find him complaining:

> Although this has many marvellous rationales in its use and practice that are very useful for painters, sculptors, and architects, it is very abandoned, not to say disdained by them, and it escapes many to whom it is more necessary than sought after.[1]

This is even truer today, when perspective views of Platonic and Archimedean solids are seen as curiosities and aesthetic objects rather than objects of serious study. This begs the question of whether a translation is necessary at all: if the topic is out of date, it is really necessary to bring the treatise to the attention of modern readers? The argument offered against translations of works like this is that the specialists and historians who seek out such treatises are almost always capable of reading them in the original.

The initial idea of translating Barbaro's perspective arose during the translation of his commentary on Vitruvius. This was partly because in Vitruvius Barbaro describes his treatise on perspective, in a passage that almost amounts to an advertisement of it:

> ... it is the work of good judgement to know how to place the point appropriately so that what is painted represents a site in a natural way of things as they are, and nothing that is forced, precipitous, deformed, or unseemly, as is seen in many scaenae where things are much too small, buildings too large, the vanishing points are drawn so low and gracelessly that they cannot be viewed with pleasure from either close up or far away. This necessity has moved me to want to improve those who study in this area as well, as far

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[1] Barbaro (1568, p. 3): *Percioche si come ella ha molte, & meravigliose ragioni nell'uso, & essercitio suo molto utile a Pittore, Scultori, & Architetti, cosi molto abandonata, per non dire sprezzata, & fuggita si trova da quelli, a i quali è piu necessaria, che ricercata.*

> as I am able, and so I have written about perspective with the reasonable ways and means aimed at the practice of what is called *scenographia*. I have laid the foundations of this knowledge and the rules of this practice, by defining, dividing, and demonstrating what is required for the rationale of this, so that there need be no doubts about how to situate the view in its proper, accommodated place, in order to avoid falling into those errors that I have described above. Here can be found the disposition of regulated planes in square and out of square; the refinement of any solid whatsoever; the way to show forms in elevation according to their heights; the rationale of the parts of columns; the three sorts of scaenae; plus all that pertains to shadows, to light, to some easy methods using instruments, and some other manners of this delightful and necessary practice.[2]

Barbaro's commentary on Vitruvius was aimed at updating the material of the Roman architect by adding to it knowledge that had been acquired during the centuries since the original treatise was written. However, rather than adding his studies on perspective to it, he kept them separate, ultimately composing a separate treatise. This may have been because the intended readership was different: where the commentary on Vitruvius was primarily aimed at architects, the treatise on perspective was primarily aimed at painters, although not to the exclusion of architects, especially those concerned with designing sets for the theatre. It may have been because his studies on perspective where quite different from the other material he studied in order to prepare his commentary on Vitruvius. Where his additions to Vitruvius could be quite lengthy (his commentary is about twice the length of the original treatise), as in the sections on proportion in Book III, on music in Book V, and most importantly on the building of sun clocks in Book IX, they were mainly glosses or summaries of the works of other scholars. Witness the discussion of sun clocks, drawn largely from the work of Commandino. In fact, for this reason Barbaro is sometimes dismissed as a compiler, and his work criticised as unoriginal. But his work on perspective is quite different, because it reflects the personal and in-depth study that preceded the treatise as finally published. Thus, in discussions during the preparation of the text and preface for Cosimo Monteleone's *La prospettiva di Daniele Barbaro*,[3] it appeared to both of us that it was desirable, even necessary, to translate the treatise on perspective in order to complete the work of presenting Barbaro as the scholar that he was: not merely a compiler, but an innovative explorer.

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[2] Barbaro (1567, p. 257); Williams (2019, pp. 421-422): … *è opra di bel giudicio di saper ponere il punto cosi accommodatamente, che tutto quello, che si vede dipinto rappresenti un sito, & un'esser naturale delle cose, & niente sia di sforzato, di precipitoso, di difforme, di sgarbato, come si vede nelle scene di molti le cose oltra modo picciole, gli edificij, che traboccano, i fuggimenti tanto al basso punto senza dolcezza tirati, che nè dappresso, nè da lontano possono essere con diletto veduti. Questa necessità mi ha mosso a voler giovare, quanto per me si puo, anche in questa parte a gli studiosi. & però io ho scritto di prospettiva con vie, & modi ragionevoli drizzati alla pratica che è detta scenografia. & ho gettato i fondamenti di questa cognitione, & le regole di questa pratica, con diffinire, dividere, & dimostrare, quanto alla detta ragione è necessario, accioche senza dubitatione l'huomo posso porre la veduta in proprio, & accommodato luogo, accioche non si caschi in quelli errori, che di sopra ho detto. ivi si vederà la dispositione de i piani regolati, & in regolati, in squadra, & fuori di squadra, & i perfetti di qualunque corpo si sia. il modo di levare i corpi secondo le altezza loro, & la ragione delle parti delle colonne, & delle tre sorti di scene, con quanto appartiene all'ombreggiare, a i lumi, ad alcuni modi facili per via d'instrumenti, & da'alcune altre maniere di questa pratica dilettevole, & necessaria.*

The use of language

In compiling his treatises, Barbaro considered the impact that the language would have. For both his Vitruvius and his perspective he produced two texts, one in Latin, incomplete and never published, intended for a scholarly, cultured readership, and one in Italian intended for a broader, less erudite and more practice-minded readership. The texts differed not only in language, but in treatment. He says, for instance, in the commentary to Vitruvius:

> It doesn't behove a wise man to argue about names when the thing is understood. We in our Latin commentaries place greater emphasis on the names pertinent to the Latins, so now it is sufficient for us to have mentioned them in the course of the interpretation.[4]

In the published treatise on perspective, he makes similar comments:

> In my Latin [treatise on] perspective I put the proofs of all the foundations of this art, but for now I leave them out so as not to slow down the scholars of the practice of perspective.[5]

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[3] Monteleone (2020).

[4] Barbaro (1567, p. 301); Williams (2019, p. 489): *…né conviene ad huomo saldo contender de nomi la, dove s'intende la cosa. Noi nei nostri commentari Latini piu ampiamente ragioniamo di questi nomi, conveniente a Latini: perche hora ci puo bastare haverli in trascorso della interpretatione accennati.*

[5] Barbaro (1568, p. 9): *Io nella Perspettiva mia Latina ho posto le dimostrationi di tutti i fondamenti di quest'arte, lequali hora io lascio per non ritardare gli studiosi della pratica della Perspettiva.*

Barbaro, like all the members of Accademia degli Infiammati,[6] thought that Italian had the same scientific dignity as Latin, in fact, in some respects it was considered to be even more advanced than the language of the ancients, which did not include all the terms coined during the Middle Ages and the Renaissance. On the other hand, Latin made it possible to communicate beyond the borders of the peninsula.

Similarly, we took into consideration the way an English translation and commentary would allow Barbaro's treatise to be appreciated the world over. Because *La pratica della perspettiva* was originally intended as a guide to perspective that was accessible even to those without a specialized background, providing a text and commentary in English today accomplishes that same purpose in the modern world.

Monteleone's *La prospettiva di Daniele Barbaro* took the ground-breaking step of including the transcription and original images of Barbaro's unpublished manuscript from which the printed treatise was derived, today conserved in Venice's Biblioteca Nazionale Marciana (Ms. It. IV, 39=5446). Because this first-ever publication brought this manuscript out of the library and into the hands of modern readers, we felt that the next step was to reference parts of it as an accompaniment to the translation of the printed treatise. This unusual approach of integrating a published treatise with parts of the unpublished manuscripts in both Italian and Latin allows us to provide a more complete portrait of Barbaro's thinking: the depth of detail and tireless effort expended in his studies show him to have truly mastered the material. A comparison of the published text with the unpublished manuscript also helps correct the many errors in the printed text, probably the fault of the publisher because the coordination of figures and text in Part III, and our choice of integrating the text makes it clear as well that the errors were not in the conception but in the execution.

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[6] The Accademia degli Infiammati aimed to promote and disseminate the Italian language with the translation of scientific Greek and Latin texts. Among its members there were Leone Orsini, Sperone Speroni, Bernardino Tomitano, Benedetto Varchi, Luigi Alemanni, Vincenzo Maggi, Giovanni Cornaro, Galeazzo Gonzaga, Cola Bruno, Pietro Bembo, Pietro Aretino, Angelo Beolco (Ruzzante), Francesco Sansovino, Fabrizio Strozzi, Alessandro Piccolomini, Lorenzo Priuli, Giovan Battista Montano, Michele Barozzi, and Paolo Manuzio. See: Girardi (1995, p. 3); Vasoli (2002, pp. 360-362); Graheli (2015, pp. 65-68).

The approach to the translation

A translation can be aimed at bringing an author into the present day, expressing his material in a way that is familiar and meaningful in a modern context – even one that is chronologically far from the author's – and expressing his ideas in an updated and tone to make it accessible to modern readers. The advantage of this is that readers do not have to accustom themselves to unfamiliar cadences and vocabulary. This is often done with the classics of Homer, for instance, or even Shakespeare, allowing modern readers to enjoy timeless stories without being hampered by unfamiliar literary styles and forms. An alternative is to bring the reader into the author's day, allowing him to travel through time and experience in some way what it must have been like to read a work as the author's contemporaries read it. This is how we decided to approach Barbaro's treatise on perspective.

Vocabulary

All translations are interpretations, and a number of decisions must be made in order to both establish correct vocabulary and find the appropriate tone. In the case of the present translation, a number of issues regarding Barbaro's voice and vocabulary had already been addressed in the translation of his commentary on Vitruvius. There is no doubt that the earlier work translating that much longer and more encyclopaedic work made the translation of this shorter and more focused work much easier. However, the fact that the treatise on perspective treats a very specialised subject calls for special attention to vocabulary. And while it was decided to translate the treatise in such a way that its language was not overly modernised, some terms had to be updated in order to be fully understandable.

Some of these terms are relatively straightforward. Barbaro's *egualmente distante* (lit. equally distant) has been translated with the modern term 'parallel'. His *campo* has been translated as 'area'.

Barbaro's *spacio* can mean two different things: in a statement such as '*sia sopra lo a. fatto il circolo spacio ad. & sia zdq*' (p. 21 in the original), *spacio* means 'radius' ('let there be drawn from point *a* the circle *zdq* with the space [radius] *ad*'). In other instances it can mean the length of a line segment, as in '*partirai lo spacio, che e tra c. & b. in due parti eguali*' (p. 26 in the original) ('you will divide that space between *c* and *b* into two equal parts'). In order to make this distinction, where *spacio* denotes 'radius', it has been translated as such; where it denotes a distance, it has been translated as 'space'.

One term that is completely out of use today is *linea diametrale* of a triangle, which we know as a median, that is, a line segment that joins a vertex to the midpoint of the side opposite it. Barbaro's usage is in keeping with the idea of a diameter of a polygon, that is, length of the line segment joining two vertices of a polygon, but in a triangle, a line segment joining two vertexes is a side, hence the use of a different term for that case. In this case the translation maintains the literal translation 'diametrical line' but indicates the modern term in square brackets for clarity.

The following terms require a bit more explanation. Here they are briefly touched on, and are discussed at greater length in the critical notes.

Barbaro first uses the term *termine* on p. 18 but only defines it on p. 114: 'By 'termination' I mean the plane on which are to be located the points of the square and the surface described in it'.[7] In other words, it is the surface on which the perspective drawing is made (in contemporary speech, the pictorial plane). It has been translated as 'termination'.

Some terms are specifically related to the science of perspective in Barbaro's day. These include *perfetto*, *degradato*,[8] *impiè*, *scenographia*, and *schiographia*.

In the case of the solids treated in Part III, the more familiar and long-studied Platonic solids Barbaro uses the names ascribed to them: pyramid (tetrahedron), cube, octahedron, dodecahedron, icosahedron. But when he comes to the more complex polyhedra, he describes them according to the figures they are derived from and the number of their faces and edges. For example, in Chap. VII in Part III, we read: …from the pyramidal body is born the body made of 4 surfaces of 6 edges, and 4 triangles of equal sides.[9] In Barbaro's day this may have been the most straightforward way to identify these solids for a readership without a formal mathematical training, and indeed, as discussed in the critical notes, many of these solids had never been studied before and did not have a name. However, today it is too much to ask of the reader to keep track of the solids simply by describing their attributes so, where appropriate, modern names have been added in square parentheses. (Did the reader figure out which figure just described above? It is the truncated tetrahedron.)

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[7] Barbaro (1568, p. 114): *Termine io intendo il piano sopra'l quale si hanno à ponere i punti del quadrato,* & *delle soperficie in esso descritte.*

[8] For a discussion of the terms *perfetto* and *degradato*, which, along with *termine*, come from Piero della Francesca's *De prospective pingendi,* see Field (1997, p. 86).

[9] Barbaro (1568, p. 56): *dal corpo piramidale nasce il corpo formato di quattro soperficie di sei lati,* & *di quattro trianguli di lati eguali.*

There are two terms link the treatise on perspective to Barbaro's Vitruvius and show a continuity of thinking. These are *compartimento* and *proportione*. We find both of these used in the first sentence of Chap. VI of Part I:

> Before I come to the division and compartition of planes, I will declare with examples that a line is to be divided according to a given ratio, and I will give other manners of proportional compartition to make the things that follow easier.[10]

The first is *compartimento*, translated as 'compartition'. The use of this particular translation first appeared in modern English translations of the treatises of Alberti and Palladio.[11] For Barbaro the terms was significant because the prefix '*com-*' (a derivation of the Italian preposition '*con*' that means '*with*') in compartition, as opposed to simply 'partition', makes it clear that the division is being done with reference or respect to something else; compartition implies non-arbitrary proportional division. A good synonym is 'apportion' – that is, divide or share out according to proportion. In the quote above, Barbaro's expression 'proportional compartition' simply drives this meaning home.

The second term is *proportione*, used not in the general sense of 'things in proportion' but in the mathematical sense of the relation or comparison of two terms or quantities. This is properly translated as 'ratio'. The use of *proportione* and the related Italian term *proportionalità* (our 'proportion', in the mathematical sense of a relation or comparison of ratios) were common in Barbaro's day. The title of a treatise by a contemporary of Barbaro's, the engineer Silvio Belli, is *Della proportione e proportionalita* ('On ratio and proportion')[12] (incidentally also first published by Francesco de' Franceschi). Therefore, in this present translation, where appropriate the mathematical term 'ratio' is used; where the notion of proportion is used in its general sense, it is maintained.

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[10] Barbaro (1568, p. 12): *Prima, che io venga alla divisione, & compartimento de i piani, io dichiarirò con esempi, come si habbia a partire una linea secondo quella proportione, che ci serà data, & ponerò altre maniere di compartimenti proportionati per rendere piu facili le cose seguenti.*

[11] The translations of both Alberti and Palladio both contain a glossary entry explaining the term 'compartition'. See Alberti (1988, p. 389) and Palladio (1997, p. 421).

[12] See Belli (2003, p. 15).

The treatment of errors in the treatise

A problem that plagues the original text is the proliferation of errors of oversight, that is, errors that occurred either during the preparation of the book from the manuscript used as a reference, or during the process of making the woodcuts from the original images. It seem as though there were no careful controls of the text against the manuscript before the book went to print. The great number of errors became particularly evident during the studies of the drawings that accompanied the instructions for constructing *perfetti* and *degradati* of the geometric solids in Part III, where the points and lines identified on the drawings may not match those point and lines in the text. We have chosen to maintain the original terms and include the cor*recte*d terms in square brackets in this form: [*recte* x]. To determine the corrections, we relied on a cross-check with the manuscripts held in the Biblioteca Nazionale Marciana in Venice: Ms. It. IV, 39=5446 (Barbaro’s long original manuscript), Ms. It. IV, 40=5447 (the menabò used to prepare the printed edition), and Ms. Lat. VIII, 41=3069 (the unpublished Latin version). We also relied on our own studies as we worked our way through Barbaro’s instructions. We believe that our painstaking efforts have resulted in a text that is as correct as possible.

The use of symbols

Barbaro’s use of symbols in his treatise is one of the most inventive aspects but also one of the most difficult to replicate in a modern treatment. In addition to Arabic numerals and Greek letters, Barbaro invented a great number – 362, to be precise – of glyphs of his own. These have been carefully catalogued by Cosimo Monteleone.[13] We don’t even know how Barbaro devised his symbols, or how he might have pronounced them. We only know that they had a significance for him. On a practical level, what this meant in Barbaro’s day was that the symbols used by the engraver in the woodcuts did not always match the symbols used by the typesetter in the text. This is one of the causes of the many errors we have discerned. We have addressed this in the present work by using modern type symbols where possible (℞, ✠, and &), and inserting an image ( ) where it is not.

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[13] Monteleone (2020, pp. 193-195).

The role of the authors

Since this present work was prepared jointly by the author of a recent book on Barbaro's perspective, and by the author of the annotated translation of Barbaro's commentary on Vitruvius, the reader might rightly ask how these works overlap and the degree to which the present work is original. Cosimo Monteleone's in-depth work, *La prospettiva di Daniele Barbaro*, was aimed at providing a contextualized investigation of the themes that Barbaro addresses in his treatise. Monteleone's approach was to comment on the treatise in much the same way that Barbaro commented on Vitruvius, by using the author's words as a point of departure, selecting passages that were held to be of particular significance: a macroscopic approach. Instead, our aim here is to enter into the detalis of the text, to read between the lines, to point out the parts that draw on the work of others as well as the parts that represent significant innovations: a microscopic approach. The present volume is thus a substantially different work, though it clearly refers to and draws on its predecessor, and on the experience of both authors with other works by Daniele Barbaro and in the field of sixteenth-century studies in general.

Our work on this present volume was truly a joint effort. The first draft of the translation was prepared by Kim Williams, reviewed by Cosimo Monteleone, and subsequently refined jointly. The critical notes that accompany the translation were authored in collaboration. All of the editorial decisions, were made jointly.

Conclusion

We were aware as we were working in 2020 that it was the year of the 450th anniversary of Barbaro's death. We were pleased to be able to make the work of this important figure accessible to new generations of readers.

Padua, May 2021

References

Alberti (1988)
Alberti, Leon Battista. 1988. *On the Art of Building in Ten Books*. Joseph Rykwert, Neil Leach, Robert Tavernor, trans. Cambridge, MA: MIT Press.
Barbaro (1567)
Barbaro, Daniele. 1567. *I dieci libri dell'architettura di M. Vitruvio. Tradotti & commentati da mons. Daniel Barbaro eletto Patriarca d'Aquileia, da lui riveduti & ampliati; & hora in piu commoda forma ridotti.* Venice: Francesco de' Franceschi.
Barbaro (1568)
Barbaro, Daniele. 1568. *La pratica della perspettiva*. Venice: Camillo e Rutilio Borgominieri.
Belli (2003)
Belli, Silvio. 2003. *On Ratio and Proportion: The Common Properties of Quantity*. Stephen R. Wassell and Kim Williams, eds. and trans. (Eng. trans. of *Della proportione e proportionalita. Comuni passione del Quanto libri tre* (1573). Fucecchio: Kim Williams Books.
Field (1997)
Field, J. V. 1997. *The Invention of Infinity*. Oxford University Press.
Girardi (1995)
Girardi, Maria Teresa. 1995. *Il sapere e le lettere in Bernardino Tomitano*. Milan: Vita e Pensiero.
Graheli (2015)
Graheli, Shanti. 2015. Daniele Barbaro e le accademie venete del Cinquecento. In: *Daniele Barbaro 1514-1570. Letteratura, scienza e arti nella Venezia del Rinascimento*, Susy Marcon and Laura Moretti, eds., pp. 65-68. Crocetta del Montello (TV): Antiga.
Monteleone (2020)
Monteleone, Cosimo. 2020. *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto it. IV, 39=5446*. Canterano (Rome): Aracne editrice.
Palladio (1997)
Palladio, Andrea. 1997. *The Four Books on Architecture*. Robert Tavernor and Richard Schofield, trans. Cambridge, MA: MIT Press.
Vasoli (2002)
Vasoli, Cesare. 2002. *Le filosofie del Rinascimento*. Milan: Bruno Mondadori.
Williams (2019)
Williams, Kim. 2019. *Daniele Barbaro's Vitruvius of 1567*. Basel: Birkhäuser.

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The images that appear in Cosimo Monteleone's essay are all by the author unless otherwise noted.

Daniele Barbaro and the Geometric Solids

Kim Williams

Introduction

When Barbaro alludes to his forthcoming treatise on perspective in his commentary on Vitruvius, it is to make two promises: to explain the ways in which perspective interweaves two distinct sciences – geometry and natural vision – into one,[1] and to show how an understanding of certain geometric figures are a great help in many things, 'and especially in perspective'.[2] He refers to his work on perspective, which was published just a year after the revised edition of his Vitruvius, as a 'treatise on *scenographia*' but this is just one of the objects that Barbaro declares in his preface to his treatise on perspective. His division of the work into nine parts include properly ordering the principles of perspective, explaining perfect and degraded planes, elevating bodies from planes, set design, anamorphosis, representing the sphere in the plane, light and shadows, measures of the human body, and instruments for constructing perspectives. What is immediately noticeable is an unevenness of treatment: we find that the section devoted to the geometric solids in Part III occupies pp. 43 to 113, or 71 of the works 193 pages, while Part V on anamorphosis is given barely 3 pages and Part VII on light and shadow only 2.

As was the case of the longest excursuses in Barbaro's commentary on Vitruvius – the discussion of proportions in Book III, the treatment of musical theory in Book V, and the description of the construction of sun clocks in Book IX – the in-depth treatment of the solids in Part III of the treatise on perspective reveals Barbaro's genuine passion for mathematical research. This is therefore a part of the work that deserves our attention. Barbaro's accomplishments here go beyond a simple didactic exposition: his mathematical research extended existing experimentation with solids[3] and presented his descriptions of new

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[1] Barbaro (1567, pp. 24-25); Williams (2019, p. 50).

[2] Barbaro (1567, pp. 398-399); Williams (2019, p. 664).

[3] Barbaro, in fact, never uses the terms polyhedron or polyhedra in either the manuscript or the printed book. He uses *solido* or *solidi* (solid or solids) and *corpo* or *corpi* (body or bodies). Fibonacci, in *De practica geometria*, used the term *solida multarum basium* (Hughes 2008, p. 278, note 7). The term 'polyhedron', from Greek *poluedrov*, for 'many-based' or 'many-sided', first

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ones. If Barbaro can be accused elsewhere of having merely gathered and distilled the information that he transmits, in the discussions of the solids he must be acknowledged of having fully mastered the material thanks to his own efforts.

In what follows I will first present a brief overview of depictions of solids leading up to Barbaro's work, and in the few years immediately following it. These sources considered include editions of ancient treatises on pure mathematics (Euclid, Apollonius, Pappus), works of practical geometry (Fibonacci, Piero della Francesca, Pacioli, and others), manuals for artists and architects (Dürer, Hirschvogel, Jamnitzer, and others), and works of art and illustration. The aim is to not by any means to present a complete survey, but to establish the currents of thinking about and visualizing solids in both mathematical and artistic contexts in order to properly situate Barbaro's treatise within them. This will be followed by a discussion of the solids described by Barbaro and his methods of studying them, which will allow me to distinguish those already well known from those that he himself discovered. I will conclude with a brief mention of developments that came after Barbaro, to show how the ideas that he was working with filtered into later research.

A visual survey of depictions of the solids in manuscripts and printed books

In the oldest surviving manuscript of Euclid,[4] dating from the ninth or tenth century and now conserved in the Vatican Library (Vat. gr. 190), is found the graphic convention for representing the solids that would be repeated in Renaissance editions. Fig. 1 shows that manuscript's representation of the regular bodies.

The illustrations of bodies use the convention of showing a key polygon in true shape and drawing from that the outlines of the solid. The tetrahedron is shown as an equilateral triangles lines extending from the vertices to a common point at the apex; the cube is shown as a square depicted the drawing technique that later came to be known as oblique axonometry;[5] the

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appears in English in the Billingsley edition of Euclid (1570, pp. 376v, 378r). The term *solidorum polyedrum* is used by Clavius in his edition of six books of Euclid (1574, Book XII, p. 188v).

[4] With regard to Euclid, since the focus of the present text is on images alone, the provenance and attribution of Books XIV and XV will not be discussed.

[5] I am grateful to José Calvo-López for discussing this kind of projection with us in private correspondence. For a brief description of this kind of axonometric, see Alonso-Rodríguez and José Calvo-López (2014).

octahedron is based on the square that is common to the two prisms with lines extending from its vertices to the apexes above and below. This means of depiction presents difficulties of both construction of the drawing and legibility. These limitations become clearer the more complex the solid, as in the depiction of the icosahedron (Book XIII, prop. 16) and especially the dodecahedron (Book XIII, prop. 17). In effect, the more complicated the solid, the less visual and more abstract the representation.

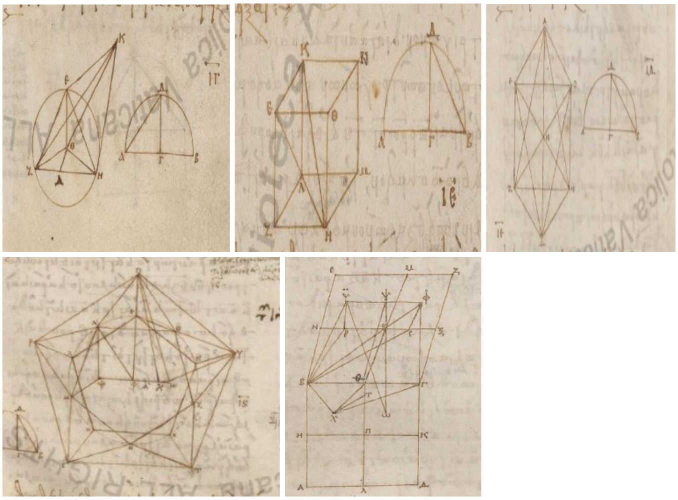

Fig. 1. Depictions of solids in the oldest manuscript of Euclid's *Elements*, the Vatican Library's manuscript Vat. gr. 190. From top left to right) the tetrahedron, fol. 240r; the cube, fol. 242r; the octahedron, fol. 241r; bottom left to right) the icosahedron, fol. 243v; the dodecahedron, fol. 245r

Book VI of the *Practica geometriae* of the 1200s by Fibonacci (Leonardo da Pisa), which includes problems of how to determine the volumes of the five Platonic solids inscribed in spheres, employs a similar graphic convention. The figures for the tetrahedron and cube allude to the third dimension, but illustrations of the dodecahedron and icosahedron avoid any overall depiction.[6] Fibonacci's method for finding the volume of the solid is to break them down into pyramids. Thus the cube is broken down into three equal triangular pyramids;[7] the dodecahedron is broken down into twelve equal pentagonal pyramids;[8] similarly, to find the volume of the icosahedron he breaks it down into twenty equal triangular pyramids.[9] These are then the figures shown in his illustrations (Fig. 2) The octahedron, though treated, is not illustrated.

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[6] See Boncompagni (1862) and Hughes (2008). Hughes relied on the Vatican Library manuscript Urb. lat. 292 but redrew the figures, commenting that '[t]he figures … are somewhat confusing because they are two-dimensional representations of solids' (2008, p. 275).

[7] See Hughes (2008, pp. 324-328).

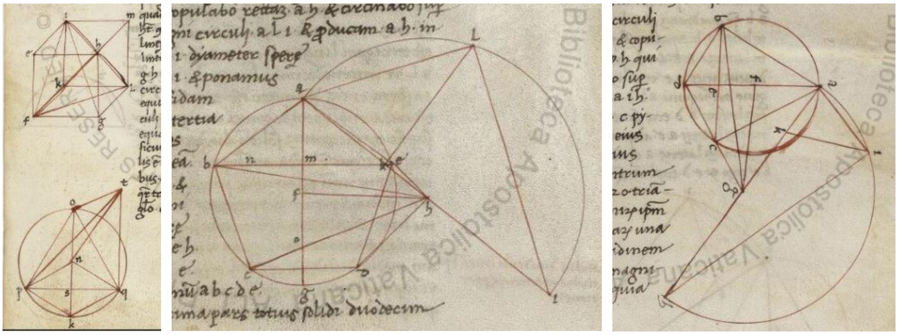

Fig. 2. Depictions of solids from Fibonacci's *De practica geometrie* in the Vatican Library's manuscript Urb. lat. 292. From left) the cube and triangular pyramid; the pentagonal pyramid used to find the volume of the dodecahedron; the triangular pyramid used to find the volume of the icosahedron

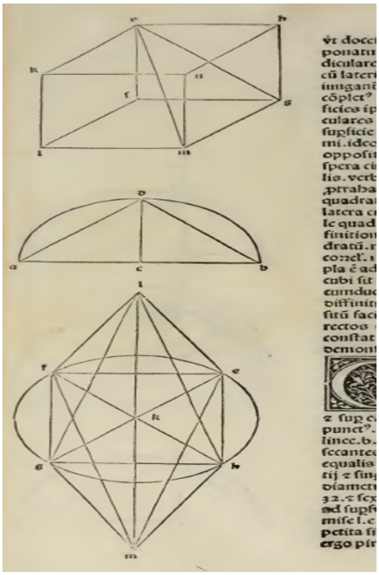

Fig. 3. The octahedron shown in the 1482 edition of the *Elements* with Campanus's text (Bk. XIII, Prop. XV, no page number) based on the translation of Adelard of Bath, printed in Venice by Erhart Ratdolt

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[8] See Hughes (2008, pp. 333-335).

[9] See Hughes (2008, pp. 337-339).

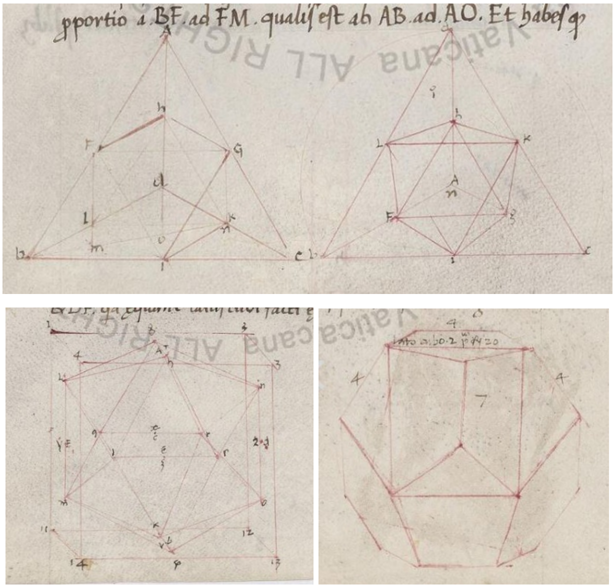

Fig. 4. Illustrations from Piero's *Libellus de quinque corporibus regularibus*. From left: nested tetrahedral (fol. 38r); icosahedron in a cube (fol. 39v); wireframe depiction of a cuboctahedron

The graphic convention found in the manuscripts of Euclid and Fibonacci is maintained in the first printed edition of Euclid, the one printed in Venice in 1482 by the German printer Erhard Ratdolt, with the text by Campanus based on the translation from the Greek by Adelard of Bath. Fig. 3 shows the octahedron from that edition. The same images then appear in the editions of Euclid that followed: the 1505 edition by Bartolomeo Zamberti (p. 439), which we know was probably studied by Barbaro,[10] and the 1509 edition by Pacioli (Bk. XIII, p. 128r).

The first depictions of representations of the solids as three-dimensional objects are found in Book III of Piero della Francesca's *Trattato d'abaco*,[11] written ca. 1460-1480. Piero produced delicate wireframe drawings of the triangular pyramid (perspective views, fols. 105r, 105v), the cube (axonometric views, fol. 106r), the truncated tetrahedron (ground plans, fol. 107v) cuboctahedron (perspective views, fols. 108r, 108v), dodecahedron (perspective views, fol. 110r, 110v), icosahedron (perspective views, fols. 112r, 112v), and octahedron (ground plan, fol. 114v). Piero's aim is like that of Fibonacci, to find the edge sizes and volumes of solids inscribed in spheres. Unlike Fibonacci, however, whose shapes were limited to those with practical uses, Piero studied solids whose purposes were mathematical rather than practical.

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[10] See Monteleone (2020, pp. 81-82).

[11] Piero della Francesca (Ash. 359); Piero della Francesca (1970); see also Daly-Davis (1977).

Piero's *Libellus de quinque corporibus regularibus*,[12] written sometime between 1480 and 1492, presents examinations of dimensions and volumes of solids inscribed in spheres as in his *Trattato d'abaco*, but in greater number and in greater depth. In fols. 23r-36v are found the five Platonic solids. In fols. 37r-44r he treats polyhedra inscribed in polyhedra. Here he employs the graphic technique of representing the faces as transparent, where all edges are visible (Fig. 4), which he did not do in the *Trattato d'abaco*, where the faces were represented as opaque.

As is well known,[13] Luca Pacioli integrated the parts dealing with the solids from Piero's *Trattato d'abaco* into his *Summa de Arithmetica Geometria Proportioni et Proportionalità*,[14] and likewise integrated Piero's *Libellus* into his *Divina proportione*.[15] In the *Summa*, although the problems posed and solved are still related to practical considerations of dimensions, there is a first hint of an interest in Platonic solids for aesthetic reasons, namely, 'the sweetness of their dimensions':

> And although above in this [part of the volume] in the 6th section of the 4th chapter on the measure of the sphere enough has been said succinctly, it nevertheless appears to me here, excellent Duke, to say something more detailed about some of the essential bodies that can be situated in the sphere, of which, one angle touching [the sphere], all angles immediately touch it. I know this principally from the information on the 5 regular [bodies] of which Euclid gives in full in the last of his books, treated scientifically. Hence it appears to me not unuseful to set out certain cases of them so that lay practitioners can sense some of the sweetness of their dimensions.[16]

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[12] Piero della Francesca (Urb. lat. 632).

[13] The classic reference about Pacioli's use of Piero's material is Mancini (1915). See also Williams (2002).

[14] Pacioli (1494). The part of the *Trattato d'abaco* that deals with the solids (Ash. 359, fols. 105r-118r) appears in the *Distinctio octava,* under the title 'Particularis tractatus circa corpora regularia et ordinaria icipit' (1494, fols. 68v-72v).

[15] Pacioli (1509). Piero's *Libellus*, translated into the vulgar, appears in the *Tractatus primus*, under the title 'Libellus in tres partiales tractatus divisus quinque corporum regularium…' (1509, fols. 1r-27r).

[16] Pacioli (1494, p. 68v): *E benche di sopra in questo [libro] nella distintion 6*^{a} *al capitolo 4*^{o} *de*

Pacioli reproduces Piero's images depicting the solids in wireframe drawings (Fig 5, left), but also includes coarse woodcuts of three perspective views of the dodecahedron rendered with shading and shadows (Fig. 5, right).

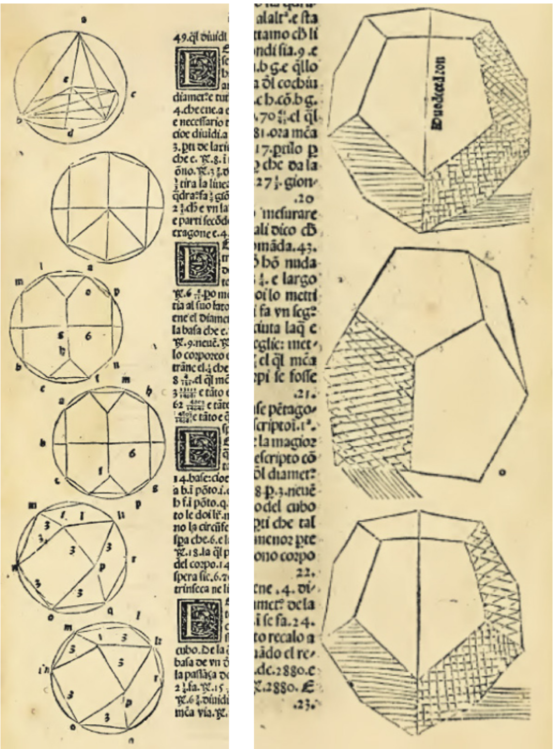

Fig. 5. Right) line drawings in the *Summa* showing solids (p. 69v); left) three shaded three-dimensional representations of the dodecahedron (p. 70r)

The rough three-dimensional views of the dodecahedron in the *Summa* lead to images that are much more refined in *Divina proportione* (1509), where the delicate originals by Leonardo da Vinci for the manuscripts[17] formed the basis for the woodcuts in the printed version (Fig. 6).

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la mesura de la spera succintamente fosse ditto abastanza: non dimeno me par qua eccelso Duca particularmente dire de alquanti corpi essentiali in ditta spera locabili. de li quali un angolo toccando subito tutti toccano: e principalmente lo so per la notitia de li 5 regulari di quali Euclide a pieno nelli ultimi soi libri scientificamente tratta. Di che mi pare non inutile a ponere de loro certi casi acio li pratici vulgari ancora essi qualche dolcezza di loro dimensioni sentino (my trans.).

[17] The two manuscripts are now conserved in Geneva, Switzerland (Bibliothèque de Genève, ms. Langues Etrangères 210) and Milan, Italy (Biblioteca Ambrosiana ms. 170sup.).

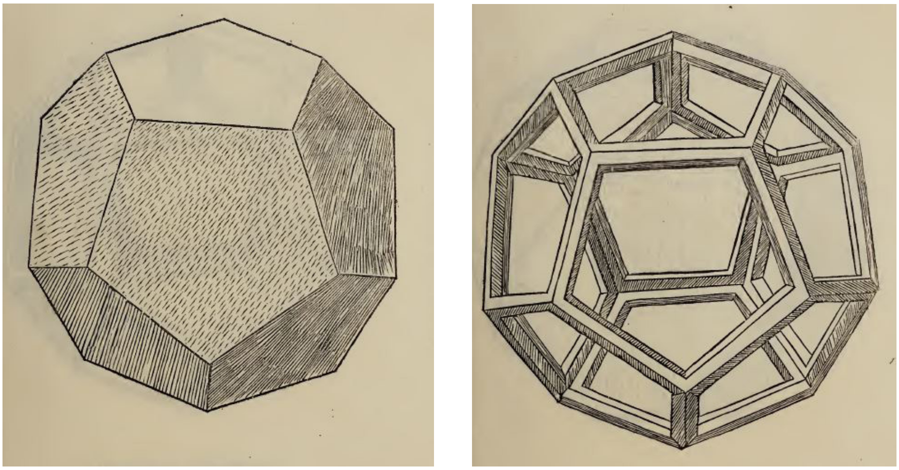

Fig. 6. The dodecahedron as shown in *Divina proportione*: 'Dodecahedron planum solidum' (plate numbered XXVII) and 'Dodecahedron planum vacuum' (plate numbered XXVIII)

Throughout the 1500s, new abacus treatises and treatises of practical geometry continued to appear, such as those by Piero Borghi (1517), Francesco Dal Sole (1526), and Francesco Feliciano (1517; 1526), but only Piero della Francesca (as disseminated by Pacioli) treated the solids in this way.

At this point the centre of activity regarding published works intended as manuals for practitioners moves out of Italy. Albrecht Dürer's *Underweysung der Messung* (1525; 1977) presents the solids by means of ground plans (that is, two-dimensional ground plans of the solids viewed orthogonally from above with all edges visible) and depictions of the polygons that comprise the faces in 'unfolded' form, a device that by most accounts[18] he is the first to use (Fig. 7). In this context he does not represent the solids in three dimensions, although he clearly could have, as shown by his *Melencolia I* (see Fig. 23 below).[19]

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[18] Although by most accounts, Dürer was the first to introduce them, two references (at least) state that it might have been Pacioli/Leonardo who first described solid bodies through their nets. Christoph Scriba and Peter Scheiber (2015, p. 303) write, 'Polyhedral net unfolding was first displayed in Pacioli's *Divina Proportione* and in more detail in Dürer's 'Instruction''. Fig. 4 in Schreiber et al. (2008, p. 462) illustrates a net for the dodecahedron presumably authored by Leonardo. I am grateful to Vera Viana for these observations.

[19] The cube is represented as a solid in Dürer's treatise on the human body, but only with the aim of enveloping the head in order to find the foreshortening (see, for example, Dürer 1528, fols. Z2v-Z3r). Piero della Francesca, in *De prospectiva pingendi* (Regg. A 41/2, fols. 52r-53r), explains how to represent a cube in various positions, for example, resting on a vertex, and Dürer's inspiration probably came this exercise. I am grateful to Cosimo Monteleone for these observations.

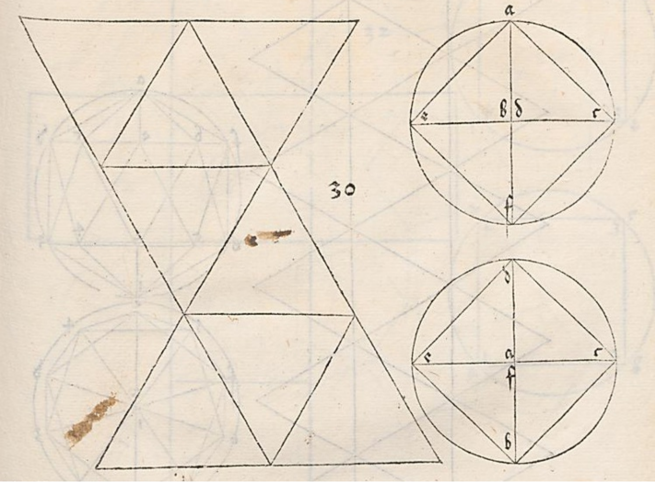

Fig. 7. Dürer's unfolding and ground plan of the octahedron

Dürer's *Underweysung* was followed by *Ein aigentliche und grundtliche anweysung in die Geometria* (A True and Thorough Instruction in Geometry) by Augustin Hirschvogel (1543). Like Dürer, he uses both nets and ground plans, but he adds three-dimensional views to facilitate understanding of the polyhedra (Fig. 8). Hirschvogel was the first to use the Latin term *rete*, or *Netz* in German.[20]

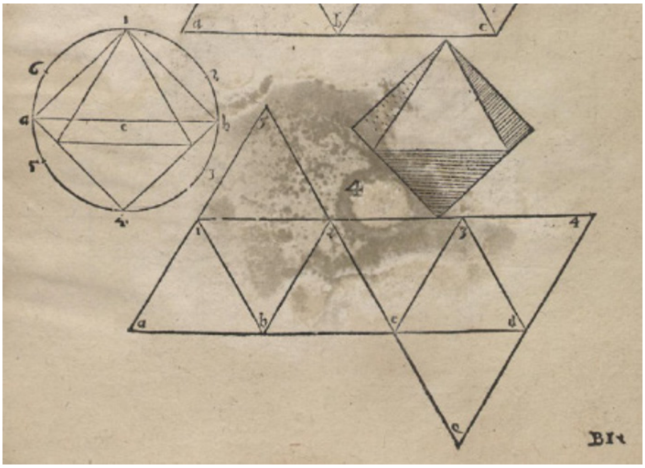

Fig. 8. Hirschvogel's presentation of the octahedron as inscribed in a sphere in a line drawing showing all edges, in a three-dimensional depiction, and as a net.

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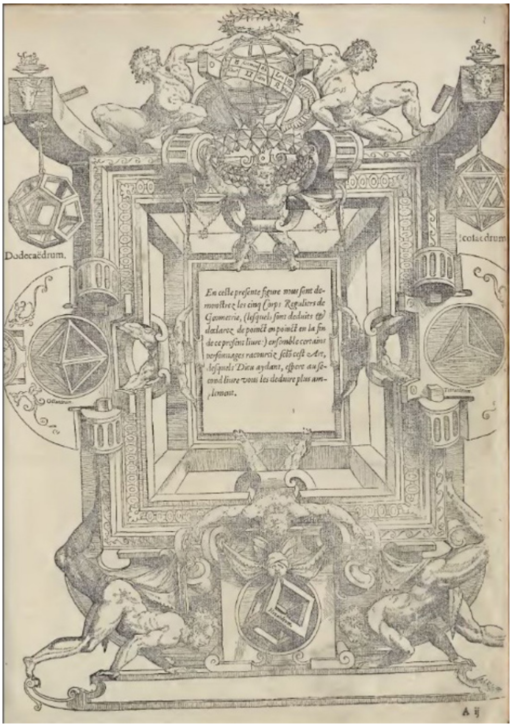
[20] See Andrews (2016, p. 420). Barbaro's term for these was *spiegatura*, literally 'unfolding'.

A closer inspection of Fig. 8 shows that in Hirschvogel's drawing the front edge of the central square is smaller than the back edge, the opposite of how it would be in a one-point perspective.

In spite of these new visions of solids, new editions of Euclid continued to reproduce the same type of images as the earlier ones. In 1543, the same year as Hirschvogel's *Geometria*, Niccolò Tartaglia published a new edition of Euclid but largely maintained the same graphic convention for representing the solids. It seems as though the mathematical treatises still maintained a graphic convention all of their own, without adopting any of the visual information possible in artistic depictions.

At this point, our examination will move from editions of Euclid to books on perspective. The opening illustration that follows the title page of Jean Cousin's *Livre de perspective* of 1560, by illustrator Aubin Olivier, already alludes to the importance of the solids for studies of perspective (Fig. 9).



En ceſte preſente figure nous ſont demonſtrez les cinq Corps Reguliers de Geometrie, (leſquels ſont deduits & declarez de poinct en poinct en la fin de ce preſent liure:) enſemble certains perſonnages raccourciz ſelõ ceſt Art, leſquels Dieu aydant, eſpere au ſecond liure vous les deduire plus amplement.

Fig. 9. The title page of Jean Cousin's *Livre de perspective* (1560) showing the five regular solids. The text in the box begins, 'In this present figure we are able to demonstrate the five Regular Bodies of Geometry, which are deduced and declared point by point at the end of this present book'

Cousin presents the first full exposition of the construction of the five regular solids in perspective. This appears about three quarters of the way into the book (the pages are not numbered), after the exposition of the five orders of architecture. Cousin provides detailed step-by-step instructions for constructing the perfect figure (*plan geometral*), the degradation in the plane (*plan perspectif*), and the elevation of the solid from the plane (*corps solides*); he also provides the net (*corps developpé*). He does this for the tetrahedron, a compound of two tetrahedra, cube, octahedron, dodecahedron, and icosahedron. He also shows multiple views of the solids, resting on either a face or a vertex, and in the case of the tetrahedron, he shows the elevation, or compound, of it (*corps ellevé*) in skeletal and solid form (Fig. 10).

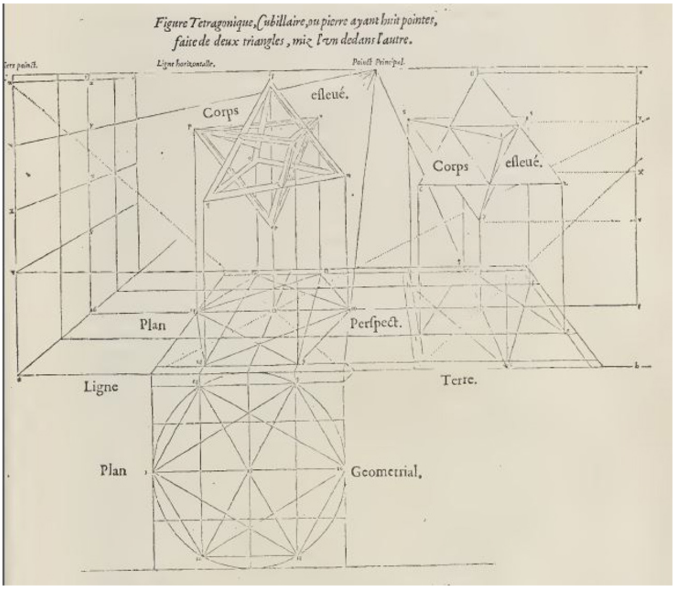


Figure Tetragonique, Cubillaire, ou pierre ayant huit pointes,
faite de deux triangles, miz l'vn dedans l'autre.
ers poinct.
Ligne horizontalle.
Poinct Principal.
Corps
eſleué.
Corps
eſleué.
Plan
Perſpect.
Ligne
Terre.
Plan
Geometrial.

Fig. 10. Cousin's presentation of the compound of two tetrahedra

In Germany, the 1560s saw a flurry publications depicting solid bodies. These include the *Des Circkels unnd Richtscheyts, auch der Perspectiva, und Porportion der Menschen…* (Of circles and right angles, also on perspective and human proportion…) by Heinrich Lautensack of 1564; *Geometria et Perspectiva* by Lorenz Stör of 1567; *Perspectiva literaria* by Johannes Lencker, with etchings by Matthias Zündt, of 1567; and *Perspectiva corporum regularium* by Wenzel Jamnitzer of 1568 (the same year as Barbaro's *Practice of Perspective*). In all of these books the emphasis is strongly visual, with different solid bodies presented in ways that are increasingly fanciful. Stör's *Geometria et Perspectiva* is a collection of 12 plates with no text (Fig. 11). Lautensack's section on perspective begins with instructions and then provides illustrations that show the solids in combinations (Fig. 12). Lencker's *Perspectiva literaria* is a collection of many different perspective views of letters of the alphabet, interspersed with depictions of three-dimensional forms (Fig. 13).

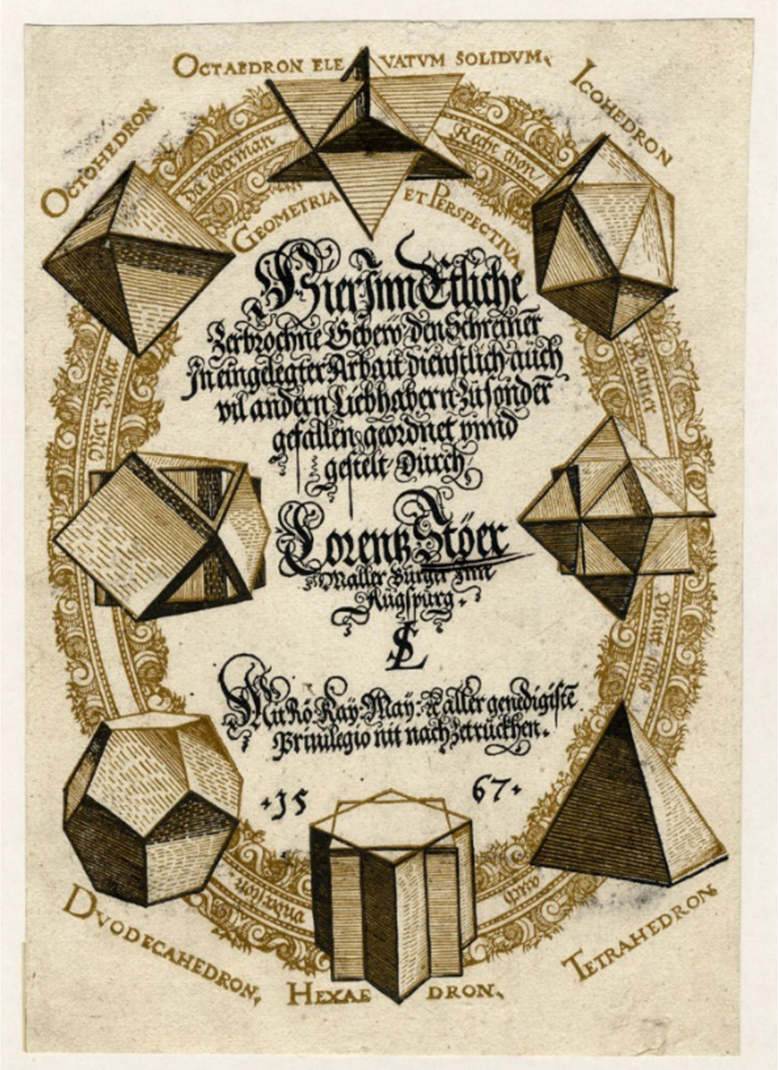

Octaedron elevatum solidum.
Icohedron
Octohedron
Da jederman
Recht thon/
Geometria et Perspectiva
Hie ſind Etliche
Zerbrochne Gebew den Schreiner
In eingelegter Arbait dienſtlich auch
vil andern Liebhabern zu ſonder
gefallen geordnet vnnd
geſtelt Durch
Lorentz Stöer
Maller Burger Inn
Augſpurg.
Mit Rö: Kay: May: allergenedigiſte
Priuilegio nit nachzetrückhen.
15 67.
Duodecahedron,
Hexaedron.
Tetrahedron.

Fig. 11. The title page of Stör's *Geometria et Perspectiva*
(this title page actually appears as the final page of the booklet)

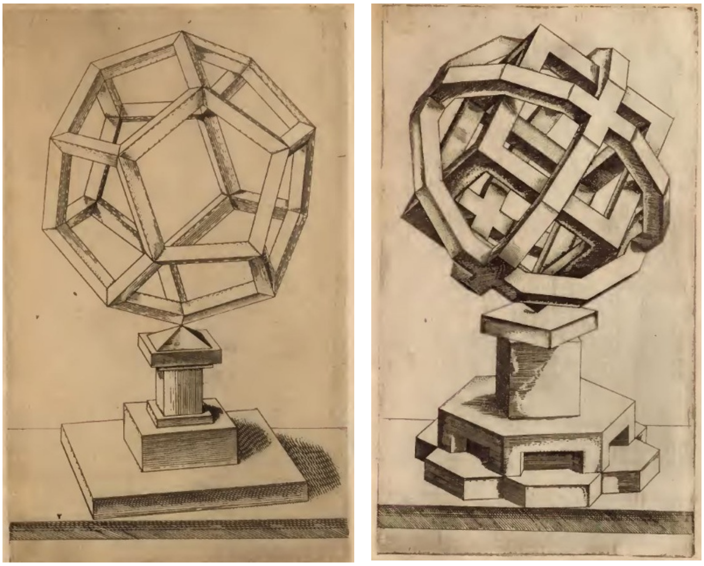
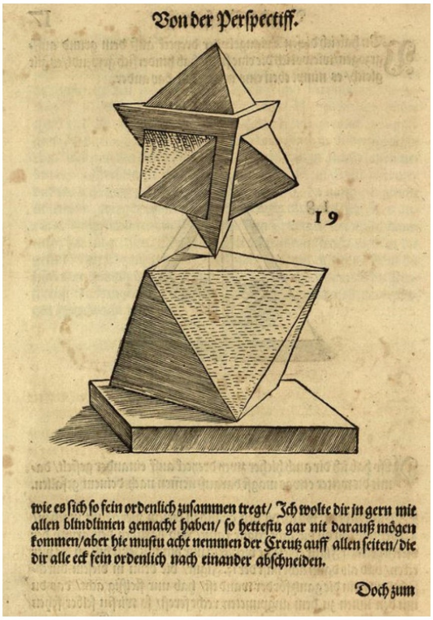


Fig. 12. Illustrations from Lencker's *Perspectiva literaria* (1567).
Left) The skeletal dodecahedron.
Right) a skeletal cube circumscribed by three great circles of 72-sided sphere



Von der Perſpectiff.

19

wie es ſich ſo fein ordenlich zuſammen tregt/ Jch wolte dir jn gern mit allen blindlinien gemacht haben/ ſo hetteſtu gar nit darauß mögen kommen/aber hie muſtu acht nemmen der Creutz auff allen ſeiten/die dir alle eck fein ordenlich nach einander abſchneiden.

Doch zum

Fig. 13. Lautensack's Fig. 19 showing a cube penetrating a tetrahedron supported by an octahedron

Jamnitzer's book only contains five pages of text, with the remaining pages filled with the splendid engravings that have made the book famous. The plates are collected into chapters devoted to the five Platonic solids, beginning with the tetrahedron. In his plates he begins with the basic solids, and then by means of truncations, elevations, stellations or excavations he transforms it into new shapes of increasingly complexity. There are points in common between Jamnitzer's explorations of the solids and those of Barbaro. For example, like Barbaro, Jamnitzer included an image of the rhombicosidodecahedron, and both Barbaro and Jamnitzer featured solids elevated with cupolas.

It is at this point in the exploration of geometric solids that Barbaro's *Practice of Perspective* is to be inserted. Barbaro positions his work as a didactic text aimed at 'the painters of our time [who] let themselves be led by simple practice',[21] and to introduce the theory and exercises that 'are very useful for painters, sculptors, and architects [but] is also very abandoned, not to say disdained, by them'.[22] However, his work on solids goes far beyond what was necessary to illustrate the theory of and exercises in perspective to practitioners. Had this been his sole aim, he could have limited himself to a few of the solids. Instead, his research was genuinely geometrical, and he pushed his work beyond where mathematicians of the time had gone. This will be discussed further below.

Barbaro's presents most of his solids by means of ground plans, nets, and perspective views. Further, in the case of solids composed of more than one type of polygonal face, he provides depictions of the solid resting on the different faces by turn. Speaking strictly from the point of view of graphic quality, in any comparison of Barbaro's depictions of the solids with others, the poor quality of the engravings published in his treatise are bound to make him come out the worst. However, an idea of the kind of image he would have liked, and was capable of obtaining, can be had from images in the manuscripts (Fig. 14). Unfortunately, the poor quality images are the ones that appeared in the published work.

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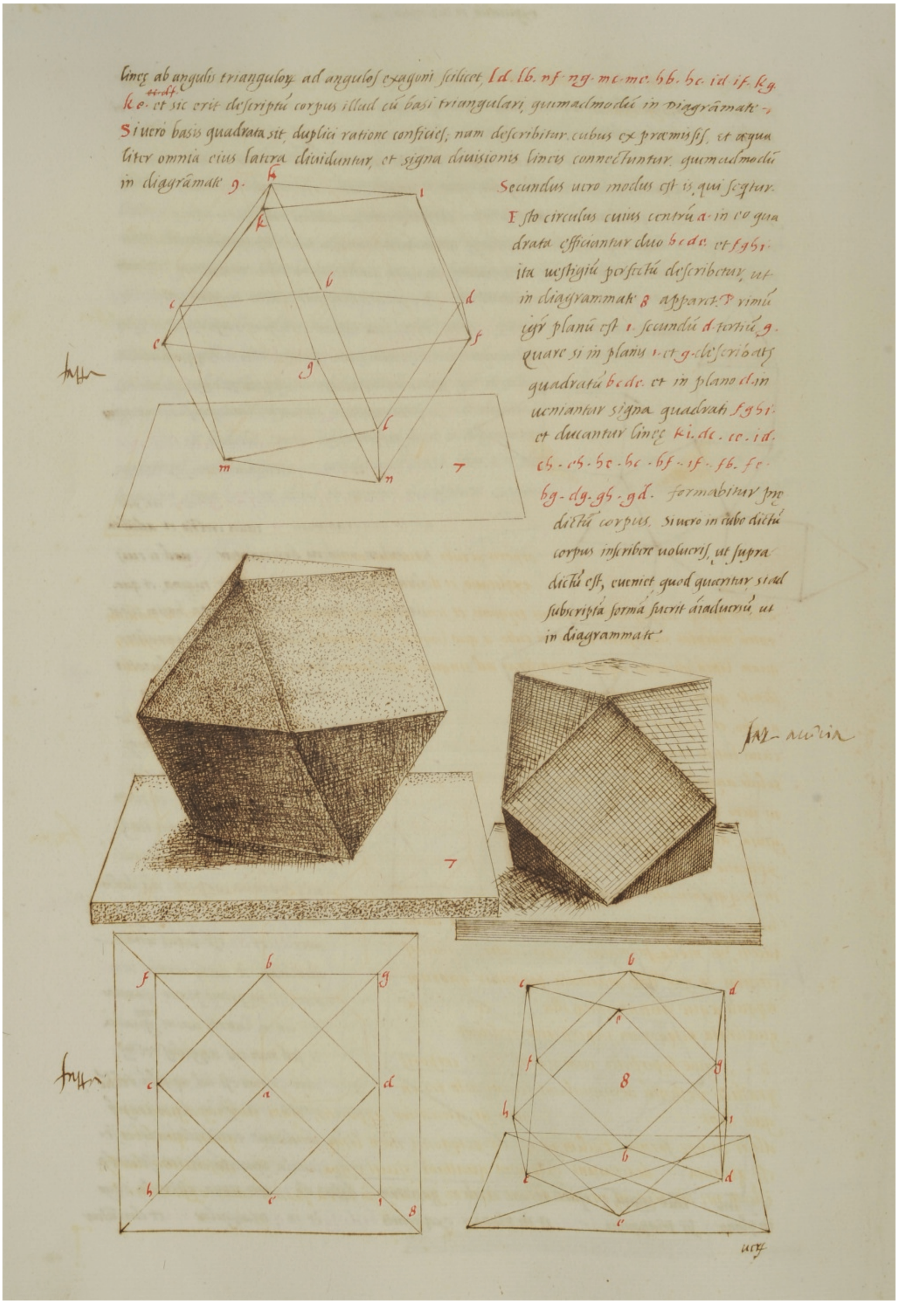
[21] Barbaro (1568, p. 1): *I pittori de i nostri tempi … si lasciano condurre da una semplice pratica*.

[22] Barbaro (1568, p. 1): *molto utili a Pittori, Scultori, & Architetti, cosi molto abandonata, per non dire sprezzata*.



lineę ab angulis trianguloꝝ ad angulos exagoni ſcilicet, ld. lb. nf. ng. mc. me. hb. hc. id. if. kg. ke. et ſic erit deſcriptú corpus illud cú baſi triangulari, quemadmodú in Diagrámate 7.

Si uero baſis quadrata ſit, duplici ratione conficies; nam deſcribitur cubus ex præmiſſis, et æqualiter omnia eius latera diuiduntur, et ſigna diuiſionis lineis connectuntur, quemadmodú in diagrámate 9.

Secundus uero modus eſt is, qui ſequitur.

Esto circulus cuius centrú a. in eo quadrata efficiantur duo bcde. et fghi. ita ueſtigiú perfectú deſcribetur, ut in diagrammate 8 apparet. Primú igr̃ planú eſt i. ſecundú d. tertiú g. quare ſi in planis i. et g. deſcribatur quadratú bcde. et in plano d. inueniantur ſigna quadrati fghi. et ducantur lineę ki. dc. ce. id. ch. eh. he. hc. bf. if. fb. fe. bg. dg. gh. gd. formabitur prędictú corpus. Si uero in cubo dictú corpus inſcribere uolueris, ut ſupra dictú eſt, eueniet quod quæritur, ſi ad ſubſcriptá formá fuerit diaduerſú, ut in diagrammate

Fig. 14. The depiction of the cuboctahedron from Barbaro's manuscript Lat. VIII, 41 =3069, fol. 33v

Barbaro's Solids

Now let us look at the collection of solids studied by Barbaro and presented in the published treatise. The solids presented by Barbaro in Part III are identified in Appendix A. A comparison of the solids presented by Barbaro with those treated by Piero della Francesca, Pacioli, Dürer, Hirschvogel, and Kepler is presented in Appendix B.

Regarding Barbaro's explorations of polyhedra, Judith Field speculated that 'he believes he is dealing with an unlimited number of solids'.[23] In fact, in manuscript It. IV, 39=5446, he states as much:

> The nature of pure angular bodies that are born of straight lines … are of a certain nature, and any of them could give birth to so many that if one wished to give a demonstration of all of them, I would never arrive to the end of the present material, because their number is infinite.[24]

As mentioned above, Barbaro indicates in the manuscript that he has studied the solids 'with the material bodies in hand'. This goes a long way towards understanding his exploration of this. He begins with the triangular pyramid (Chap. I) before going on with the Platonic solids, then proceeds, by means of truncations and elevations, to derive new bodies.

As is his rule in treating any subject, the concept of order organises Barbaro's thinking.[25] He begins his descriptions of the solids with this justification: 'Proceeding with some order, I will begin with the pyramidal body, that is, the one formed of four equilateral triangles, since it is less difficult'.[26] Then after he presents the first two, most elementary – the tetrahedron and the cube – he pauses before going further to render an account of the order he has decided upon, which is based on the number of faces rather than on the configuration of the angles:

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[23] Field (1997, p. 273).

[24] Barbaro (Ms. It. IV, 39=5446, fol. 29v); Monteleone (2020, p. 217): *le nature de puri corpi angulari, che da le line rette nascono … sono di certe nature, et qualunque ne partorisse tanti, che se de tutte ne volesse far dimonstratione non aggiungerei mai al fine de la presente materia, per esser il numero loro infinito* (my trans.).

[25] For a detailed discussion of the importance of order in Barbaro's work, see Williams (2020, esp. pp. XII-XIII).

[26] Barbaro (Ms. It. IV, 39=5446, fol. 30r); (Monteleone 2020, p. 217): *Procedendo con qualche ordine principierò dal corpo pyramidale, cio, è, da quello, che, è, formato di quatri triangoli equilateri per essere meno difficile* (my trans.).

> The body demonstrated above [the tetrahedron], wishing to proceed in an orderly fashion, would be followed by the demonstration of the body subject to obtuse angles, but so that the octahedral body, which has right and acute angles, is not represented first, it seems better to me to place the aforementioned [cube] and demonstrate its rule.[27]

One thing that is exciting about Barbaro's explorations is that he was working in 'free territory' where the rules had not yet been fixed. Kepler did not name and describe the Archimedean solids until half a century after the publication of Barbaro's treatise. Further, some of the solids that Barbaro discusses are neither Platonic nor Archimedean. But this does not disqualify his work as 'unmathematical'; rather, it shows that his explorations were not limited by a set of pre-established definitions.

A classification of Barbaro's solids

The solid bodies presented by Barbaro can be classified in different ways. In order to shed light on his studies, I have chosen to organize them into three sets. This does not imply that he followed this order in his studies, or that he derived each from the solid first mentioned;[28] it is only meant to facilitate our understanding of the collection as a whole. A 'family tree' showing the way in which Barbaro's explorations developed from solid to solid is shown in Appendix C.

The first set of Barbaro's solids is arranged according to the Platonic solids they are derived from:

Polyhedra based on the tetrahedron. There are two of these:

- the tetrahedron itself, which Barbaro calls the pyramid, as was usual in his day (Chap. II);
- the truncated tetrahedron (Chap. VII).

Polyhedra based on the cube. There are eight of these:

- the cube itself (Chap. III);
- the octahedron (Chap. IV);

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[27] Barbaro (Ms. It. IV, 39=5446, fol. 33v); Monteleone (2020, p. 219): *Seguitarebbe al corpo sopra dimostrato, volendo proceder ordinatamente la demonstratione del corpo sottoposto alli angoli obtusi: ma perchè non si rapresenta avanti il corpo ottaedro, che, è, de li angoli retti, et acuti: mi par meglio sia posto poner il predetto, et la regola di questo dimostrar* (my trans.).

[28] In private correspondence, Vera Viana noted that Barbaro did not obtain the octahedron from the cube, nor the icosahedron from the dodecahedron, although he understood that the cuboctahedron and the icosidodecahedron derived from each of the pairs. She furthermore remarked that, according to Field (1997), these are Barbaro's findings.

- the cuboctahedron (Chap. VIII);
- the truncated cube (Chap. IX);
- the small rhombicuboctahedron (Chap. X);
- the truncated octahedron (Chap. XI);
- the great rhombicuboctahedron (Chap. XV);
- the rectified truncated octahedron (Chap. XVI).

Polyhedra based on the dodecahedron. There are six of these:

- the dodecahedron itself (Chap. V);
- the icosahedron (Chap. VI);
- the icosidodecahedron (Chap. XII);
- the truncated dodecahedron (Chap. XIII);
- the truncated icosahedron (Chap. XIV);
- the small rhombicosidodecahedron (Chap. XVII).

Whether by plan on Barbaro's part or by coincidence, the above classifications account for all of the Platonic and Archimedean solids[29] except for two: the snub cube, and the snub dodecahedron. The snub cube appears as Fig. 40 in Dürer's *Underweysung* of 1525. The first appearance of the snub dodecahedron is found in Fig. 13 on p. 62 in Kepler's *Harmonices mundi* of 1619 (see also Appendix B).

A second set represents the solids that were not treated by Dürer or any other previous author, and appear to be Barbaro's own discoveries. There are seven of these:

- the rectified truncated icosahedron (Chap. XVIII);
- the truncated pentakis dodecahedron (Chap. XIX);
- the truncated truncated icosahedron (Chap. XX);
- the great rhombicosidodecahedron (Chap. XXI, an Archimedean solid based on the dodecahedron, published for the first time ever by Barbaro).
- the truncated truncated octahedron (Chap. XXII);
- the great rhombicosidodecahedron augmented with pentagonal cupolas (Chap. XXIII);
- the great rhombicuboctahedron augmented with square cupolas (Chap. XXIV).

Two of the solids treated by Barbaro do not fall into any of the above sets. These are the triangular pyramid used to illustrate the way to raise a body from a plan in Chap. I, and the solid treated in Chap. XXV, which remains unidentified, despite all best attempts.

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[29] In private correspondence, Vera Viana remarked that it is essential to point out (before defining them as solids) that Barbaro's descriptions of the non-uniform polyhedra can only be considered as such if we assume that there were serious mistakes (from Barbaro and/or the engraver) on the description of the faces, all of which were drawn as regular.

The third set of solids, appearing in Chaps. XXVI through XXXIV, are elevations of some of the previously presented solids. There are nine of these, five of which are based on the five Platonic solids, and a further four based on Archimedean solids (again see Appendix A). While Barbaro was not the first to present elevated solids, he was the first to present nets of them.

Judith Field remarks, with regard to Leonardo's three-dimensional depiction of the cuboctahedron, that the artist has oriented the object in a way to provide optimum legibility, which she says was 'likely to be the result of an artist having a sharp eye for noticing a good viewpoint'.[30] Barbaro's studies went beyond presenting a 'good viewpoint'. For the Platonic solids, a single viewpoint may be sufficient, but the Archimedean solids with two types of faces shown in Chaps. VIII to XIV (the cuboctahedron, truncated cube, small rhombicuboctahedron, truncated octahedron, icosidodecahedron, truncated dodecahedron and truncated icosahedron) are shown resting on their different faces.[31] Further, he quantifies the solids, describing not only their faces, but numbering their edges, and solid and plane angles. This is already challenging with a relatively simple solid, but as the solids grow more complex it must have been truly mind-bending. His description of the truncated cuboctahedron with the addition of square cupolas presented in Chap. XXIV is typical: 'It has 24 triangles, 72 solid angles, 144 edges, 72 acute plane angles, 168 right plane angles, and 48 obtuse plane angles'.[32] It is worth noting that he did this for the solids that were previously unpublished, so he had no sources except his own explorations to depend on.

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[30] Field (1997, p. 263).

[31] The exception is the truncated tetrahedron (Chap. VII), which is only shown resting on a hexagonal face.

[32] Barbaro (1568, p. 103): *ha trianguli 24. anguli solidi 72. lati 144. anguli piani stretti 72. anguli giusti piani 168. anguli larghi piani quarantotto.*

Barbaro's methods for obtaining solids

Barbaro begins his explorations with the Platonic solids, and then applies processes of subtraction, mutation, and addition to derive new solids.

Subtraction. Used the most by Barbaro, the subtractive process is truncation and involves dividing the edges into a certain number of parts, and cutting off the solid vertices of the original solid. This operation changes the original faces into different polygons, and gives rise to new polygons along the section of the cut.

Barbaro truncates his solids by dividing the edges into either halves or thirds. In the first instance he divides the edges into two, and removes the solid angles defined by at those midpoints. This is now known as 'rectification'. Using this process he correctly obtains the cuboctahedron (Chap. VIII) and the icosidodecahedron (Chap. XII). In the second instance, he divides the edges into three equal or unequal segments, and removes the solid angles defined by the division into thirds. He uses the division into three equal segments to obtain the truncated octahedron (Chap. XI) and the small rhombicosidodecahedron (Chap. XXII) the truncated icosahedron (Chap. XIV). He uses the division into three unequal segments to obtain the truncated cube (Chap. IX), the small rhombicuboctahedron (Chap. X), and the great rhombicuboctahedron (Chap. XV). Here, however, we encounter the problem that in some cases the subtractive process that he claims to have used result in solids whose faces are not regular polygons, thus necessitating a second step of distortion to make the faces regular.[33] This occurs when the edges are divided into both halves – the icosidodecahedron (Chap. XII) and the small rhombicosidodecahedron (Chap. XVII) – and equal thirds – the great rhombicosidodecahedron (Chap. XXI) and the truncated truncated octahedron (Chap. XXII).

The type of polygonal faces and number of parts that its edges of the original are divided into determines whether or not the resulting faces are regular polygons. For instance, if the edges of regular triangles are divided into 3 equal parts, regular hexagons are obtained; but the edges of squares are divided into 3 equal parts, the resulting octagons are irregular. Field remarks, 'In fact, it is not clear whether he has noticed this defect: his nets merely show regular faces'.[34] But to the contrary, Barbaro must have been aware of the fact that in some cases the operation results in irregular polygons. This is shown by the net of the truncated cube (Chap. IX), in which we see triangles that are clearly isosceles (Fig. 15).

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[33] For more on this distortion, see Field (1997, p. 272).

[34] Field (1997, p. 272).

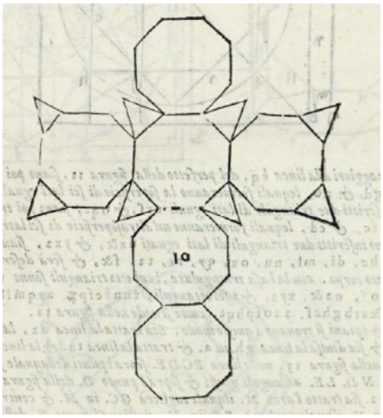

Fig. 15. The net of the truncated cube (Barbaro 1568, p. 61)

Here we must note another problem in some way related to the question of regular vs. irregular polygons. That is, in some cases the nets that Barbaro presents include vertices surrounded by angles whose sum is equal to 180°. Judith Field noted that there are two nets depicted by Barbaro that do not work because in his illustrations of them it is taken for granted that all polygons in the net are regular.[35] She cites the rectified truncated octahedron (Chap. XVI) and the truncated pentakis dodecahedron (Chap. XIX), but this defect also applies to the rectified truncated icosahedron (Chap. XVIII), the truncated truncated icosahedron (Chap. XX) and the truncated truncated octahedron (Chap. XXII).

Essentially, the problem is this: no vertex in a net can be surrounded by angles that add up to 180°, since in that case the vertex would be flat. In the case of the rectified truncated octahedron (Chap. XVI), beginning with a truncated octahedron with regular squares and octagons, dividing the sides of these into two parts would result in triangles that are not equilateral but isosceles, the base being slightly shorter than the sides. More precisely, the angles, rather than 60°-60°-60°, would measure approximately 66°-66°-48°. If the triangles were equilateral, a vertex surrounded by two hexagons and two triangles would be flat. The case of the net depicting the truncated pentakis dodecahedron in Chap. XIX is similar. The net only works if there are tiny spaces between three contiguous hexagons; if not, the vertices are flat. In addition to the two cases noted by Field, it may be added that the nets shown in Chaps. XVIII, XX and XXII also contain flat vertices.

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[35] Field (1997, p. 271).

Mutation. The next process that he uses is that of *mutare*, 'mutation'. In Chap. XIX he claims to have produced the truncated pentakis dodecahedron by having mutated the 60 triangles of the rectified truncated icosahedron (presented in the previous Chap. XVIII) into 60 hexagons.[36] Similarly, in Chap. XXV he says to transform the 24 triangles of the truncated truncated octahedron (Chap. XXII) into 24 hexagons. In this case, however, the result is not a valid one, as this solid defies attempts to identify it.

Addition. The final process that he uses is addition, involving 'elevation' and 'augmentation'. Here it is necessary to clarify the difference between them. Pacioli, in *De divina proportione*, spoke of 'elevation' when pyramids were added to the faces of polyhedral to make new bodies. Rinus Roelofs has defined this process as 'pulling each midpoint of all the faces outward until the triangles formed by those midpoints with two adjacent vertices of the original face form are equilateral', as opposed to 'stellation', as defined by Kepler in 1619, which is the process of 'extending edges or faces until they meet to form a new polygon or polyhedron'.[37] Thus the addition of cupolas to polygonal faces, as in Chaps. XXIII and XXIV are examples of augmentation, while in Chaps. XXVI through XXXIV the addition of pyramids to polygonal faces are examples of elevation.

Let us close this discussion of Barbaro's exploration of solids by noting some remarkable results.

One of Barbaro's solids – the truncated pentakis dodecahedron (Chap. XIX) – is a Goldberg polyhedron, and two of his solids – the rectified truncated octahedron (Chap. XVI) and the truncated truncated iscosahedron (Chap. XX) – are symmetrohedra.[38] That is, he was discovering solids that would only be classified hundreds of years after his work.

In Chaps. XXIII and XXIV, Barbaro begins with two earlier solids – the truncated icosidodecahedron (Chap. XXI) and the truncated cuboctahedron (Chap. XV) – and adds to their faces new shapes that would come to be known as cupolas. In Chap. XXIII, beginning with the truncated icosidodecahedron, each decagonal face becomes the base of a pentagonal cupola formed of one pentagon, five squares and five triangles. The gaps between the triangles and the squares show that when folded it forms a three-dimensional shape (Fig. 16).[39] Similarly, in Chap. XXIV, to each of the octagonal faces of the truncated cuboctahedron is added a square cupola formed of five squares and four triangles (Fig. 17). Thus he enlarges his family of bodies to include concave solid forms.

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[36] In private correspondence Vera Viana pointed out that the solid from which the Archimedean great rhombicosidodecahedron of Chapter XIX derives is the truncated truncated icosahedron described in Chapter XX.

[37] Roelofs (2014).

[38] For Goldberg polyhedra, see Goldberg (1937). For symmetrohedra, see Kaplan and Hart (2001). I am grateful to Vera Viana for verifying and advancing my identifications.

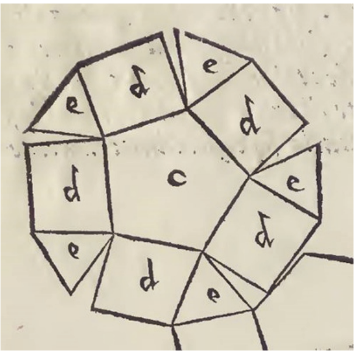

Fig. 16. Detail of the net shown in Chap. XXIII, showing the pentagonal cupola

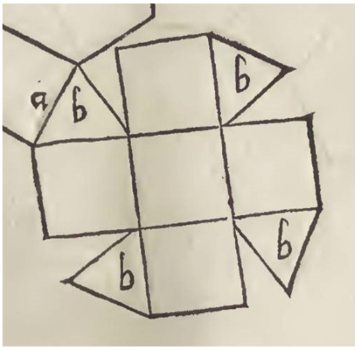

Fig. 17. Detail of the net shown in Chap. XXIV, showing the square cupola

These cupolas are Johnson solids,[40] which differ from the Platonic and Archimedean solids and the convex semiregular prisms and antiprisms in that while each face is a regular polygon, not all faces are the same polygon (as in the Platonic solids) and not all vertices have the same configuration (that is, they are not transitive).

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[39] Jamnitzer (1568, pl. V), regarding bodies based on the dodecahedron, shows the basic form with a pentagonal cupola added to each face.

[40] A Johnson solid is a non-uniform convex polyhedron (that is, each of its faces is a regular polygon but the faces do not have to be the same polygon, nor must the same polygons join at all vertices. See Johnson (1966).

Material Bodies

Artistic depictions of material bodies

From the end of the fifteenth century, artists were concerned with representing the solids not for their mathematical properties but as material bodies, sometimes with aesthetic aims, others with symbolical overtones.

One of the most iconic representations of a solid is a marble intarsia depicting a stellated dodecahedron that appears in in the pavement of the Basilica of St. Mark in Venice (Fig. 18). If the attribution of the pavement inset to Paolo Uccello (1397-1475) is correct, and it was executed during his stay in Venice in 1425, it is one of the earliest three-dimensional renderings ever of a solid.[41]

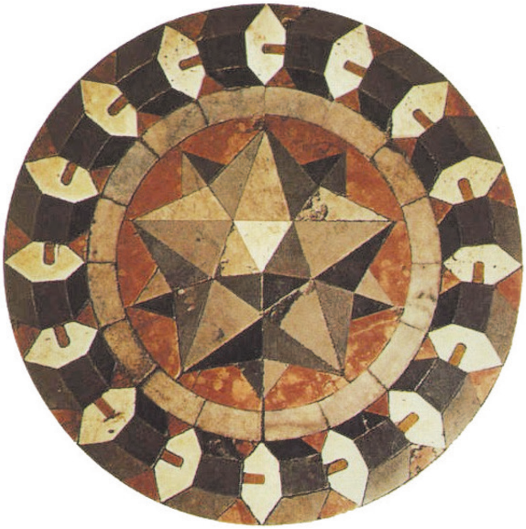

Fig. 18. Pavement inset in St. Mark's depicting the elevated dodecahedron

Other late fifteenth-century perspective representation of the solids appeared in the marquetry work of Fra' Giovanni da Verona (c.1457-1525) in the choir of S. Maria in Organo in Verona[42] (Fig. 19) and in the abbey of Monte Olivetto Maggiore in the province of Siena (Fig. 20).

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[41] See Hudson (2008, pp. 52 and 314). The reference on p. 52 to Kepler's description of the elevated dodecahedron being the earliest is incorrect, as we have seen, as Barbaro describes it in Chap. XXIX of Part III.

[42] Thomas and Williams (2000).

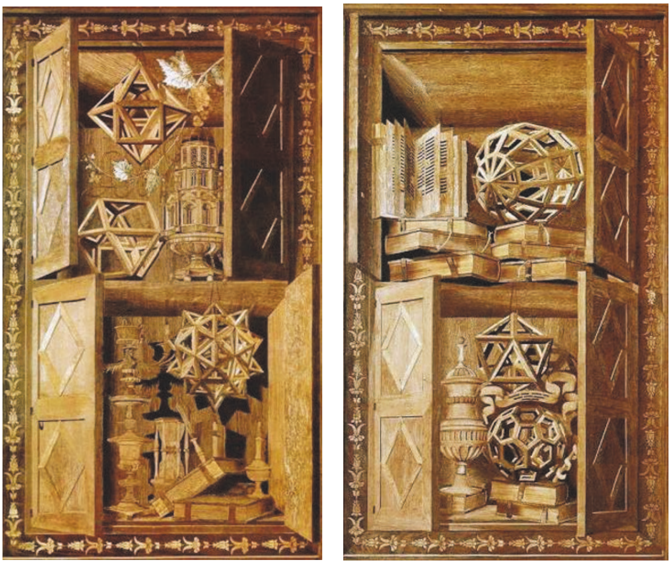

Fig. 19. Marquetry panels in S. Maria in Organo. Left, above) the elevated cube and cuboctahedron; left, below) elevated icosidodecahedron; right, above) Leonardo's 72-sided sphere; below) icosahedron and truncated icosahedron

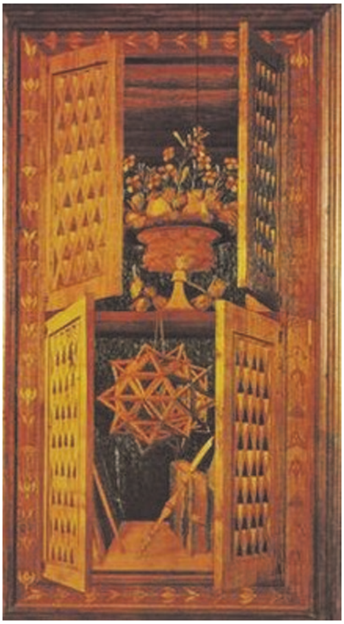

Fig. 20. The elevated icosidodecahedron in the marquetry work in abbey of Monte Olivetto Maggiore

The famous portrait of Luca Pacioli, said to have been painted by Jacopo de Barbari around 1495, shortly after the publication of the *Summa*, shows a transparent rhombicuboctahedron that appears to be filled halfway with water (Fig. 21).

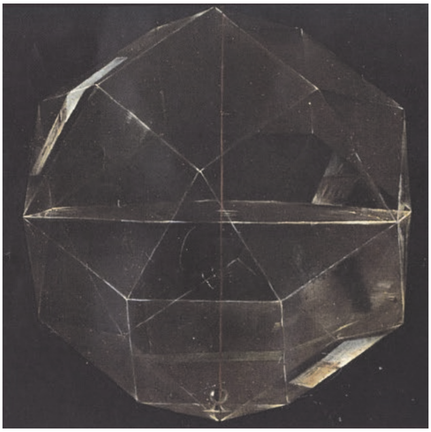

Fig. 21. Detail of Jacopo de Barbari's portrait of Luca Pacioli showing the rhombicuboctahedron

Pacioli twice wrote about having himself made models of the solids. In the introduction to his section of the solids in the *Summa*, Pacioli writes about having shown a collection of solids 'with many adornments' to Guidobaldo of Urbino:

> These are the ones, Magnanimous Duke, of which the material forms (with much adornment) you held in your own hands. In the sublime palace of the most reverend cardinal, our protector monsignor of San Pietro in Vincoli [Giuliano della Rovere], when he came during the visit of Pope Innocent VIII, in the year of our health 1489, in the month of May, which is already five years ago. And together with those there were many others that depended on said regular solids. Those I made for the reverend monsignor lord Pierre de Valetariis of Genova, the most worthy bishop of Carpentras... .[43]

---

[43] Pacioli (1494, p. 68r): *Questi son quelli Magnanimo Duca di quali le forme materiali (con assai adornezze nelle proprie mani di V.D.S. Nel sublime palazzo del Reverendissimo cardinale nostro protectore Monsegnor de san piero in vincula: quando quella venne ala visitazione del summo pontifice Innocentio octavo: negli anni de a salute nostra 1489. Del mese de aprile che gia sonno 5 anni elapsi. E insiemi con quelli vi foron molti altri da ditti regulari dependenti. Quali fabricai*

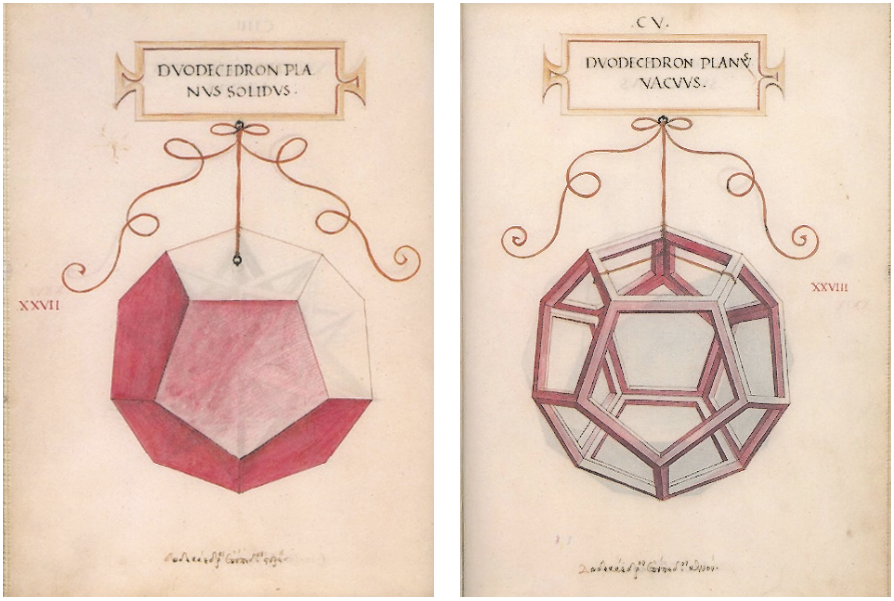

Fig. 22. The dodecahedron as shown in *Divina proportione*: 'Dodecahedron planum solidum' (plate numbered XXVII) and 'Dodecahedron planum vacuum' (plate numbered Fig. XXVIII)

He mentions this again in *Divina proportione*:

> And the most beautiful forms of said material bodies with all grace here in Milan with my own hands I made, coloured and adorned, and these were 60 in number between the regular bodies and those that depended on them. And as many other similar ones I made for my patron lord Galeazzo Sanseverino in that place. And then as many others in Florence at the order of our gonfaloniere for life Piero Solderini… .[44]

It is in this location that Pacioli credits Leonardo with three-dimensional renderings representing the solids as physical bodies suspended by graceful wires (Fig. 22):

---

per lo Reverendo monsegnor meser Pietro de valetarij de Genoa dignissimo vescovo de carpentras… (my trans.).

[44] Pacioli (1509, Pars Prima, p. 28v): *E le forme de dicti corpi materiali bellissime con tutta ligiadria quivi in Milano de mie proprie mani disposi colorite e adorne e forono numero 60 tra regulari e lor dependenti. Et simile altre tanti ne disposi per lo mio patrone S. Galeazzo Sanseverino in quel luogo. E poi altre tante in firenze ala ex.a del nostro S. Confalonieri perpetuo P. Soderino…* (my trans.)

> As you fully see in the disposition of all the regular bodies and those that depend on them in this book, which were made by the most worthy painter, perspectivist, architect, musician and gifted with all the virtues: Leonardo da Vinci, Florentine, in the city of Milan when in the service of the most excellent duke of that city, Ludovico Maria Sforza.[45]

Leonardo's depictions were followed by another famous depiction of a material solid by Albrecht Dürer, the truncated triangular trapezohedron shown in his engraving *Melencolia I* of 1514 (Fig. 23).

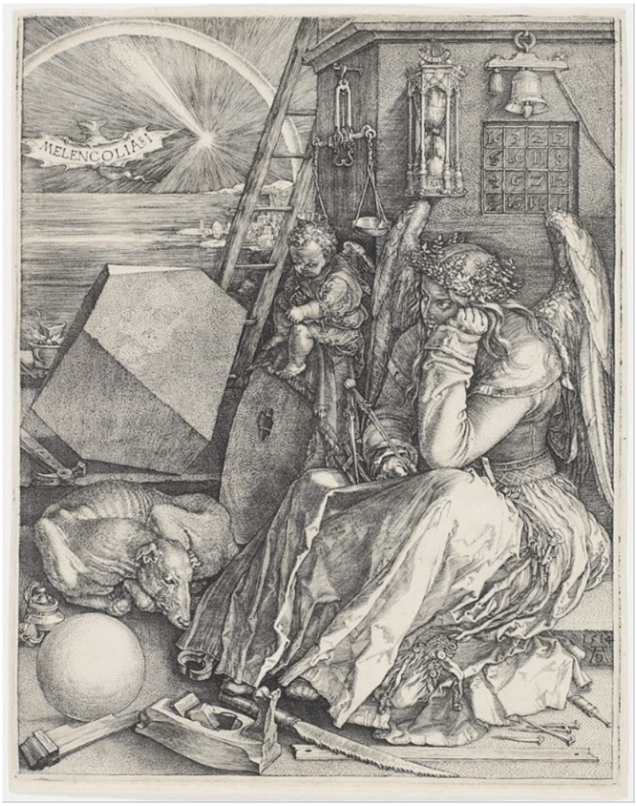

Fig. 23. Dürer's *Melencolia I* of 1514

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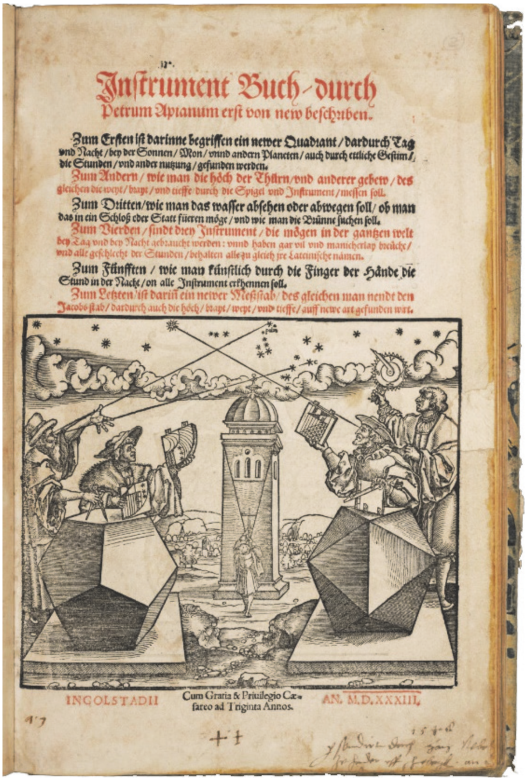
[45] Pacioli (1509, Pars Prima, p. 28v): *Comme apien in le disposition de tutti li corpi regulari e dependenti di sopra in questo vedete quali sono stati facti dal degnissimo pictore prospective architetto musico. E de tutte virtu doctato: Lionardo davince fiorentino nella cita de Milano quando ali stipendii dello Excellentissimo Duca di quello Ludovico Maria Sforza* (my trans.).

Solids also began appear in the role of symbols. The woodcut that adorns the title page of Peter Apian's *Instrument Buch* of 1533 features a large dodecahedron and a large icosahedron on which the scientists rest their instruments: the implication is that everything rests on geometry (Fig. 24).



Inſtrument Buch durch
Petrum Apianum erſt von new beſchriben.

Zum Erſten iſt darinne begriffen ein newer Quadrant/dardurch Tag vnd Nacht/bey der Sonnen/Mon/vnnd andern Planeten/auch durch etliche Geſtirn/die Stunden/vnd ander nutzung/gefunden werden.

Zum Andern/wie man die höch der Thürn/vnd anderer gebew/desgleichen die weyt/brayt/vnd tieffe durch die Spigel vnd Inſtrument/meſſen ſoll.

Zum Dritten/wie man das waſſer abſehen oder abwegen ſoll/ob man das in ein Schloß oder Statt füeren möge/vnd wie man die Brünne ſuchen ſoll.

Zum Vierden/ſindt drey Inſtrument/die mögen in der gantzen welt bey Tag vnd bey Nacht gebraucht werden: vnnd haben gar vil vnd manicherlay breüche/vnd alle geſchlecht der Stunden/behalten alle zu gleich jre Lateiniſche nämen.

Zum Fünfften/wie man künſtlich durch die Finger der Hände die Stund in der Nacht/on alle Inſtrument erkhennen ſoll.

Zum Letzten/iſt dariñ ein newer Meßſtab/des gleichen man nendt den Jacobs ſtab/dardurch auch die höch/brayt/weyt/vnd tieffe/auff newe art gefunden wirt.

INGOLSTADII
Cum Gratia & Priuilegio Cæſareo ad Triginta Annos.
AN. M.D.XXXIII.

Fig. 24. The dodecahedron and icosahedron on the title page of Apian's *Instrument Buch*

The German artist and printmaker Heinrich Vogtherr the younger, in his collection of portraits of 48 famous astronomers of about 1545, used polyhedra to characterise the astronomers: a truncated cube for John of Saxony and a cuboctahedron for Georg Purbach (Fig. 25).[46]

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[46] As Cosimo Monteleone has pointed out, the tie between geometry, polyhedra and astronomy was not only symbolic but practical as well. Among the astronomers of the Renaissance it was usual to derive instruments for measuring by taking advantage of the various inclinations of the faces of polyhedra, as shown by the portrait of Nikolaus Kratzer by Hans Holbein of 1528 or the multiple meridian of Stefano Buonsignori of 1597. See Monteleone (2020, pp. 78-79).

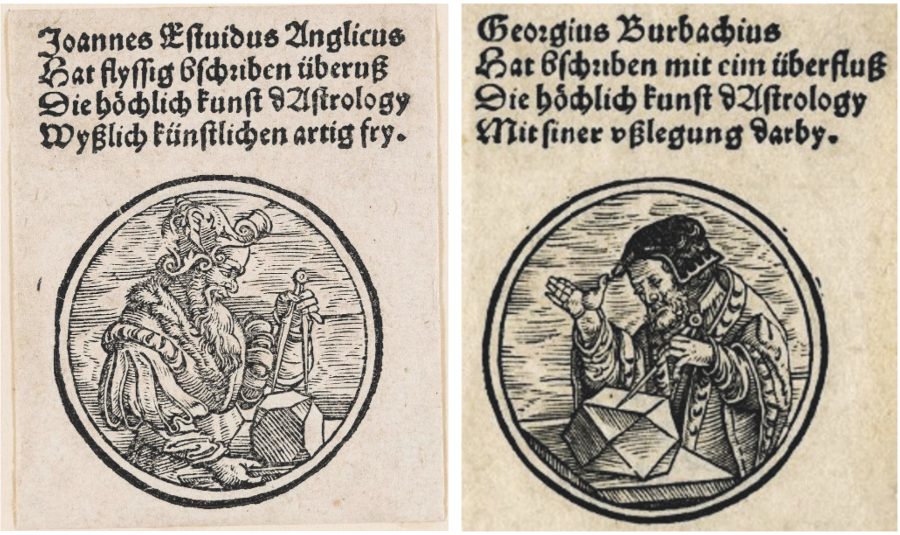
Joannes Eſtuidus Anglicus
Hat flyſſig bſchriben überuß
Die höchlich kunſt d Aſtrology
Wyßlich künſtlichen artig fry.
Georgius Burbachius
Hat bſchriben mit eim überfluß
Die höchlich kunſt d Aſtrology
Mit ſiner vßlegung darby.

Fig. 25. Vogtherr's portraits of astronomers with polyhedral, ca. 1545: left) John of Saxony with the truncated cube; right) Georg Purbach with the cuboctahedron

The solids depicted in the 1561 portrait of Johannes Neudörfer by Nicolaus Neufchatel, conserved in the Alte Pinakothek in Munich, serve the same purpose of underlining the importance of geometry in a liberal education, but with a painterly skill that rivals that of Jacopo de Barbari.

The concept of the geometrical beauty and perfection of polyhedra was so deeply rooted in Renaissance thinking that it is possible to find representations of solids used in decorative ways. In Germany, the production of books regarding geometry and perspective by Lencker, Jamnitzer, Hirschvogel and others took place in contemporary with the production of objects that use depictions of the solids as ornament. The lid of a recently displayed court casket produced in Nuremberg in 1565[47] is adorned marquetry representations of skeletal views of the Platonic solids.

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[47] Moore (2019).

Mathematical studies based on material bodies

Even those whose written works are mentioned above studied the solids with the aid of material bodies. Describing the octahedron, Dürer writes:

> The other body is pointed like a diamond and has equal angles, six equal corners and twelve sharp edges. Then I opened it completely and laid it on the ground, and raised it again, drew it all.[48]

Hirschvogel mentions making a model of some light material such as paper or cardboard:

> This figure arises from the previous example, only it is put together with more triangles. If one wants to have such a body with eight bases, it is put together in this way. It is basically cut open. This and others want to be called simply a *rete* or a net that is spread out. Make it in this way.
>
> There is a line *a*, *b*, *c*, *d* divided into three equal parts with an open compass. Draw from it in black three triangles according to what was given in the 12th chapter of the first part [instructions on how to construct a triangle with a compass], marked with the number 1.2.3. Following the number 3 is another triangle marked with 4. [Other triangles are made] on segment *cd*, marked with *e* and on 1.2. marked with 5. Cut this out, put it together, and make a material body out of it.[49]

That Barbaro thought of solids as material bodies and not simply as abstract forms is indicated by his introduction to Part III, Chap. II:

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[48] Dürer (1525, Book IV, pages not numbered): *Das ander corpus ist wie ein diamant punct / und hat gleych wincklen / und sechs gleicher viereecte unnd zwelf scharffer seyten. Wie ich das hernach ganz offen / darnach zūgethan nider in grund gelegt / und darnach aufgezogen / alles hab ausgerissen* (my trans.).

[49] Hirschvogel (1543, Vol. 1 (text), Part 2, Chap. 4, pages not numbered): *Auß forichem Exempel entspringt dir dise Figur / allein das es von mer Driangelen züsamen gesetzt wirt / Und wil solches Corpus ein acht sechzig Corpus genent haben / so es züsamen gemacht wirt. Aber also im grundtligent auffgerissen / mag es und andere wol genant / un verteurscht werden / ein Rete / oder ein Netz / das außgebreyt wirt / Solchs mach also.*

Es sey ein lini a/b/c/d/ geteilt in drey gleicher teil mit unverricktem zirckel / Zeuch darauß ausswartz drey Driangel nach der aufgab des 12. Capitels / des ersten teils gezeychnet mit der zal 1.2.3. henct an die ziffer 3. Und / noch einen driangel / gezeichnet mit 4. Des gleichen an c/d/ mit e/ und auß 1.2. mit 5 gezeichnet. Solches schneyd auß / legs züsamen / und mach darauß ein materlich Corpus.

> The following order will serve in describing the bodies: in the first place we will set forth their unfoldings, then their perfect and degraded plans, and then their shadowings. By 'unfolding' I mean the descriptions of the opened figures from which the solid figures are made by folding them together to demonstrate the true form, something that is truly convenient for practice and delightful for forming many bodies in lanterns and other uses for pleasure.[50]

In his manuscript It. IV, 39=5446 he writes:

> Perspective is extending the surfaces of any [regular] body onto planes … so that it can then be reduced from that dilatation into corporal form. Then, with the material body in hand it is possible to plainly see of what order this body is and understand its heights, in order to then be able to place its perfect in the square, and from that into the plane.[51]

Thus, *with the material body in hand*, Barbaro the scholar and master of solids, is able to achieve two ends. He is able to visualize the solids in different positions (resting on different faces), and he is able to define the process of determining the distance between the parallel faces (horizontally placed) and between one of these and the vertices of the other faces. Twice he assumes that his students will also have material bodies at hand, mentioning properties 'as seen in the material body'.[52]

It seems likely that the 'material bodies in hand' included both models made of solid materials and models constructed as wireframes. Such wire models have three advantages: for the generation of ground plans or side views, they make it easy to see the three-dimensional volume as depicted on a two-dimensional plane; for three-dimensional representations their transparency makes it possible to see all edges in relationship to each other in space; they are easily made, modified, and unfolded into nets. I personally believe that Barbaro produced his own drawings rather than relying on a draftsman; if he studied the solids from wireframe models, he could also have made those models on his own.

---

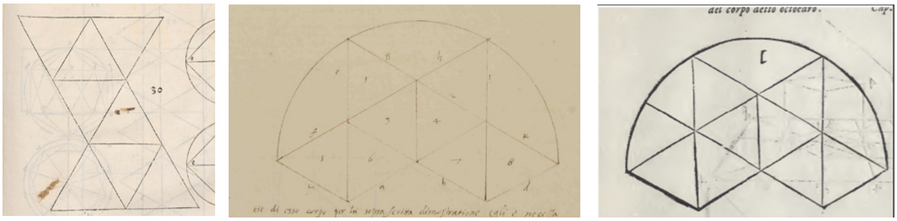
[50] Barbaro (1568, p. 45): *Nel descrivere i corpi si serverà a quest'ordine, che nel primo luoco poneremo le lloro* [sic] *spiegature, dapoi le loro piante perfette,* & *digradate,* & *finalmente i dritti,* & *le loro adumbrationi. Spiegature io intendo le descrittioni delle figure aperte, dellequali si fanno i corpi sodi piegandole insieme per dimonstratione del vero, cosa veramente commoda per a practica,* & *dilettevole per formare molti corpi in lanterne,* & *altri usi di piacere.*

[51] Barbaro (Ms. It. IV, 39=5446, fol. 30r); Monteleone (2020, p. 217): *perspettive estender di qualunque corpo la supeficie sua in piano … che la si possa poi da quella dilatazione redur in forma corporale: accioche poi con il corpo materiale in mano apertamente di che ordine sia esso corpo veder si possa, et intender la sue altitudine per poter poi metter il perfetto suo nel quandrato, e da quello nel piano.*

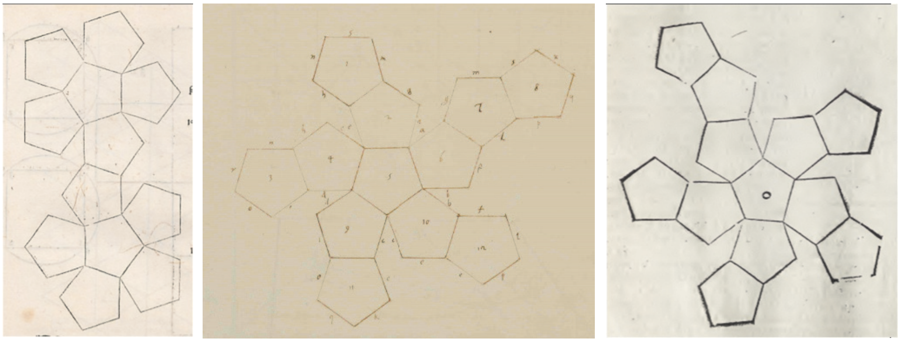
[52] In explaining the dodecahedron (1568, Chap. V, p. 51), and the icosahedron (1568, Chap. VI, p. 54).

Although the nets shown in Barbaro's treatise as published in 1568 are identical to those shown in Dürer's *Underweysung*, the nets drawn in his manuscript It. IV, 39=5446 make it clear that Barbaro was not merely copying but was unfolding and refolding the solids for himself. This can be seen in the nets for the octahedron, dodecahedron, icosahedron, truncated tetrahedron, and small rhombicuboctahedron (Fig. 26). It is unclear why some of Dürer's arrangements of the unfoldings were chosen for the published treatise rather those in the manuscript, but this does not diminish Barbaro's originality of research.

Octahedron



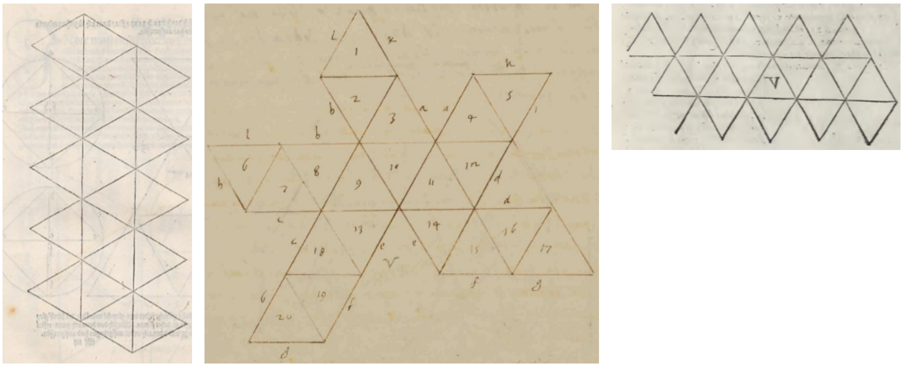
Dodecahedron



Dürer 1525, fig. 33 Ms. IV, 39=5446, fol. 35v Barbaro 1568, p. 49

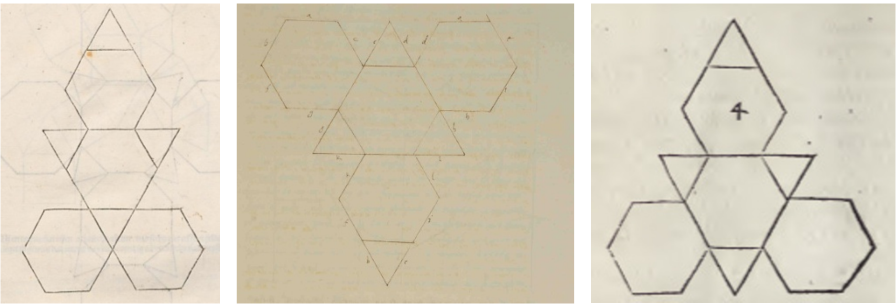
Fig. 26. Comparison of the nets of Dürer and Barbaro

Icosahedron



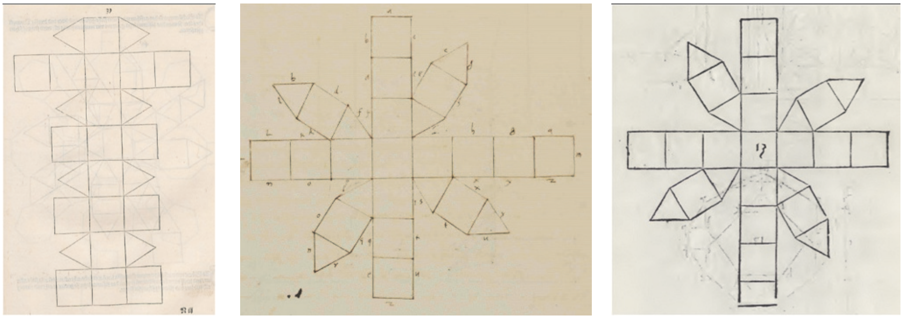
Dürer 1525, fig. 31 Ms. IV, 39=5446, fol. 39v Barbaro 1568, p. 53

Truncated tetrahedron



Dürer 1525, fig. 35 Ms. IV, 39=5446, fol. 44r Barbaro 1568, p. 57

Small rhombicuboctahedron



Dürer 1525, fig. 39 Ms. IV, 39=5446, fol. 51v Barbaro 1568, p. 64

Fig. 26 (cont). Comparison of the nets of Dürer and Barbaro

Clues that Barbaro was studying from material bodies come from some of this descriptions. For instance, in Chap. IV about the octahedron he writes:

> Line *bh* being the diametrical line [median] of the triangle *bdg*, and *bl*, equal to that, the line *lk*, will be the true height of that body formed of eight triangles equal to triangle *cef*, since the line *bh* is the width but not at right angles, since if it were at right angles, the point *k* would be over the point *b*.[53]

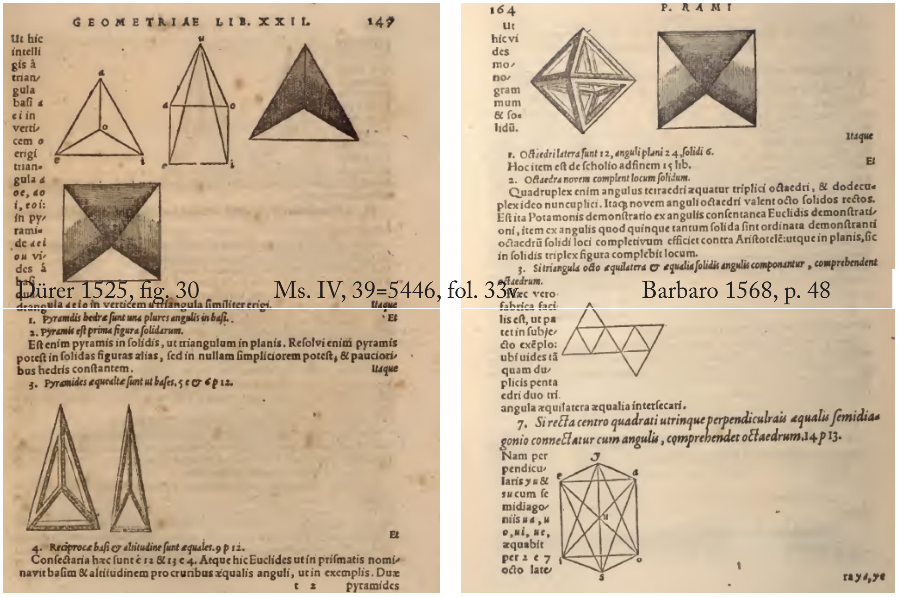
Being able to hold a material body in hand and rotate it makes it possible to understand how relationships between points change in a way that is very difficult indeed in a two-dimensional representation.

Studies on solids after Barbaro

To conclude, I will briefly survey of representations of the solids that appeared in the years immediately following Barbaro's treatise. This will show how research on perspective and the solids gradually made its way into mathematics treatises. What is noteworthy about these mathematical treatises is that they continue to reproduce the former figurative conventions but also introduce new ways to represent the solids. While most of the depictions were not rigorous in perspective but were closer to axonometric, like the ones used for representing machines, it in any case appears that the mathematicians felt the need to show solids in a way that was less abstract and much more similar to what the eye sees. In 1569, the French philosopher Pierre de la Ramée, who published as Petrus Ramus, published his *Arithmeticae libri duo Geometriae septem et viginti*. Shown in Fig. 27 are some of the illustrations use the same convention as in earlier editions of Euclid's *Elements*, but complementing these are shaded three-dimensional solids, skeletal representations, and nets.

---

[53] Barbaro (1568, p. 48): *essendo la linea* bh, *diametrale del triangulo* bdg, & bl, *eguale a quella linea* lk, *serà la vera altezza del detto corpo, formato di otto trianguli eguali al triangulo* cef, *perche la linea* bh, *è la largehezza ma non ad angulo giusto*.



GEOMETRIAE LIB. XXII. 149

Ut hic intelligis à triangula baſi aei in verticem o erigi triangula aoe, aoi, eoi: in pyramide aei ou vides à baſi quadrangula aeio in verticem u triangula ſimiliter erigi.

Itaque

1. *Pyramdis hedræ ſunt una plures angulis in baſi.*

Et

2. *Pyramis eſt prima figura ſolidarum.*

Eſt enim pyramis in ſolidis, ut triangulum in planis. Reſolvi enim pyramis poteſt in ſolidas figuras alias, ſed in nullam ſimpliciorem poteſt, & paucioribus hedris conſtantem.

Itaque

3. *Pyramides æquealtæ ſunt ut baſes. 5 e & 6 p 12.*

Et

4. *Reciprocæ baſi & altitudine ſunt æquales. 9 p 12.*

Conſectaria hæc ſunt è 12 & 13 e 4. Atque hic Euclides ut in priſmatis nominavit baſim & altitudinem pro cruribus æqualis anguli, ut in exemplis. Duæ

t 2 pyramides

164 P. RAMI

Ut hic vides monogrammum & ſolidū.

Itaque

1. *Octaedri latera ſunt 12, anguli plani 24, ſolidi 6.*

Hoc item eſt de ſcholio ad finem 15 lib.

Et

2. *Octaedra novem complent locum ſolidum.*

Quadruplex enim angulus tetraedri æquatur triplici octaedri, & dodecuplex ideo nuncuplici. Itaq; novem anguli octaedri valent octo ſolidos rectos. Eſt ita Potamonis demonſtratio ex angulis conſentanea Euclidis demonſtrationi, item ex angulis quod quinque tantum ſolida ſint ordinata demonſtranti octaedrū ſolidi loci completivum efficiet contra Ariſtotelē: utque in planis, ſic in ſolidis triplex figura complebit locum.

3. *Si triangula octo æquilatera & æqualia ſolidis angulis componantur, comprehendent octaedrum.*

Hæc vero fabrica facilis eſt, ut patet in ſubjecto exēplo: ubi uides tā quam duplicis pentaedri duo triangula æquilatera æqualia interſecari.

7. *Si recta centro quadrati utrinque perpendicularis æqualis ſemidiagonio connectatur cum angulis, comprehendet octaedrum. 14 p 13.*

Nam perpendicularis yu & ſu cum ſemidiagoniis ua, uo, ui, ue, æquabit per 2 e 7 octo late-

ra ya, ye

Dürer 1525, fig. 30 Ms. IV, 39=5446, fol. 33v Barbaro 1568, p. 48

Fig. 27. Pages 149 and 164 and from the *Geometriae septem et viginti* by Petrus Ramus showing representations of the tetrahedron and octahedron

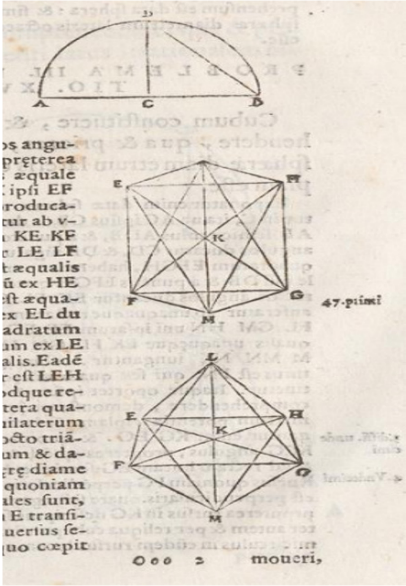


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Fig. 28. Representations of the octahedron in Commandino's 1572 edition of Euclid, p. 238, showing the conventional representation (above) and the new perspective view

Federico Commandino, in his 1572 edition of Euclid, was the first to use line representations that convincingly depict solids in true perspective (Fig. 28). Christopher Clavius, in his 1574 *Euclidis Posteriores Elementorum Libri XV*, reproduces on p. 257 (Book XIV, prob. 4, prop. 4) an older depiction, but then on p. 286 provides an axonometric view (Fig. 29). Maurolico, in his *Opuscola matematica* of 1575 showed the axonometric view of the octahedron with the addition of allusions to the circumsphere (Fig. 30).

Christopher Clavius's 1574 *Euclidis Posteriores Elementorum Libri XV* introduced nets in scholia, and cites Dürer as the source (Fig. 31).

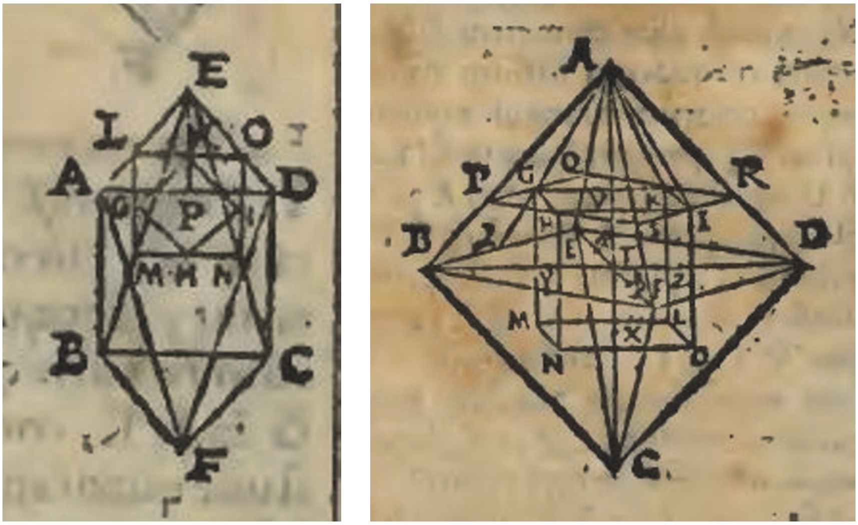
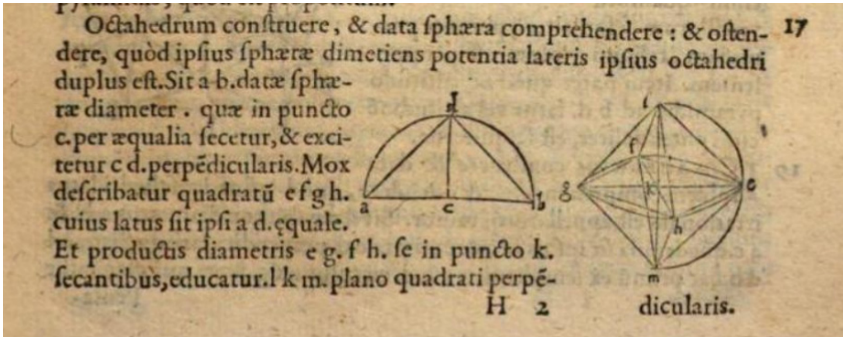


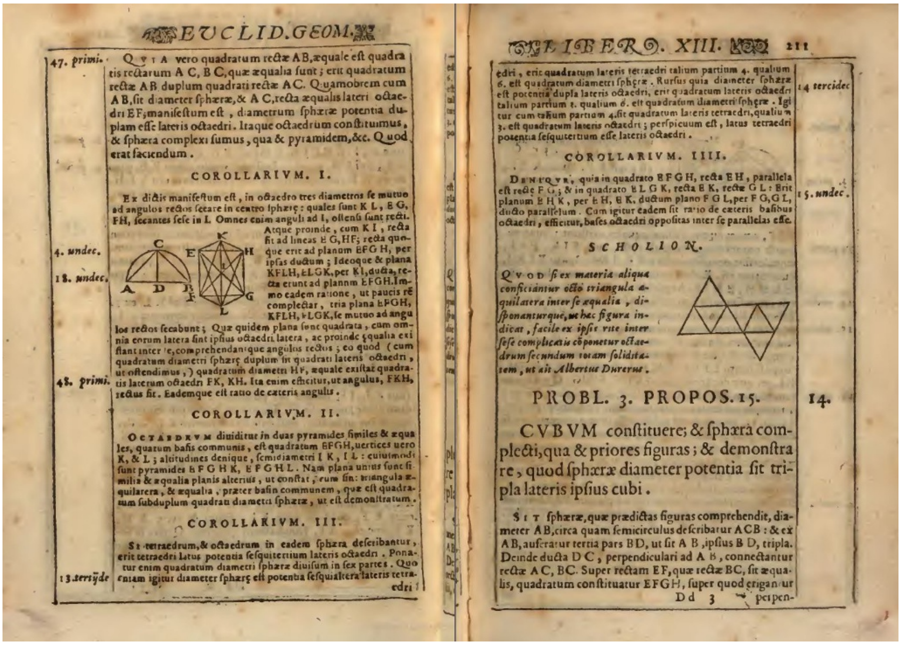
Fig. 29. The cube in the octahedron as represented in Clavius's 1574 edition of Euclid. Left) as shown in Book XV (prob. 4, prop. 4, p. 257r; right) as shown in Book XVI (theor. 23, prop. 23, p. 286r)



Octahedrum conſtruere, & data ſphæra comprehendere: & oſtendere, quòd ipſius ſphæræ dimetiens potentia lateris ipſius octahedri duplus eſt. Sit a b. datæ ſphæræ diameter. quæ in puncto c. per æqualia ſecetur, & excitetur c d. perpẽdicularis. Mox deſcribatur quadratũ e f g h. cuius latus ſit ipſi a d. çquale. Et productis diametris e g. f h. ſe in puncto k. ſecantibus, educatur. l k m. plano quadrati perpẽdicularis. 17

H 2

Fig. 30. Maurolico 1575, p. 115



EVCLID. GEOM.

47. primi. QVIA vero quadratum rectæ AB, æquale eſt quadratis rectarum A C, B C, quæ æqualia ſunt; erit quadratum rectæ AB duplum quadrati rectæ AC. Quamobrem cum AB, ſit diameter ſphæræ, & A C, recta æqualis lateri octaedri EF; manifeſtum eſt, diametrum ſphæræ potentia duplam eſſe lateris octaedri. Itaque octaedrum conſtituimus, & ſphæra complexi ſumus, qua & pyramidem, &c. Quod erat faciendum.

COROLLARIVM. I.

Ex dictis manifeſtum eſt, in octaedro tres diametros ſe mutuo ad angulos rectos ſecare in centro ſphærę: quales ſunt K L, E G, FH, ſecantes ſeſe in I. Omnes enim anguli ad I, oſtenſi ſunt recti. Atque proinde, cum K I, recta ſit ad lineas E G, HF; recta quoque erit ad planum EFG H, per ipſas ductum; Ideoque & plana KFLH, ELGK, per KI, ducta, recta erunt ad planum EFGH. Immo eadem ratione, ut paucis rē complectar, tria plana EFGH, KFLH, ELGK, ſe mutuo ad angulos rectos ſecabunt; Quæ quidem plana ſunt quadrata, cum omnia eorum latera ſint ipſius octaedri latera, ac proinde ęqualia exiſtant inter ſe, comprehendantque angulos rectos; eo quod (cum quadratum diametri ſphærę duplum ſit quadrati lateris octaedri, ut oſtendimus,) quadratum diametri HF, æquale exiſtat quadratis laterum octaedri FK, KH. Ita enim efficitur, ut angulus, FKH, rectus ſit. Eademque eſt ratio de cæteris angulis.

4. undec.

18. undec.

48. primi.

COROLLARIVM. II.

OCTAEDRVM diuiditur in duas pyramides ſimiles & æquales, quarum baſis communis, eſt quadratum EFGH, uertices uero K, & L; altitudines denique, ſemidiametri I K, I L: cuiuſmodi ſunt pyramides E F G H K, E F G H L. Nam plana unius ſunt ſimilia & æqualia planis alterius, ut conſtat, cum ſint triangula æquilatera, & æqualia, præter baſin communem, quæ eſt quadratum ſubduplum quadrati diametri ſphæræ, ut eſt demonſtratum.

COROLLARIVM. III.

SI tetraedrum, & octaedrum in eadem ſphæra deſcribantur, erit tetraedri latus potentia ſeſquitertium lateris octaedri. Ponatur enim quadratum diametri ſphæræ diuiſum in ſex partes. Quoniam igitur diameter ſphærę eſt potentia ſeſquialtera lateris tetra-

13. tertijde

edri

LIBER. XIII. 211

edri, erit quadratum lateris tetraedri talium partium 4. qualium 6. eſt quadratum diametri ſphęræ. Rurſus quia diameter ſphæræ eſt potentia dupla lateris octaedri, erit quadratum lateris octaedri talium partium 3. qualium 6. eſt quadratum diametri ſphęræ. Igitur cum talium partium 4. ſit quadratum lateris tetraedri, qualium 3. eſt quadratum lateris octaedri; perſpicuum eſt, latus tetraedri potentia ſeſquitertium eſſe lateris octaedri.

14 tercidec

COROLLARIVM. IIII.

DENIQVE, quia in quadrato EFGH, recta EH, parallela eſt rectę FG; & in quadrato ELGK, recta EK, rectæ GL: Erit planum EHK, per EH, EK, ductum plano FGL, per FG, GL, ducto parallelum. Cum igitur eadem ſit ratio de cæteris baſibus octaedri, efficitur, baſes octaedri oppoſitas inter ſe parallelas eſſe.

15. undec.

SCHOLION.

QVOD ſi ex materia aliqua conficiantur octo triangula æquilatera inter ſe æqualia, diſponanturque, ut hæc figura indicat, facile ex ipſis rite inter ſeſe complicatis cōponetur octaedrum ſecundum totam ſoliditatem, ut ait Albertus Durerus.

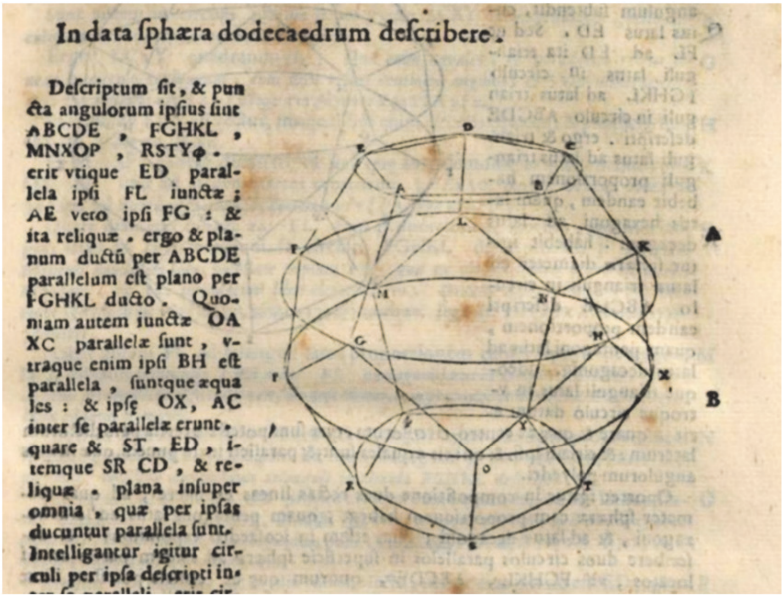
PROBL. 3. PROPOS. 15. 14.

CVBVM conſtituere; & ſphæra complecti, qua & priores figuras; & demonſtrare, quod ſphæræ diameter potentia ſit tripla lateris ipſius cubi.

SIT ſphæræ, quæ prædictas figuras comprehendit, diameter AB, circa quam ſemicirculus deſcribatur ACB: & ex AB, auferatur tertia pars BD, ut ſit A B, ipſius B D, tripla. Deinde ducta D C, perpendiculari ad A B, connectantur rectæ A C, BC. Super rectam EF, quæ rectæ BC, ſit æqualis, quadratum conſtituatur EFGH, ſuper quod erigantur

Dd 3 perpen-

Fig. 31. Clavius, 1574, Book XIII regarding the octahedron, with the conventional representation on p. 210v and the net in a scholion on p. 211r, citing Dürer



In data ſphæra dodecaedrum deſcribere.

Deſcriptum ſit, & puncta angulorum ipſius ſint ABCDE, FGHKL, MNXOP, RSTYφ. erit vtique ED parallela ipſi FL iunctæ; AE vero ipſi FG: & ita reliquæ. ergo & planum ductū per ABCDE parallelum eſt plano per FGHKL ducto. Quoniam autem iunctæ OA XC parallelæ ſunt, vtraque enim ipſi BH eſt parallela, ſuntque æquales: & ipſę OX, AC inter ſe parallelæ erunt. quare & ST, ED, itemque SR CD, & reliquæ. plana inſuper omnia, quæ per ipſas ducuntur parallela ſunt. Intelligantur igitur circuli per ipſa deſcripti in-

Fig. 32. Figure from Commandino's 1588 edition of Pappus showing the dodecahedron

Commandino's posthumous edition of the *Mathematical Collection* of Pappus of 1588, the first translation to make the work of the Greek mathematician widely known, also contains figures that reflect perspective views of solids. The treatment of the solids is found in Book VIII, and they are depicted in skeletal form with perspective views. These are particularly interesting in relation to Barbaro's depictions of the solids in perspective because they indicate circumcircles that connect all of the points lying in the same plane (Fig. 32). This demonstrates that *The Practice of Perspective* was circulating among the mathematicians in Venice.

Finally, a half century after the publication of Barbaro's treatise, Johannes Kepler, in his *Harmonices Mundi* of 1619, presented and named the Archimedean solids (Fig. 33).

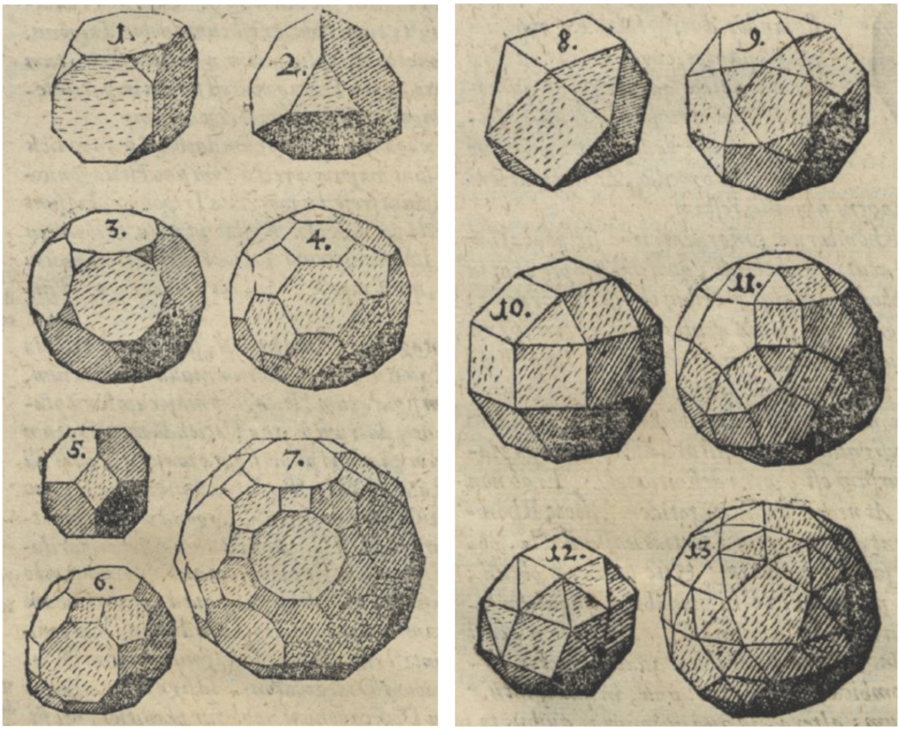


Fig. 3. Johannes Kepler, *Harmonices mundi*, 1619, pp. 62 (right), 64 (left):
1) truncated cube; 2) truncated tetrahedron; 3) truncated dodecahedron;
4) truncated icosahedron; 5) truncated octahedron; 6) great rhombicuboctahedron;
7) truncated icosidodecahedron; 8) cuboctahedron; 9) icosidodecahedron;
10) small rhombicuboctahedron; 11) small rhombicosidodecahedron;
12) snub cube; 13) snub dodecahedron

Did Kepler know Barbaro's work on the solids? He never mentions it, but it is fun to speculate that he might actually have seen it. The latter part of the sixteenth century saw the rise of cabinets of curiosities, known as *Wunderkammer*, which housed objects and books devoted to the mathematical and natural sciences. One of the most famous of these was the Kunstkammer of the Elector of Saxony. Kepler twice mentions experiences in the Kunstkammer that inspired him. He first mentions an experience that involved a solid:

> For I have seen at Dresden, in the Royal Palace known as the Stables, a panel of silver ore, from which a dodecahedron the size of a small hazelnut blossomed forth so that half of it stood out.[54]

His later experience involved a room that had been outfitted as a camera obscura:

> …an experiment of which I saw at Dresden in the Elector's Theater of Wonders. A disk thicker in the middle, or a crystalline lens, a foot in diameter, was standing at the entrance of a closed chamber against a little window, which was the only thing that was open, slanted a little to the right. Thus when the eyesight travelled through the dark emptiness, it also, fortuitously, hit upon the place of the image, nearer, in fact, than the lens. And so since the lens was weakly illuminated, it did not particularly attract the eyes. The walls were also not particularly conspicuous through the lens, because they were in deep darkness.[55]

As we have seen, the dodecahedron was treated in Barbaro's treatise, but so was the camera obscura.[56] Barbaro was the first to have described the shape of the lens of the camera obscura as convex: 'the eyeglass of an old man – that is, one that has a bit of thickness in the middle'.[57]

We also know that Kepler was familiar with Vitruvius,[58] and the intriguing fact is that both Barbaro's *Practice of Perspective* and his translation and commentary of Vitruvius were part of the collection of scientific books in the Kunstkammer, as they are catalogued in the inventory of 1587.[59] We'll never know if Kepler knew the works of Barbaro, but their inclusion in the Saxon collection indicates the esteem in which they were held, and that they would have been available to Kepler.

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[54] Kepler (2010, pp. 111-113).

[55] Kepler (2000, p. 194).

[56] See Monteleone (2020, pp. 153-154), and the saggio by Cosimo Monteleone in this present volume, pp. 97-98.

[57] Barbaro (1568, p. 192): *Et piglia un occhiale da vecchio, cioè che habbia alquanto di corpo nel mezzo….*

[58] See Eastwood (1982, esp. p. 378 and note 50).

References

Alonso-Rodríguez and Calvo-López (2014)
Alonso-Rodríguez, Miguel Ángel, José Calvo-López, Jose. 2014. *Prospettiva Soldatesca*: An Empirical Approach to the Representation of Military Architecture in the Early Modern Period. *Nexus Network Journal* 16 (2014): 543-567.
Andrews (2016)
Andrews, Noam. 2016. Albrecht Dürer's personal *Underweysung der Messung*. *Word* & *Image* 32(4), pp. 409-429.
Apian (1533)
Apian, Petrus. 1533. *Instrument Buch*. Ingolstadt.
Barbaro (1567)
Barbaro, Daniele. 1567. *I dieci libri dell'architettura di M. Vitruvio. Tradotti & commentati da mons. Daniel Barbaro eletto Patriarca d'Aquileia, da lui riveduti & ampliati; & hora in piu commoda forma ridotti.* Venice: Francesco de' Franceschi.
Barbaro (1568)
Barbaro, Daniele. 1568. *La pratica della perspettiva*. Venice: Camillo e Rutilio Borgominieri.
Barbaro (It IV, 39=5446)
Barbaro, Daniele. n.d. [La pratica della perspettiva, manuscript in Italian]. Ms. It IV, 39=5446. Biblioteca Nazionale Marciana, Venice.
Barbaro (It. IV, 40=5447)
Barbaro, Daniele. n.d. [La pratica della perspettiva, menabò]. Ms. It. IV, 40=5447. Biblioteca Nazionale Marciana, Venice.
Barbaro (Lat. VIII, 41=3069)
Barbaro, Daniele. n.d. Scenographia [manuscript in Latin]. Ms. Lat. VIII, 41=3069. Biblioteca Nazionale Marciana, Venice.

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[59] See Syndram and Minning, eds. (2010): 'Mag. Vitruvii Pollionis de architectura lib. X cum commentaris Danielis Barbari' is no. 72 on fol. 174v/184v, and 'Perspectiva Danielis Barbari in lingua italiana' is no. 83 on fol. 175v/185v.

Billingsley (1570)

Billingsley, Henry, *The Elements of Geometrie of the most auncient Philospher Euclid of Megara*. London: John Daye.

Boncompagni (1862)

Boncompagni, Baldassarre. 1862. *Scritti de Leonardo Pisano. Vol. II, Practica Geometriae ed Opuscoli*. Rome: Tipografia Delle Scienze Matematiche e Fisiche.

Clavius (1574)

Christophoro Clavio [Christopher Clavius]. 1574. *Euclidis Posteriores libri sex a x. ad xv*. Rome: Vincenzo Accolti.

Clavius (1607)

Clavius, Christopher. 1607. *Evclidis Elementorvm libri XV: Accessit liber XVI. de solidorum regularium cuiuslibet intra quodlibet comparatione. Omnes perspicvis demonstrationibus, accuratísque scholiis illustrati: nunc quarto editi, ac multarum rerum accessione post primam editionem locupletati, Parte 2*. Frankfurt: N. Hoffmanni.

Commandino (1572)

Commandino, Federico. 1572. *Euclidis Elementorum libri XV*. Pesaro: Jacobus Chriegher German.

Commandino (1588)

Commandino, Federico. 1588. *Pappi Alexandrini Mathematicae collectiones. A Federico Commandino Urbinatae in latinum conversae, & commentarijs illustratae*. Venice: Francesco de' Franceschi.

Cousin (1560)

Cousin, Jean 1560. *De perspective de Iehan Cousin*. Paris: De l'imprimerie de Iehan Royer Imprimeur le Royés Mathematiques.

Daly-Davis (1977)

Daly-Davis, Margaret. 1977. *Piero Della Francesca's Mathematical Treatises: The Trattato D'abaco and Libellus de Quinque Corporibus Regularibus*. Ravenna: Longo.

Dal Sole (1526)

Dal Sole, Francisco. 1526. *Libretto di abaco: novamente stampato / composto per lo eccellente maestro Ioanne Francisco dal Sole*. Venice: Francisco Bindoni e Mapheo Pasini.

Dürer (1525)

Dürer, Albrecht. 1525. *Underweysung der Messung, mit dem Zirckel und Richtscheyt, in Linien, Ebenen unnd gantzen corporen*. Nuremberg. Rev. ed. 1538.

Dürer (1528)

Dürer, Albrecht. 1528. *Hierinn sind begriffen vier Bucher von menschlicher Proportion durch Albrechten Durer von Nurerberg [sic.] erfunden und beschuben zu nutz allen denen so zu diser kunst lieb tragen*. Nuremberg: Hieronymus Andreae Formschneider.

Dürer (1977)

Dürer, Albrecht. 1977. *The Painter's Manual: A Manual of Measurement of Lines, Areas, and Solids by Means of Compass and Ruler Assembled by Albrecht Dürer for the Use of All Lovers of Art with Appropriate Illustrations Arranged to be Printed in the Year MDXXV*, Walter S. Strauss, trans. Abaris Books.

Eastwood (1982)

Eastwood, Bruce S. 1982. Kepler as Historian of Science: Precursors of Copernican Heliocentrism according to 'De Revolutionibus' I, 10. *Proceedings of the American Philosophical Society* 126(5), pp. 367-394.

Feliciano (1517)

Feliciano, Francesco. 1517. *Libro d'abaco*. Venice: per Iacobo Pentio da Lecho.

Feliciano (1526)

Feliciano, Francesco. 1526. *Libro di arithmetica* & *geometria speculativa* & *praticale ... Intitulato Scala gramaldelli*. Venice: per Francesco di Alessandro Bindoni & Mapheo Pasini.

Field (1997)

Field J. V. 1997. Rediscovering the Archimedean Polyhedra: Piero della Francesca, Luca Pacioli, Leonardo da Vinci, Albrecht Dürer, Daniele Barbaro, and Johannes Kepler. *Archive for History of Exact Sciences* 50(3/4), pp. 241-289.

Goldberg (1937)

Goldberg, Michael. 1937. A Class of Multisymmetric Polyhedra. *Tohoku Mathematical Journal*, First Series, 43, pp. 104-108.

Hirschvogel (1543)

Hirschvogel, Augustin. 1543. *Ein aigentliche und grundtliche anweysung in die Geometria* (A True and Thorough Instruction in Geometry) (Vol. I, text; Vol. II, figures). [Nuremberg]: [Vom Berg und Neuber].

Hudson (2008)

Hudson, Hugh. 2008. *Paolo Uccello: Artist of the Florentine Renaissance Republic*. Riga: Vdm Verlag Dr Müller.

Hughes (2008)

Hughes, Barnabas. 2008. *Fibonacci's* De practica geometrie. Heidelberg: Springer.

Jamnitzer (1568)

Jamnitzer, Wenzel. 1568. *Perspectiva corporum regularium.* [Nuremberg: Christoph Heussler].

Johnson (1966)

Johnson, Norman W. 1966. Convex Polyhedra with Regular Faces. *Canadian Journal of Mathematics* 18, pp. 169-200.

Kaplan and Hart (2001)

Kaplan, Craig, Hart, George. 2001. Symmetrohedra: Polyhedra from Symmetric Placement of Regular Polygons. In: *Proceedings of Bridges 2001: Mathematical Connections in Art, Music, and Science,* pp. 21-29. http://archive.bridgesmathart.org/2001/bridges2001-21.pdf, accessed 13/02/2021.

Kepler (1611)

Kepler, Johannes. 1611. *Strena seu De nive sexangula*. Frankfurt-am-Main: Godfrey Tampach.

Kepler (1619)

Kepler, Johannes. 1619. *Harmonices Mundi*. Linz: Johann Planck for Gottfried Tampach.

Kepler (2000)

Kepler, Johannes. 2000. *Optics: Paralipomena to Witelo* & *Optical Part of Astronomy*. W. H. Donahue, trans. Santa Fe: Green Lion Press.

Kepler (2010)

Kepler, Johannes. 2010. *The Six-Cornered Snowflake*, Jacques Bromberg, trans. Paul Dry Books.

Lautensack (1564)

Lautensack, Heinrich. 1564. *Des Circkels unnd Richtscheyts, auch der Perspectiva, und Porportion der Menschen und Rosse, kurtze, doch gründtliche underweisung, deß rechten gebrauchs*. [Frankfurt am Main]: n.p.

Lencker (1567)

Lencker, Johannes [Hans or Hansen]. 1567. *Perspectiva literaria.* [Nuremberg]: n.p.

Mancini (1915)

Mancini, Girolamo. 1915. L'opera 'De corporibus regularibus' di Piero Franceschi detto Della Francesca, usurpata da fra Luca Pacioli. *Memorie della Reale Accademia dei Lincei, Classe di scienze morali, storiche e filologiche* XIV, pp. 441-580.

Maurolico (1575)

Maurolico, Francesco. 1575. *Opuscula mathematica*. Venice: Francesco de' Franceschi.

Monteleone (2020)

Monteleone, Cosimo. 2020. *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto it. IV, 39=5446*. Canterano (Rome): Aracne editrice.

Moore (2019)

Moore, Susan. 2019. A rare Renaissance casket at London Art Week. Apollo Magazine, 19 June 2019. https://www.apollo-magazine.com/newbattle-abbey-casket-london-art-week/

Pacioli (1494)

Lucæ de Burgo [Luca Pacioli]. 1494. *Summa de Arithmetica Geometria Proportioni et Proportionalità*. Venice: Paganino Paganini, 2nd ed., 1523.

Pacioli (1509)

Lucas Patiolus Burgensis [Luca Pacioli]. 1509. *Divina proportione*. Venice: Paganino Paganini.

Piero della Francesca (Regg. A 41/2)

Piero della Francesca. n.d. *De prospectiva pingendi*, Mss. Regg. A 41/2, Biblioteca Panizzi, Reggio Emilia.

Piero della Francesca (Ash. 359)

Piero della Francesca. n.d. *Trattato d'abaco*, Ms. Ashburnham 359, Biblioteca Medicea Laurenziana, Florence.

Piero della Francesca (1970)

Piero della Francesca. 1970. *Trattato d'abaco*, ed. Gino Arrighi, Pisa: Domus Galileana.

Piero della Francesca (Urb. lat. 632)

Piero della Francesca. n.d. *De quinque corporibus regularibus* [*et de corporibus irregularibus*], [*Libellus*] Ms. Urb. lat. 632, Vatican Library, Rome

Piero della Francesca (1517)

Piero Borghi da Venetia [Piero della Francesca]. 1517. *Libro d'abacho*. Venice: Iacomo Pentio da Lecho ad instãtia de Marchio Sesia & Piero di Rauani compagni.

Ramus (1569)

Ramus, Petrus [Pierre de la Ramée]. 1569. *Arithmeticae libri duo Geometriae septem et viginti*. Basel: n.p.

Roelofs (2014)

Roelofs, Rinus. 2014. Elevations and Stellations. Proceedings of Bridges 2014: Mathematics, Music, Art, Architecture, Culture', eds. Gary Greenfield, George W. Hart, Reza Sarhangi, pp. 235-242. http://archive.bridgesmathart.org/2014/bridges2014-235.pdf, accessed 13/02/2021.

Scriba and Schreiber (2015)

Scriba, Christoph J., Schreiber, Peter. 2015. *5000 Years of Geometry: Mathematics in History and Culture*. Basel: Birkhäuser.

Schreiber et al. (2008)

Schreiber, Peter, Gisela Fischer and Maria Luise Sternath. 2008. New light on the rediscovery of the Archimedean solids during the Renaissance. *Archive for History of Exact Sciences* 62, 4, pp. 457-467.

Stör (1567)

Stör, Lorenz. 1567. *Geometria e Perspectiva*. Augsburg: Hans Rogel.

Syndram and Minning (2010)

Syndram, Dirk, Minning, Martina, eds. 2010. *Die Inventare der kurfürstlich-sächsischen Kunstkammer in Dresden. Band I: Das Inventar del 1587*. Dresden: Sandstein Verlag.

Thomas and Williams (2000)

Thomas, Hartwig, Williams, Kim. 2000. Fra' Giovanni's intarsias in Verona. *The Mathematical Intelligencer* 22, pp. 38-40.

Williams (2002)

Williams, Kim. 2002. Plagiary in the Renaissance. *The Mathematical Intelligencer* 24 (2002), pp. 45–57.

Williams (2019)

Williams, Kim. 2019. *Daniele Barbaro's Vitruvius of 1567*. Basel: Birkhäuser.

Williams (2020)

Williams, Kim. 2020. Prefazione. In: Cosimo Monteleone, *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto It. IV, 39=5446*. pp. VII-XIII Canterano (Rome).

Appendix A: Catalogue of the solids presented by Barbaro

The table here provides identifications of the solid bodies presented by Barbaro in Part III from Chap. I through Chap. XXXIV. Further remarks regarding the modern classification of the solids are added.

Chapter	Barbaro's description	Name	Composition	Remarks
I	Pyramid	Triangular pyramid	4 triangles	
II	Pyramid	Tetrahedron	4 triangles	Platonic
III	Cube	Cube	6 squares	Platonic
IV	Octahedron	Octahedron	8 triangles	Platonic
V	Dodecahedral body	Dodecahedron	12 pentagons	Platonic
VI	Icosahedron	Icosahedron	20 triangles	Platonic
VII	Irregular body born from the pyramid	Truncated tetrahedron	4 triangles 4 hexagons	Archimedean
VIII	A body that is born from the cube and from the octahedron	Cuboctahedron	8 triangles 6 squares	Archimedean
IX	Irregular body that is born from the cube	Truncated cube	8 triangles 6 octagons	Archimedean
X	Another body that is born from the cube	Small rhombicuboctahedron (rhombitruncated cuboctahedron)	8 triangles 18 squares	Archimedean
XI	A body that is born from the octahedron	Truncated octahedron	6 squares 8 hexagons	Archimedean
XII	A body that is born from the dodecahedron and from the icosahedron	Icosidodecahedron	20 triangles 12 pentagons	Archimedean
XIII	Another body that is born from the dodecahedron	Truncated dodecahedron	20 triangles 12 decagons	Archimedean

XIV	Another body that is born from the icosahedron	Truncated icosahedron	12 pentagons 20 hexagons	Archimedean
XV	Another body that is born from a composite body	Great rhombicuboctahedron (rhombitruncated cuboctahedron)	12 squares 8 hexagons 6 octagons	Archimedean
XVI	Another body that is born from a composite body	Rectified truncated octahedron	24 [isosceles] triangles 6 squares 8 hexagons	Symmetrohedron
XVII	Another composite body	Small rhombicosidodecahedron (rhombicosidodecahedron)	20 triangles 30 squares 12 pentagons	Archimedean
XVIII	Another body	Rectified truncated icosahedron	60 [isosceles] triangles 12 pentagons 20 hexagons	Near-miss Johnson solid
XIX	Another body	Truncated pentakis dodecahedron	12 pentagons 80 hexagons [20 regular and 60 irregular]	Goldberg polyhedron
XX	Another irregular body	Truncated truncated icosahedron	60 triangles 12 decagons 20 dodecagons	Symmetrohedron
XXI	Another irregular body	Great rhombicosidodecahedron (rhombitruncated icosidodecahedron)	30 squares 20 hexagons 12 decagons	Archimedean
XXII	Another irregular body	Truncated truncated octahedron	24 [isosceles] triangles 6 octagons 8 dodecagons	Convex polyhedron
XXIII	Another irregular body	Great rhombicosidodecahedron augmented with pentagonalcupolas	60 triangles 90 squares 12 pentagons 20 hexagons	Concave polyhedron

XXIV	Another irregular body	Great rhombicuboctahedron augmented with square cupolas	24 triangles 42 squares 8 hexagons	Concave polyhedron
XXV	Another irregular body	Unidentified	36 squares 24 hexagons 6 octagons 8 dodecagons	Not a polyhedron
XXVI	A body based on the pyramid	Elevated tetrahedron	12 triangles	Elevated Platonic
XXVII	A body based on the cube	Elevated cube	24 triangles	Elevated Platonic
XXVIII	A body based on the octahedron	Elevated octahedron	24 triangles	Elevated Platonic
XXIX, XXXIV	A body based on the dodecahedron	Elevated dodecahedron	60 triangles	Elevated Platonic
XXX, XXXIV	A body based on the icosahedron	Elevated icosahedron	60 triangles	Elevated Platonic
XXXI	Another body based on an irregular body	Elevated cuboctahedron	48 triangles	Elevated Archimedean
XXXII	A body resting on an underlying body	Elevated small rhombicuboctahedron	96 triangles	Elevated Archimedean
XXXIII	Another body	Elevated icosidodecahedron	120 triangles	Elevated Archimedean
XXXIV	Some previous bodies with pyramids added:			
	A body based on the dodecahedron	Partially elevated dodecahedron (only some faces elevated)		
	A body based on the icosahedron	Partially elevated icosahedron (only some faces elevated)		
	A body based on the icosidodecahedron	Partially elevated icosidodecahedron (only the triangular faces elevated)		

Appendix B: Comparison of studies of polyhedra

Solid	**Piero, *Trattato d'abaco* (Ash. 359)**	**Piero, *De prospettiva pingendi,* (Parm. 1576)**	**Piero, *Libellus* (Vat. Urb.632)**	**Pacioli, *Summa* 1494 (Chap. 10)**	**Pacioli, *Divina Proportione* 1509 (Pars Prima, Tractatus secundus)**	**Dürer, *Underweysung* 1525 (Book IV)**	**Hirschvogel *Geometria* 1543 (Part II)**	**Barbaro, *Pratica* 1568 (Part III)**	**Kepler, *Harmonices Mundi* 1619**
Tetrahedron	Fols. 105r-v **3D-L**	Fols. 17v-18r, 43r-v **3D-L**	Fols. 24v-29r **3D-L**	pp. 68v-69r **P, 3D-L** pp. 72r-72v **3D-L**	pp. 9r-10v **3D-L** pp. 10v-11v **P, 3D-L** Figs. I-II **3D-S**	Fig. 29 **N, P**	Figs. 1-2 **P, N, 3D-S**	Chap. II **P, N, 3D-L, 3D-S**	
Cube	Fols. 106r-v **3D-L**		Fols. 29v-30r **3D-L**	p. 69r **3D-L** p. 72v **3D-L**	pp. 10v-11r **P** Figs. VII-VIII **3D-S**	Fig. 32 **N, P**	Figs. 7-8 **P, N, 3D-S**	Chap. III **P, N, 3D-L, 3D-S**	
Octahedron	Fol. 107v **P** Fols. 108r-v **P3D-L** Fol. 114v **P**		Fols. 30v-31v **P**		p. 11v **P** Figs. XV-XVI **3D-S**	Fig. 30 **N, P**	Figs. 3-4 **P, N, 3D-S**	Chap. IV **P, N, 3D-L, 3D-S**	
Dodecahedron	Fol. 110v **3D-L**		Fols. 32r-34r **3D-L**	p. 70r **3D-S** p. 70v **3D-L**	pp. 12r-12v **3D-L** Figs. XXVII-XXVIII **3D-S**	Fig. 33 **N, P**	Fig. 9-11 **P** (4), **3D-S** (2)	Chap. V **P** (3), **N, 3D-L, 3D-S**	

Legend: **N**, net; **P**, plan; **3D-L**, line representation in three dimensions; **3D-S**, shaded representation in three dimensions

Icosahedron	Fol. 112r **3D-L**		Fols. 35r-37r **3D-L**		pp. 13r-v **3D-L** Figs. XXI-XXII **3D-S**	Fig. 31 **N, P**	Figs. 5-6 **P, N, 3D-S**	Chap. IV **P** (3), **N, 3D-L, 3D-S**	
Truncated tetrahedron				p. 69v **3D-L** (3) ?	Figs. III-IIII **3D-S**	Fig. 35 **N**	Figs. 21-23 **P** (2), **N, 3D-S**	Chap. VII **P, N, 3D-L**	p. 64
Cuboctahedron					Figs. XXXV-XXXVI **3D-S**	Fig. 37 **N**	Fig. 27-29 **P** (2), **N, 3D-S** (2)	Chap. VIII **P** (3), **N, 3D-L** (2) (+ 3D-L solid in a cube)	p. 62
Truncated cube					Figs. XIIII-X **3D-S**	Fig. 36 **N**	Figs. 24-26 **P** (2), **N, 3D-S** (2)	Chap. IX **P** (2), **N, 3D-L** (2)	p. 64
Small rhombicuboctahedron				p. 70r **3D-L**		Fig. 39 **N**	Figs. 35-37 **P** (2), **N, 3D-S**	Chap. X **P** (3), **N, 3D-L** (3) (+ 3D-L solid in a cube)	p. 62
Snub cube						Fig. 40 **N**			p. 62
Snub dodecahedron									p. 62
Truncated octahedron					Figs. XVI-XVII **3D-S**	Fig. 38 **N**	Figs. 32-24 **P** (2), **N, 3D-S** (2)	Chap. XI **P** (2), **N, 3D-L** (2)	p. 64

Legend: **N**, net; **P**, plan; **3D-L**, line representation in three dimensions; **3D-S**, shaded representation in three dimensions

Icosidodecahedron							Fig. 18-19 **P** (2), **N**, **3D-S** (2)	Chap. XII **P** (2), **N**, **3D-L** (2)	p. 62
Truncated dodecahedron					Figs. XX-VIIII-XXX **3D-S**			Chap. XIII **P** (2), **N**, **3D-L** (2)	p. 64
Truncated icosahedron					Figs. XXIII-XXIIII **3D-S**			Chap. XIV **P** (2), **N**, **3D** (2)	p. 64
Great rhombicuboctahedron						Fig. 41 **N**		Chap. XV **P, N, 3D-L**	p. 64
Truncated truncated cube						Fig. 42 **N**			
Rectified truncated octahedron								Chap. XVI **P, N, 3D-L**	
Small rhombicosidodecahedron								Chap. XII **P, N, 3D-L**	p. 62
Rectified truncated icosahedron								Chap. XVIII **N**	
Truncated pentakis dodecahedron								Chap. XIX **N**	

Legend: **N**, net; **P**, plan; **3D-L**, line representation in three dimensions; **3D-S**, shaded representation in three dimensions

Truncated truncated icosahedron								Chap. XX **N**	
Great rhombiicosidodecahedron								Chap. XXI **N**	p. 64
Truncated truncated octahedron								Chap. XXII **N**	
Great rhombicosidodecahedron augmented with pentagonal cupolas								Chap. XXIII **N**	
Great rhombicuboctahedron augmented with square cupolas								Chap. XXIV **N**	
Unidentified								Chap. XXV **N**	
Elevated tetrahedron					Figs. V-VI **3D-S**			Chap. XXVI	
Elevated cube					Figs. XI-XII **3D-S**			Chap. XXVII **N**	

Legend: **N**, net; **P**, plan; **3D-L**, line representation in three dimensions; **3D-S**, shaded representation in three dimensions

Elevated octahedron					Figs. XIX-XX **3D-S**			Chap. XXVIII **N**	
Elevated dodecahedron								Chap. XXIX **N**	
Elevated icosahedron					Figs. XXXI-XXXII **3D-S**			Chap. XXX **N**	
Elevated cuboctahedron					Figs. XXXVII-XXXVIII **3D-S**			Chap. XXXI **N**	
Elevated small rhombicuboctahedron								Chap. XXXII **N**	
Elevated icosidodecahedron					Figs. XXV-XXVI **3D-S**			Chap. XXXIII **N**	
								Chap. XXXIV **3D** (3)[1]	
Elevated truncated cube					Figs. XIII-XIIII **3D-S**				
Elevated truncated icosahedron					Figs. XXXIII-XXXIV **3D-S**				

Legend: **N**, net; **P**, plan; **3D-L**, line representation in three dimensions; **3D-S**, shaded representation in three dimensions

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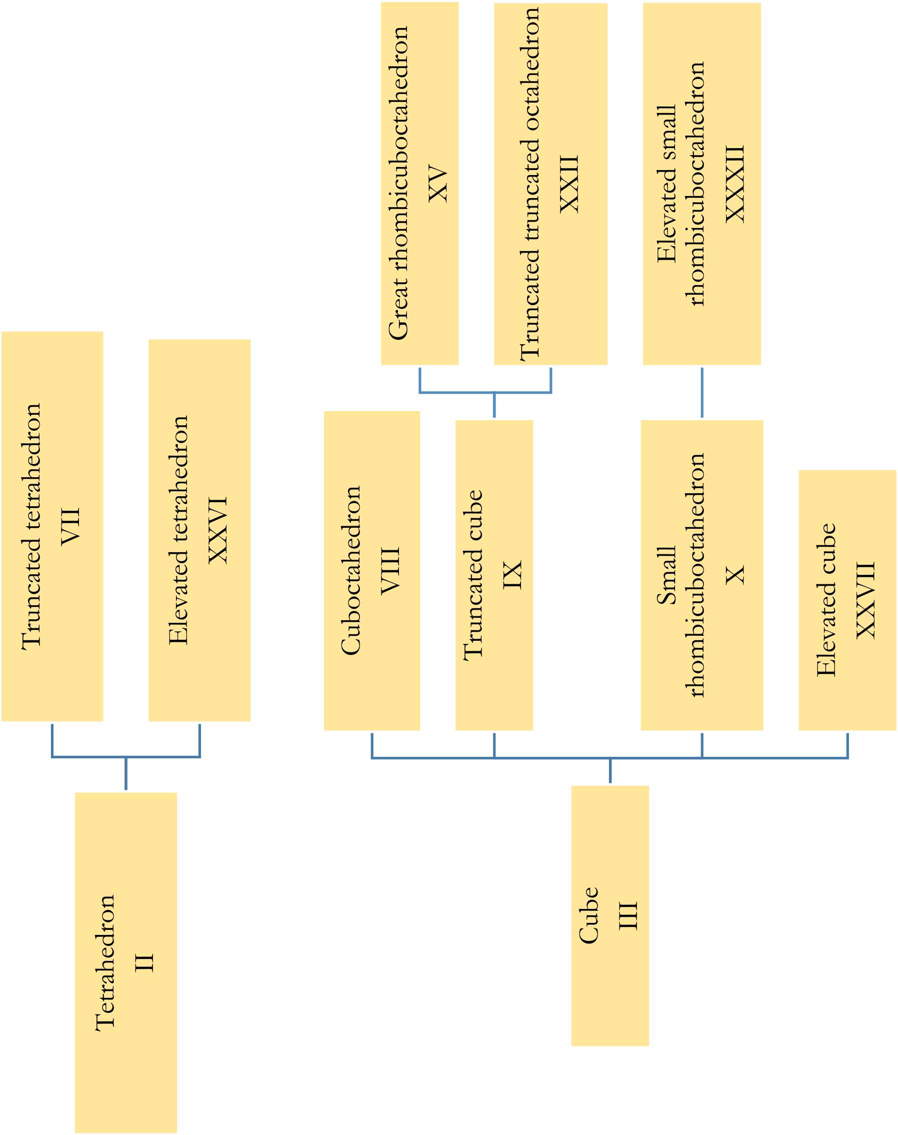
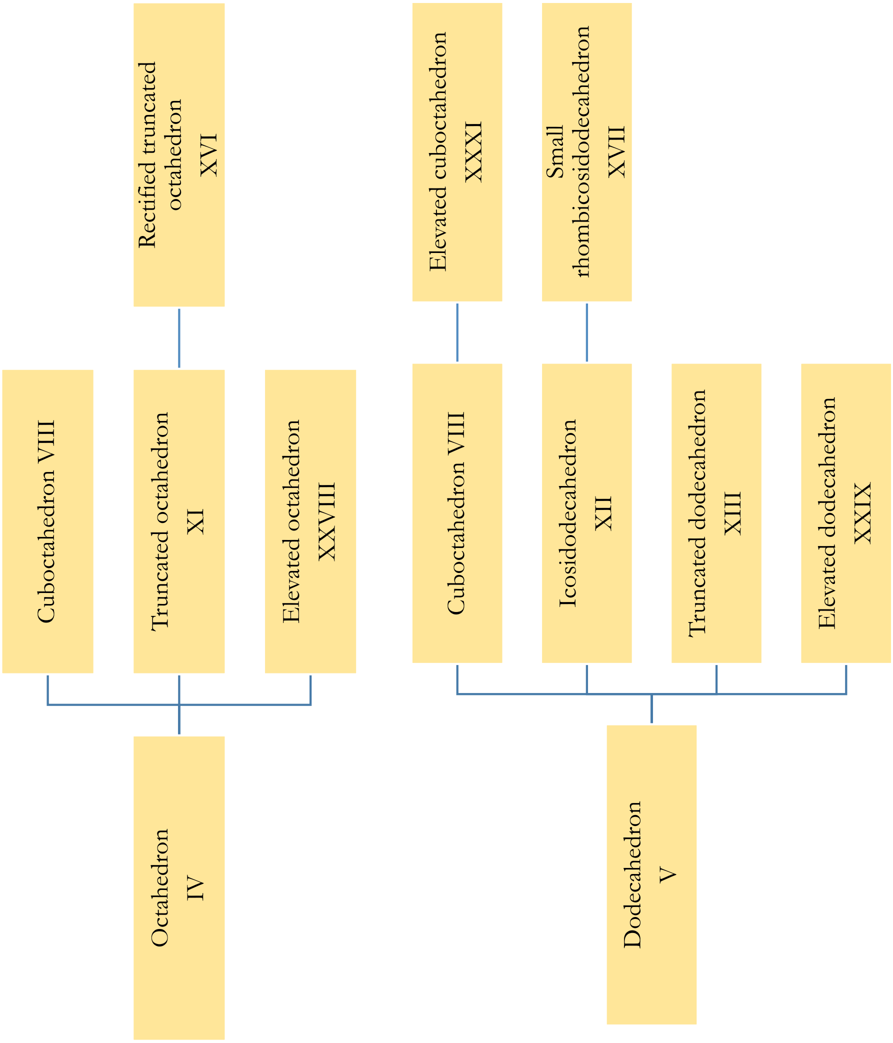
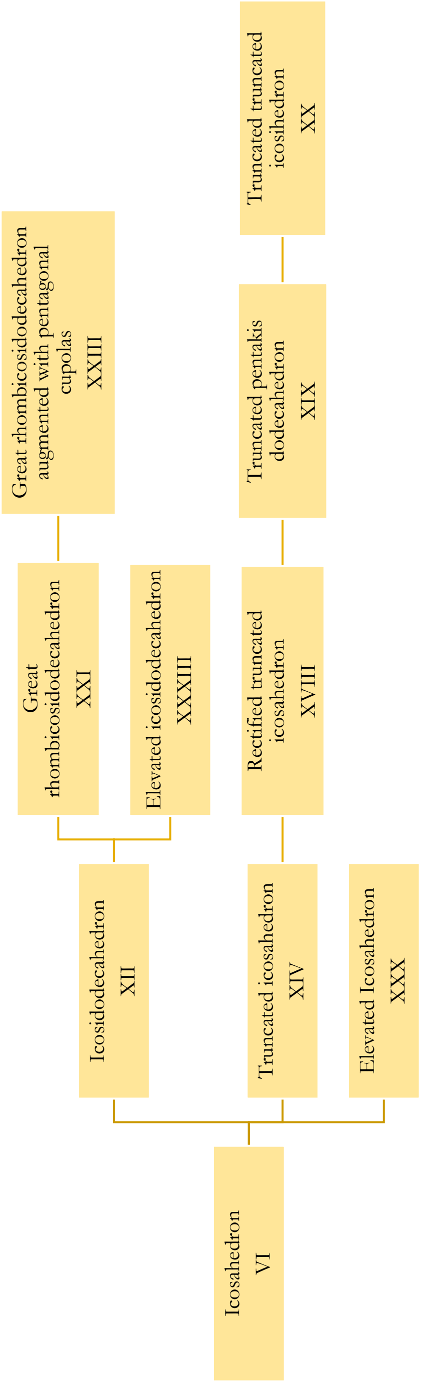
[1] The 3D figures in Chap. 34 show the dodecahedron with pentagons elevated with pyramids; the icosahedron with triangular faces elevated with pyramids; the icosidodecahedron with triangular faces elevated with pyramids.

Appendix C: The 'family tree' of Barbaro's solids

The following chart shows the hierarchy of solids presented in Barbaro's Part Three (Roman numerals indicate the relative chapter)


Tetrahedron II
Truncated tetrahedron VII
Elevated tetrahedron XXVI
Cube III
Cuboctahedron VIII
Truncated cube IX
Small rhombicuboctahedron X
Elevated cube XXVII
Great rhombicuboctahedron XV
Truncated truncated octahedron XXII
Elevated small rhombicuboctahedron XXXII


Dodecahedron
V
Cuboctahedron VIII
Icosidodecahedron
XII
Truncated dodecahedron
XIII
Elevated dodecahedron
XXIX
Elevated cuboctahedron
XXXI
Small
rhombicosidodecahedron
XVII
Octahedron
IV
Cuboctahedron VIII
Truncated octahedron
XI
Elevated octahedron
XXVIII
Rectified truncated
octahedron
XVI


Icosahedron
VI
Icosidodecahedron
XII
Truncated icosahedron
XIV
Elevated Icosahedron
XXX
Great rhombicosidodecahedron
XXI
Elevated icosidodecahedron
XXXIII
Rectified truncated icosahedron
XVIII
Great rhombicosidodecahedron augmented with pentagonal cupolas
XXIII
Truncated pentakis dodecahedron
XIX
Truncated truncated icosihedron
XX

Daniele Barbaro's Innovations in Perspective Studies

Cosimo Monteleone

Introduction

When Daniele Barbaro published his treatise in 1568 there had not yet appeared a text on perspective that was without errors and ambiguities. To be sure, the excellent manuscript of Piero della Francesca circulated among the ateliers of the artists, at least in Italy, but both the book by Sebastiano Serlio (1545) and that by Albrecht Dürer (1538), to quote Barbaro, 'stopped at the threshold of the portal'.[1] In any case, no one had yet highlighted all the possible applications, both practical and theoretical, of perspective.

There is no need to analyse in detail Barbaro's debt to his predecessors; this has been done in a recent publication.[2] Here it is sufficient to reiterate that *The Practice of Perspective*, while gathering the theoretical principles formulated by other authors, is situated within the editorial panorama of the Cinquecento as a manual of an encyclopaedic nature, presenting applications of the material, even in other disciplinary fields, for the first time. Furthermore, his contemporaries expected from Barbaro, the age's most celebrated translator and commentator of Vitruvius, an original contribution, as the patriarch of Aquileia, Giovanni Grimani, made clear in a conversation that took place in Trento in 1563 among a group of friends:

> …the world will have much more to say, seeing his book on perspective, which will soon be brought to light, in which with such diligence he has treated that science which is most noble in itself and necessary to diverse arts, that it seems that his studies

---

[1] Barbaro (1568, Foreword, p. 1): *si sono fermati sopra il limitare della porta*. This and all translations of passages from the treatise are from this present edition.

[2] Monteleone (2020), in particular on Piero della Francesca see pp. 59, 71-72, 76-78, 83-84, 85-87, 142-144, 182; on Dürer pp. 57-60, 85-87, 111, 119-123, 142-144, 149-153, 167-174; on Serlio pp. 61-62, 84-85, 94, 137-144.

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0_2

> have produced the ultimate amplification, and arrived at its perfection.[3]

These words demonstrate the high opinion generally held by his contemporaries of Barbaro's capacity to deepen and advance scientific knowledge.

What then were the innovations that Barbaro brought to perspective? After many years of studying this subject, I believe we can compile a list of arguments that are distinguished for their originality:

- the *giusta distanza* (correct distance) and the unicity of the viewing point (end of Part I);
- the representation of polyhedra (Part III);
- the creation of theatrical scenes according to the method of Pompeo Pedemonte (Part IV);
- anamorphosis and the *tabula scalata* (Part V);
- perspective machines (Part IX);
- the *horario universale* (universal clock) (Part IX).

This list does not include other themes addressed in the treatise that are important but less original, such as: basic perspective theory, drawn from *perspectiva naturalis* (Part I) and the manuscript of Piero della Francesca (Part II); geographic representation (Part VI), derived from Federico Commandino's Latin translation of Ptolemy's *Planisphaerium*; questions relative to light, shadow and colour (Part VII), which retrace considerations that were widespread at the time;[4] the representation of the human body (Part VIII), inspired by Dürer and Piero della Francesca.

In what follows we will examine in detail these six most original contributions of Barbaro.

The *giusta distanza* and the unicity of the viewing point

Before concluding Part I, dedicated to the optical and geometrical foundations of perspective, Barbaro addresses the problem of the *giusta distanza* (correct distance), a question intimately tied to the unicity of the viewing point. His reflections constitute an answer to the (unresolved) anomaly that had been raised by Piero della Francesca regarding distortion at the edges generated by an angle of vision greater than 90°. By means of drawings that were completely new, Barbaro provides the reader with the means to avoid this inconvenient problem in all situations, both when the principle point falls within the area of the painting as well as when it falls outside of it. These diagrams, very simple to carry out graphically, synthesise all that occurs in space after a series of rotations of the eye of the observer in such a way that the pictorial representation is always maintained within an optical cone that does not exceed 90°. The rotations of geometric entities are mathematical operations that Barbaro had been accustomed to since his university days, when he studied Euclid and astronomy with Federico Delfino.[5]

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[3] Paruta (1579, p. 28): …*et molto piu'* ne *dira'* il mondo, *veggendo i suoi libri della Perspettiva, che tosto usciranno alla luce: ne'quali con tal diligenza ha trattato di quella scienza per sé nobilissima, et a diverse arti necessaria, che pare, che gli studij di lui le habbiano recato l'ultimo accrescimento, et la compiuta sua perfettione.*

[4] For example, similar discussions are found in the *De pictura* of Leon Battista Alberti (2004, pp. 44, 82-83).

The concept can be made clear by looking at one of Barbaro's diagrams, for example, *Figura 6* of the treatise (Fig. 1). Contextualising it in space (Fig. 2), the hypotenuse *ha* of the isosceles triangle *hfg*, whose right angle is in *f*, coincides with the outermost visual ray coming from the eye *h* and passing through *a*, the most distant point of the painting. Rotating hypotenuse *ha* about the cathetus *hf* forms an optical cone of 90° that always comprises within itself the picture observed, avoiding aberrations at the edges.

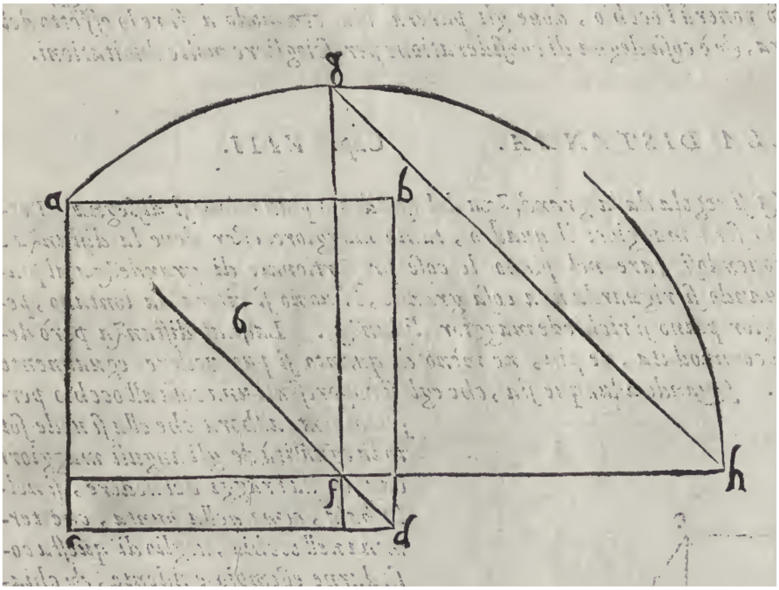

Fig. 1. *Figura 6*: the *giusta distanza* (Barbaro 1568, p. 20)

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[5] Rose (1969, p. 198); Rose (1975, pp. 11, 22, 53, 162, 284ff., 290).

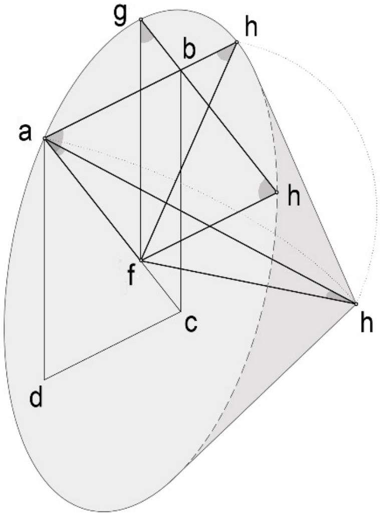

Fig. 2. Interpretation in space of *Figura 6*

The problem of the *giusta distanza* is followed by the discussion of the unicity of the viewing point. Barbaro is the first in the history of perspective to treat this subject, presented in his *Figura 11* (Fig. 3), inspired by the illustrations of Euclid’s *Elements*, widespread among the schools of artists, the so-called abacus schools.[6]

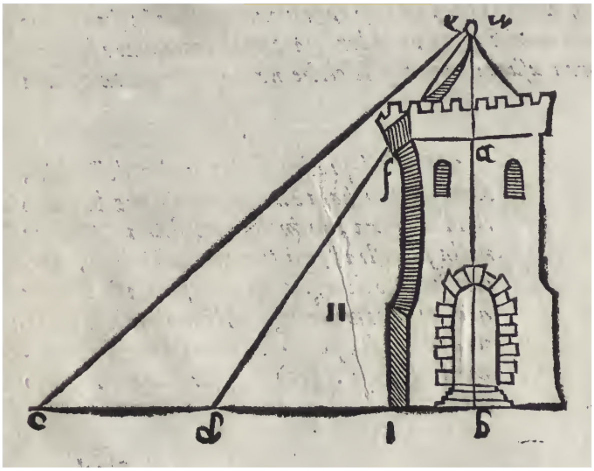

Fig. 3. Barbaro’s *Figura 11*, demonstrating the unicity of the viewing point (Barbaro 1568, p. 22)

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[6] Camerota (2006, pp. 11-14).

For Barbaro, there exists a perfect superimposition of the retinal image of reality with the perception of a painting that depicts it. This condition is verified only when the viewing point of the painter and the observer coincide. The problem of the unicity of the viewing point had been intuited by other scholars but the problem had never been given such a clear and detailed analytical exposition. If the painter does not fix the *giusta distanza* from which the painting is to be perceived, each time the observer moves away from the established point his gaze will fall upon certain details that are erroneous, and therefore, in incorrect perspective. In his *Figura 11*, the optimal distance corresponds to the segment *bc*: it should be noted that from point *c* the painter sees the cross at the top of the tower, marked with point *e*. If the observer occupies the same position, then he too correctly sees the cross. The situation changes if instead the observer moves to point *d*, because in this case he will continue to see the cross in the painting, even though in reality it will be hidden by the projection of the cornice (point *f*). The chapters dedicated to the *giusta distanza* and the unicity of the viewing point are fundamental in terms of theory because, as we shall see below, it is precisely the discussions about these topics that will pave the way in the 1600s for the dissemination of anamorphosis.

The representation of polyhedra

Part III is the section of the treatise in which Barbaro undoubtedly presents the greatest number of innovations in various senses. Kim Williams in this same volume deals with important scientific aspects of this, such as the manner of deriving semi-regular polyhedra and their elevations and augmentations.[7] Here instead we will analyse the novel didactic strategy that Barbaro associates with the exercises with the solids.

After having indicated the basic rules of perspective in Parts I and II, Barbaro addresses the representation of the solids, by means of *spiegatura* (unfolding), *perfetto* (the perfect), *digradato* (the degraded), *dritto* (the rising), and *adombratione* (shading), each of which has a precise role to play. The *spiegatura* corresponds to the development of the solid and provides the reader with physical information such as the shape and number of faces, and the characteristics of the edges and angles. It also makes it possible to construct a material solid, starting from 'descriptions of the opened figures from which the solid figures are made by folding them together to demonstrate the true form'.[8] The *perfetto* is the plan of the solid, a drawing that can be executed mechanically with ruler and compass according to the instructions that Barbaro provides. The *digradato* corresponds to the perspective of the plan, which can be obtained by applying the rules of perspective described in Part II. The *dritto* is a completely new kind of diagram that is derived from the *perfetto* by means of a rotation onto a vertical line of the significant dimensions of the solid, in order to find the heights of the planes that contain its vertices. Practically speaking, the *dritto* is a drawing that augments the perfect, making a representation of the elevation superfluous. Finally, the *adombratione* is an image of synthesis that provides a perspective representation of the solid that alludes to reality thanks to hatching.

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[7] In this present volume, pp. 16-23.

Except for the *dritto* the elements listed above are not drawings that were completely new in Renaissance manuals. However, here for the first time we find them gathered together and, most importantly, organised according to a precise order. Further, while it is true that the unfolding of polyhedra had been introduced in Albrecht Dürer's *Underweysung der Messung*, the *perfetto* and the *digradato* in Piero della Francesca's *De prospectiva pingendi*, and the shading in Luca Pacioli's *Summa* and *Divina proportione*, it must be acknowledged that the substitution of the elevation by the *dritto* is a genuine innovation in the world of representation.

Before Barbaro the perspectives of complex objects (such as polyhedra) could be carried out with a method described by Piero della Francesca in *De prospectiva pingendi* to foreshorten the mazzocchio and human heads (Fig. 4).[9] Barbaro, by substituting the elevation with the *dritto*, provides an alternate strategy.

---

[8] Barbaro (1568, p. 45): *descrittioni delle figure aperte, delle quali si fanno i corpi sodi piegandole insieme per dimostrazione del vero…*

[9] After having drawn the plan and elevation of an object separately, its significant points must be united in the eye of the observer by means of lines; the intersection of these lines with the picture plane make it possible to derive two pieces of information for each point considered, a horizontal coordinate (in plan) and a vertical coordinate (in elevation); finally, by transferring and intersecting these measures in the picture plane it is possible to draw the perspective of the object. Barbaro describes Piero della Francesca's third method at the end of Part III, after dealing with polihedra (Barbaro 1568, pp. 114-128).

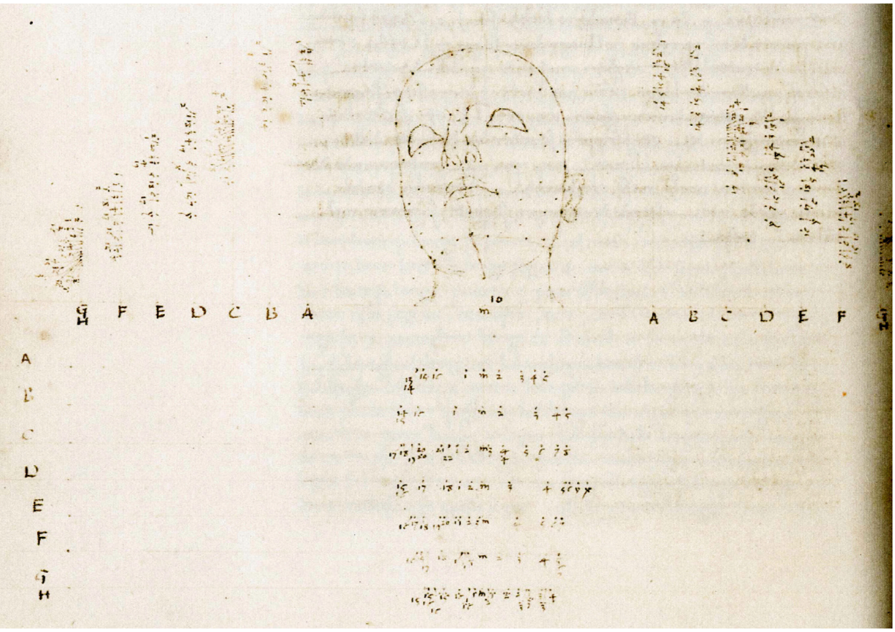


Fig. 4. Perspective of a head (Piero della Francesca, Mss. Regg. A 41/2, fol. 91v)

It will be helpful for the modern reader to provide a complete explanation of at least one of the exercises with polyhedra proposed by Barbaro in order to make it possible to fully understand this new method of constructing a perspective. To do this we will use the icosahedron, the Platonic solid that he presents in Part III, Chap. VI, starting our analysis from the beginning.

The exercise with the icosahedron, composed of 20 identical triangular faces, opens with the description of the *spiegatura*, providing an opportunity to highlight the duality of the icosahedron and the dodecahedron: each contains 30 edges, but the dodecahedron has 12 faces and 20 vertices, while the icosahedron has 20 faces and 12 vertices. These specular numerical characteristics translate geometrically in a process that makes it possible to derive one solid from another when the vertices of one polyhedron coincide with the centres of the faces of its dual (Fig. 5).

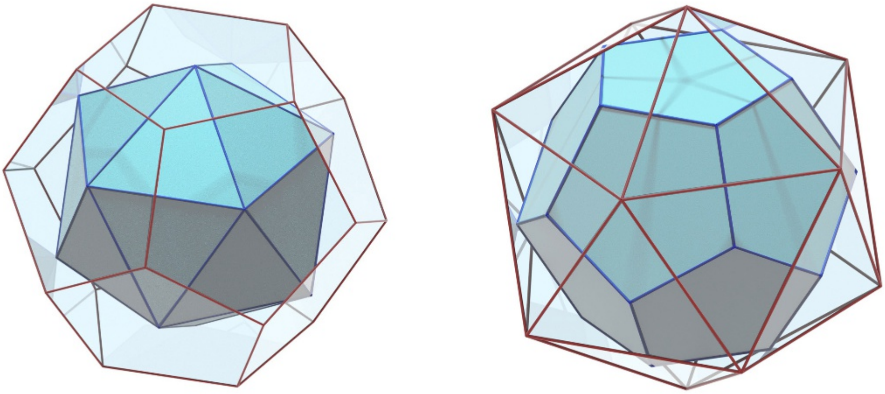


Fig. 5. The duality of the icosahedron and the dodecahedron

Barbaro then goes on to execute the *perfetto* of the icosahedron by means of two methods illustrated by his *Figure X* and *Figure Y*, providing a series of mechanical procedures to be carried out with ruler and compass. The sequence of operations to carry out the *perfetto* of *Figura X* (Fig. 6) consists of 8 steps (Fig. 7):

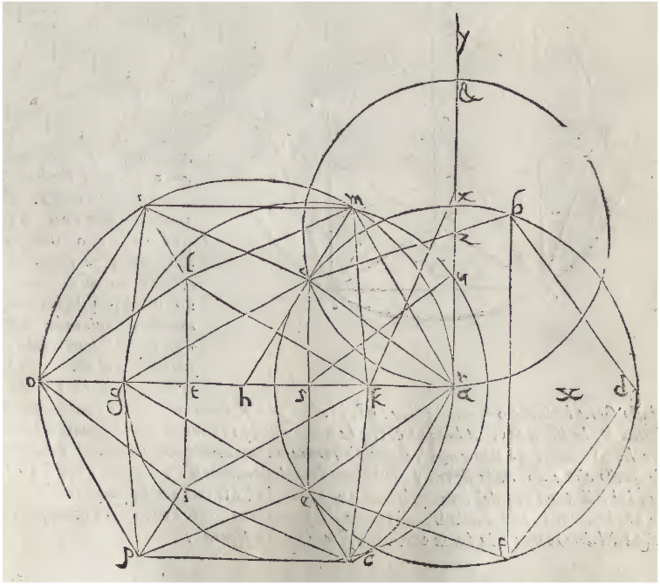
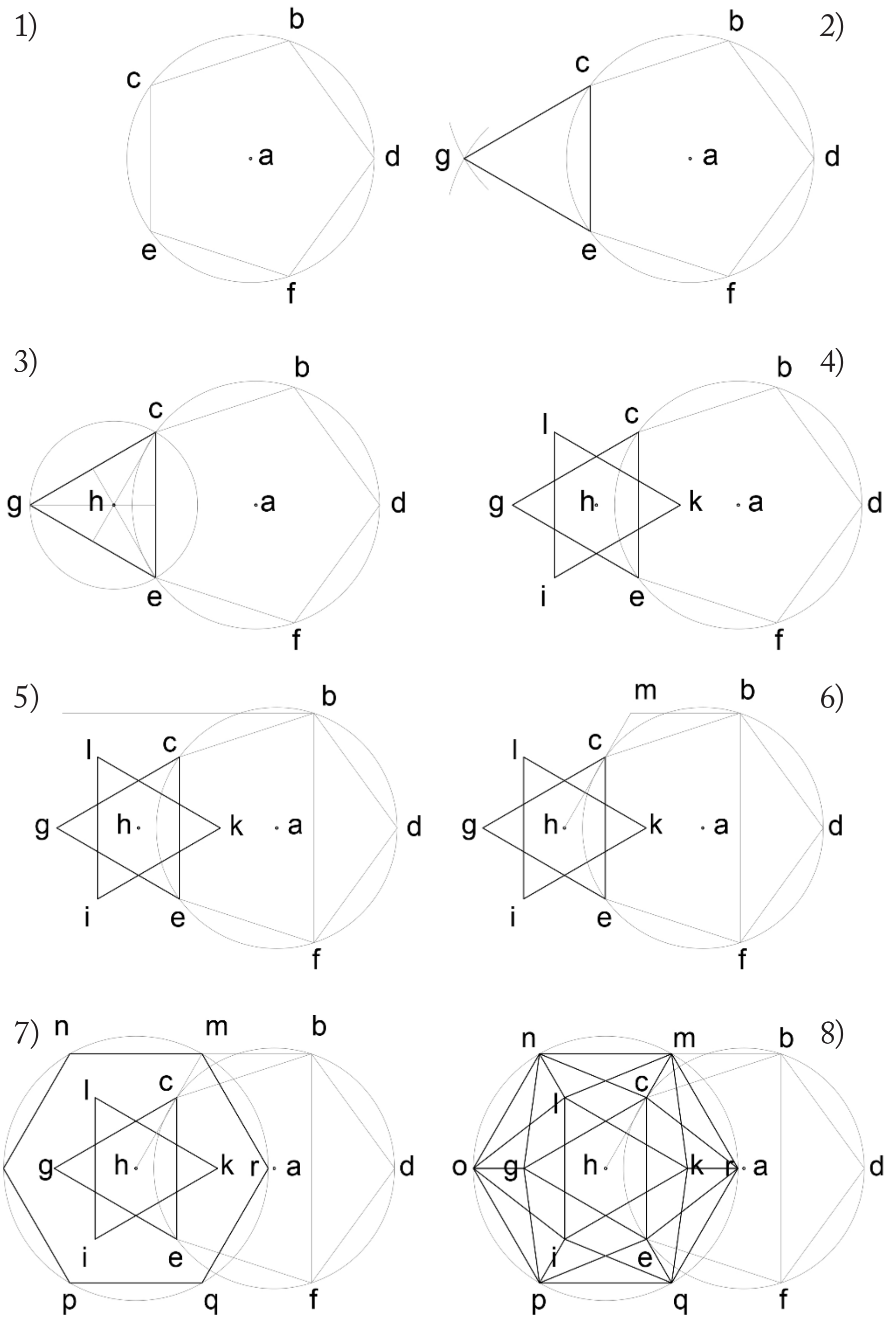


Fig. 6. Barbaro's *Figura X*: the *perfetto* and the *dritto* of the icosahedron resting on a triangle (Barbaro 1568, p. 53)


1)
b
c
a
d
e
f
2)
b
c
g
a
d
e
f
3)
b
c
g
h
a
d
e
f
4)
b
l
c
g
h
k
a
d
i
e
f
5)
b
l
c
g
h
k
a
d
i
e
f
6)
m
b
l
c
g
h
k
a
d
i
e
f
7)
n
m
b
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c
g
h
k
r
a
d
i
e
p
q
f
8)
n
m
b
c
l
o
g
h
k
r
a
d
i
e
p
q
f

Fig. 7. Construction of the *perfetto* of Barbaro's *Figura X*

1) Draw a pentagon *bcdef* inscribed in a circle with centre *a* (the construction begins with this geometric figure because each vertex of an icosahedron can be considered as the apex of a pyramid whose base is a regular pentagon).
2) Draw an equilateral triangle *ceg* starting from side *ce* of the pentagon (this effectively involves rotating one of the five lateral faces of the pentagonal pyramid onto a horizontal plane).
3) Inscribe equilateral triangle *ceg* in a circle whose centre is *h* (Barbaro calls this circle an 'occult circle'[10] and does not draw it; further, he does not tell the reader how to locate point *h*, which is the orthocentre of triangle *ceg*, that is, the intersection of the three altitudes of the triangle).
4) Draw the equilateral triangle *ikl* symmetrical to *ceg* with respect to orthocentre *h* (*ikl* corresponds to the equilateral triangle on which the icosahedron rests, while *ceg* is the equilateral triangle at its top. Since *ikl* rests on the plane of projection and *ceg* is parallel to it, both are in true form, that is, they do not undergo deformation with the projection).
5) Draw the lines *bf* and *bm* so as to form a 90° angle at *b* (line *bm* must be extended indefinitely in order to allow point *m* to be located in the next step).
6) Extend segment *hc* to intersect the indefinite line that starts at *b* to find point *m*.
7) Draw a circle with centre *h* and radius *hm* in which is inscribed a hexagon *mnopqr*.
8) In addition to the sides of triangles *ceg* and *ikl*, the *perfetto* of *Figura X* of the icosahedron is completed by the lines *rk*, *rc*, *re*, *rm*, *rq*, *nl*, *nc*, *nm*, *no*, *ng*, *pi*, *po*, *pg*, *pq*, *pe*, *mk*, *mc*, *ml*, *og*, *oi*, *ol*, *qi*, *qe*, and *qk.*[11]

Our Fig. 8 shows the *perfetto* (plan) of the icosahedron in a modern rendering, distinguishing with dotted lines the edges that are hidden from view.

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[10] An occult circle is one that is 'understood', or drawn for convenience and then erased; see Barbaro (1568, p. 38).

[11] The final step contains the correct list of the sides of the *perfetto* of *Figura X*, as we have identified some gaps in both the treatise as published (1568, p. 53) and in the manuscripts held in the Biblioteca Nazionale Marciana that preceded the publication: Barbaro (Ms. It. IV, 40=5447, fol. 29r); (Ms. It IV, 39=5446, fol. 40r and fol. 284r); (Ms. Lat. VIII, 41=3069, fol. 27r).

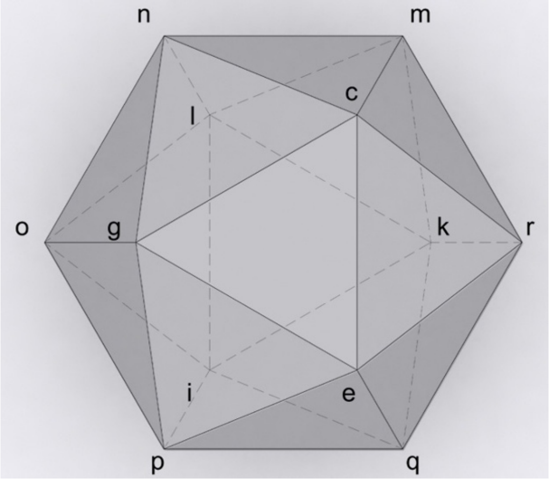

Fig. 8. The *perfetto* of Barbaro's *Figura X* using today's graphic conventions

Barbaro then goes forward, constructing, along with the *perfetto* of his *Figura X*, the *dritto*; that is, he locates the heights of the planes that contain the vertices. This time the mechanical procedure consists of 5 steps (Fig. 9):

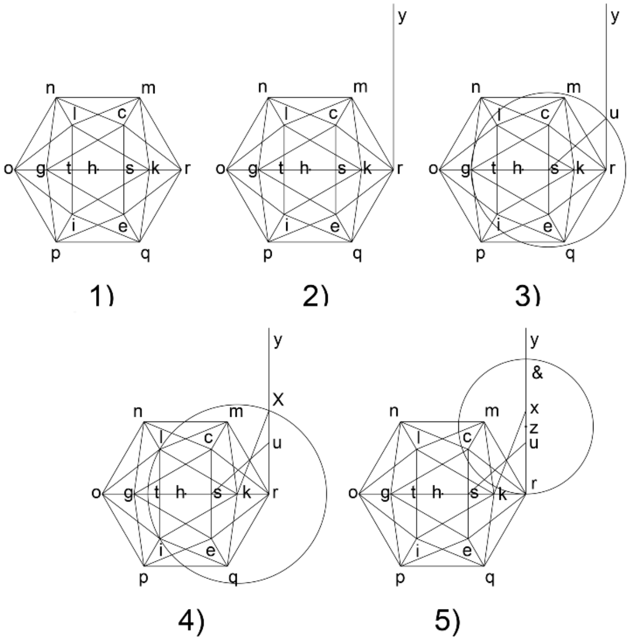

Fig. 9. Construction of the *dritto* of Barbaro's *Figura X*

1) Consider line *rg* that intersects sides *ce* and *il* respectively in points *s* and *t*.
2) Draw, starting at point *r*, a straight line *ry* perpendicular to *rg*.
3) Placing the compass at point *s*, rotate line *sg* onto *ry* to locate point *u* (*sg* is equal to the measure of the height of any equilateral triangle of the icosahedron; in this specific case it is the height of *ceg*).
4) Placing the compass in *k*, rotate line *ki* onto *ry* to locate point *x* (*ki* is equal to the measure of the side of any equilateral triangle of the icosahedron; in this specific case it is the side of *ikl*).
5) Divide segment *ux* into two at point *z* and draw the circle of radius *zr* that intersects the vertical segment *yz* at point & (*zr* is the distance between the triangular face *ikl* resting on the horizontal plane and the centre of the icosahedron; in consequence, *z*& is the distance from the centre to the uppermost triangle *ceg*).

In this way Barbaro has located on vertical segment *yz* the heights of the four planes on which lie, three by three, the twelve vertices of the icosahedron. Point *z* corresponds to the plane located at quota zero, and on it are found the three vertices of triangle *ikl*. Then follow points *m*, *o* and *q*, located at quota *u* and obtained by rotating the height of one of the triangles. At the next quota, *x*, are found the vertices *n*, *p* and *r*, located thanks to the rotation of a side of one of the triangles. Finally, if the lowest triangle resting on the horizontal plane is *ikl*, which is at distance *zx* from the centre of the icosahedron, then the uppermost triangle *ceg* will be at a height double that, and thus it will be found at quota &.

Barbaro then introduces his *Figura Y* with the aim of confirming the previous results. In terms of representation this approach makes Barbaro's treatment of the polyhedra unique. The reasoning is quite simple: when the position of a solid is modified in space, the relative distances between its faces must remain unchanged. Let us consider, for example, two opposing triangles in the icosahedron, a pink one *ikl* and a red one *ceg* (Fig. 10): the distance between them remains constant no matter whether the solid rests on *ikl* (position *a*) or on edge *il* (position *b*).

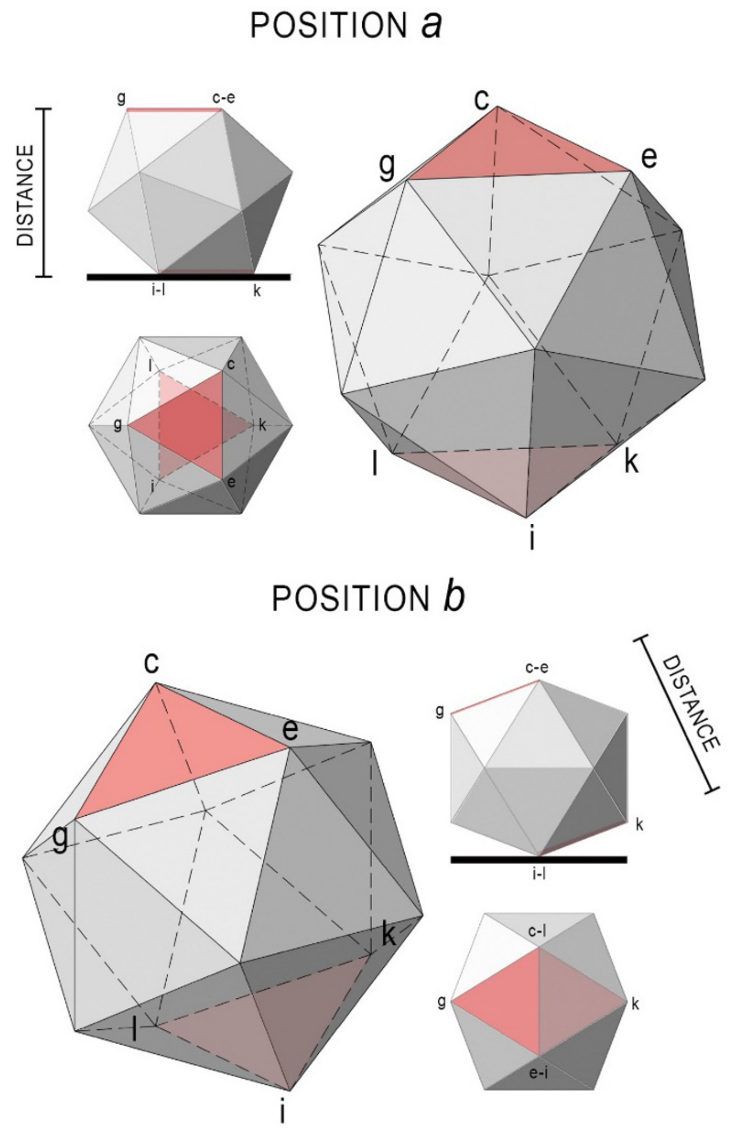
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Fig. 10. The distance between opposing triangles remains constant no matter what the position of the icosahedron

This is the reason why Barbaro, in addition to the first *perfetto* of *Figura X* (in which the solid rests on one of its faces, provides the construction of the *perfetto* of his *Figura Y* (in which the solid rests on one of its edges) (Fig. 11). The 4 steps to construct this second *perfetto* are (Fig. 12):

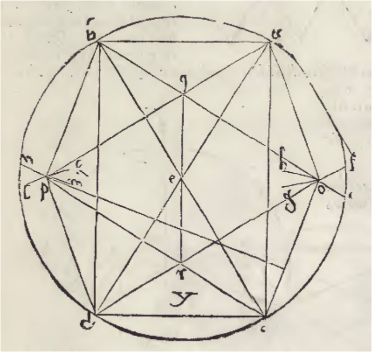

Fig. 11. Barbaro's *Figura Y:* the *perfetto* and the *dritto* of the icosahedron resting on an edge (Barbaro 1568, p. 54)

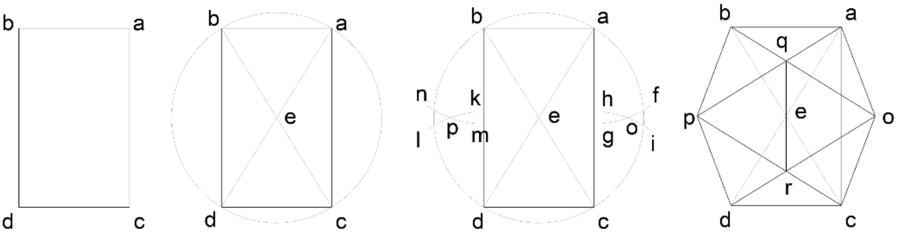

Fig. 12. Construction of the *perfetto* of Barbaro's *Figura Y*

1) From the construction of the *perfetto* of *Figura X* consider the measures *bf* and *ce*, respectively the diagonal of the pentagon[12] and the side of the equilateral triangles. With these measures construct rectangle *abcd*.
2) Locate centre *e* of rectangle *abcd* by means of its diagonals and draw the circle of radius *ae* (this circle corresponds to a great circle of the sphere that circumscribes the icosahedron).
3) From the construction of the *perfetto* of *Figura X* consider the distance *gs* (height of the triangles) and, placing the compass at the vertices of rectangle *abcd*, draw arcs *hi*, *gf*, *kl*, and *mn*, whose intersections locate points *p* and *o*.
4) Draw lines *ob*, *od*, *pa* and *pc*, whose intersections locate the points *q* and *r*, which mark the ends of the edge on which the solid rests, and in consequence, also the ends of the uppermost edge.

---

[12] Recall that each vertex of an icosahedron can be considered as the apex of a pyramid whose base is a regular pentagon.

Fig. 13 is a modern rendering of the plan of the icosahedron of *Figura Y*, showing 8 visible triangles that conceal another 8, while the 4 remaining triangles are represented by the sides *ao*, *oc*, *bp* and *pd*. To confirm the results obtained, Barbaro notes that the measure of segment *ps* (the red line in Fig. 13), drawn perpendicular to side *oc*, is equal to the total height *&r* of the icosahedron analysed when resting on a face in *Figura X*. The author specifies that this occurs because the *perfetto* of *Figura Y* was constructed using the same measures of the *perfetto* of *Figura X*, such as the diagonal of the pentagon, and the side and height of the triangles. The *perfetto* of *Figura Y*, apparently secondary, constitutes incontrovertible proof of Barbaro's ability to manage orthogonal projections, because this drawing coincides with the elevation of the icosahedron considered in the first position (Fig. 14). Logic thus leads to the conclusion that the measure *&r*, the total height of the icosahedron in the *dritto* of *Figura X*, corresponds to the distance *ps* of the *dritto* of *figura Y*.

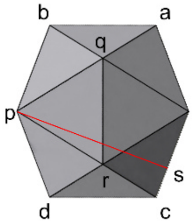

Fig. 13. The *perfetto* of Barbaro's *Figura Y* using today's graphic conventions

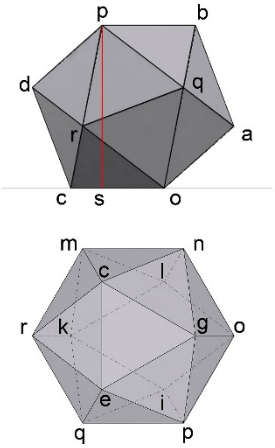

Fig. 14. Plan (the *perfetto* of *Figura X*) and elevation (the *perfetto* of *Figura Y*) of the icosahedron

The chapter on the icosahedron proceeds with the description of *Figura* &, a third way of finding the *perfetto* and the *dritto*, completely similar to that of *Figura X* (Fig. 15), and closes without further ado with two synthesising images: the wireframe perspective of the solid and its shaded representation (Fig. 16).

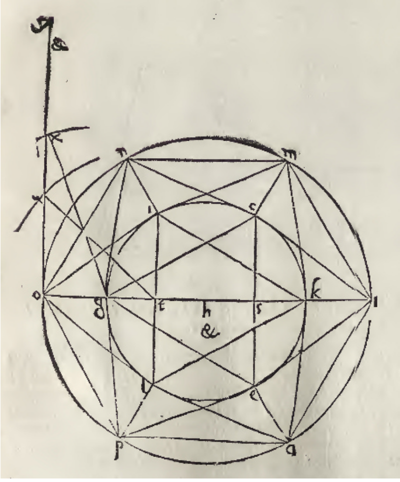

Fig. 15. Barbaro’s *Figura* &: a third way to find the *perfetto* and the *dritto* of the icosahedron (Barbaro 1568, p. 55)

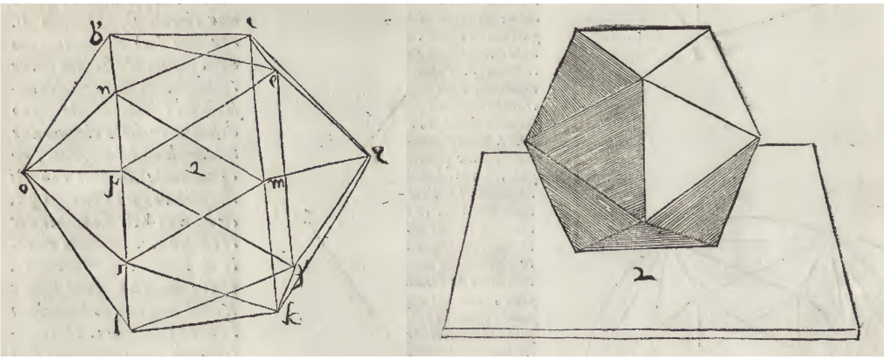

Fig. 16. Wireframe perspective and shaded representation of the icosahedron (Barbaro 1568, p. 56)

Contrary to what the reader probably expects, when Barbaro deals with the representation of the solids we do not find any trace of a construction to draw the actual appearance. Evidently, he holds it superfluous to go into the procedure of constructing the perspective itself, since this can be done thanks to the *digradato*, that is, by foreshortening the plane figures defined by the vertices of the icosahedron. Barbaro explains how to delineate the *digradato* of a plane figure at the end of Part II, in Chap. IX, entitled 'The plan of the triangle, and how the degraded is made from the perfect.[13] In essence, the *digradato* is a figure that results from a method that uses the diagonal of a square to delineate a foreshortening of any given plane figure, a method invented by Piero della Francesca[14] and later reworked by Albrecht Dürer[15] in order to avoid having to rotate the *perfetto* to below the ground line. Barbaro preferred the solution of the German artist (Fig. 17), perhaps not to confuse his readers with the rotation of the plane. In fact, it can be noted that in Barbaro's drawing, in contrast to the drawing of Piero, but in agreement with that of Dürer, Barbaro keeps the diagonals oriented in the same way in both the *perfetto* and *digradato*.

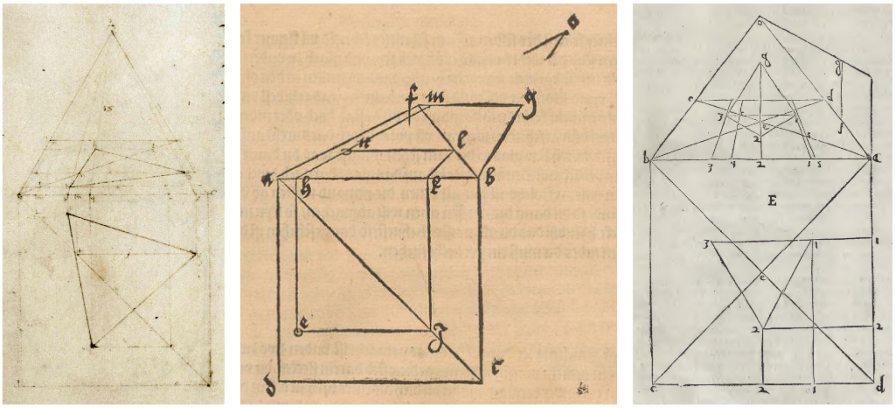

Fig. 17. The method of the diagonal. Left) Piero della Francesca (Mss. Regg. A 41/2, fol. 10v); centre) Dürer (1525, p. 90r); right) Barbaro (1568, p. 38)

In light of what has been said, let us complete the exercise of the icosahedron by decomposing the *perfetto* of *Figura X* into four smaller *perfetti*, each of which contains the vertices of the solid that that are found at the same quota (Fig. 18):

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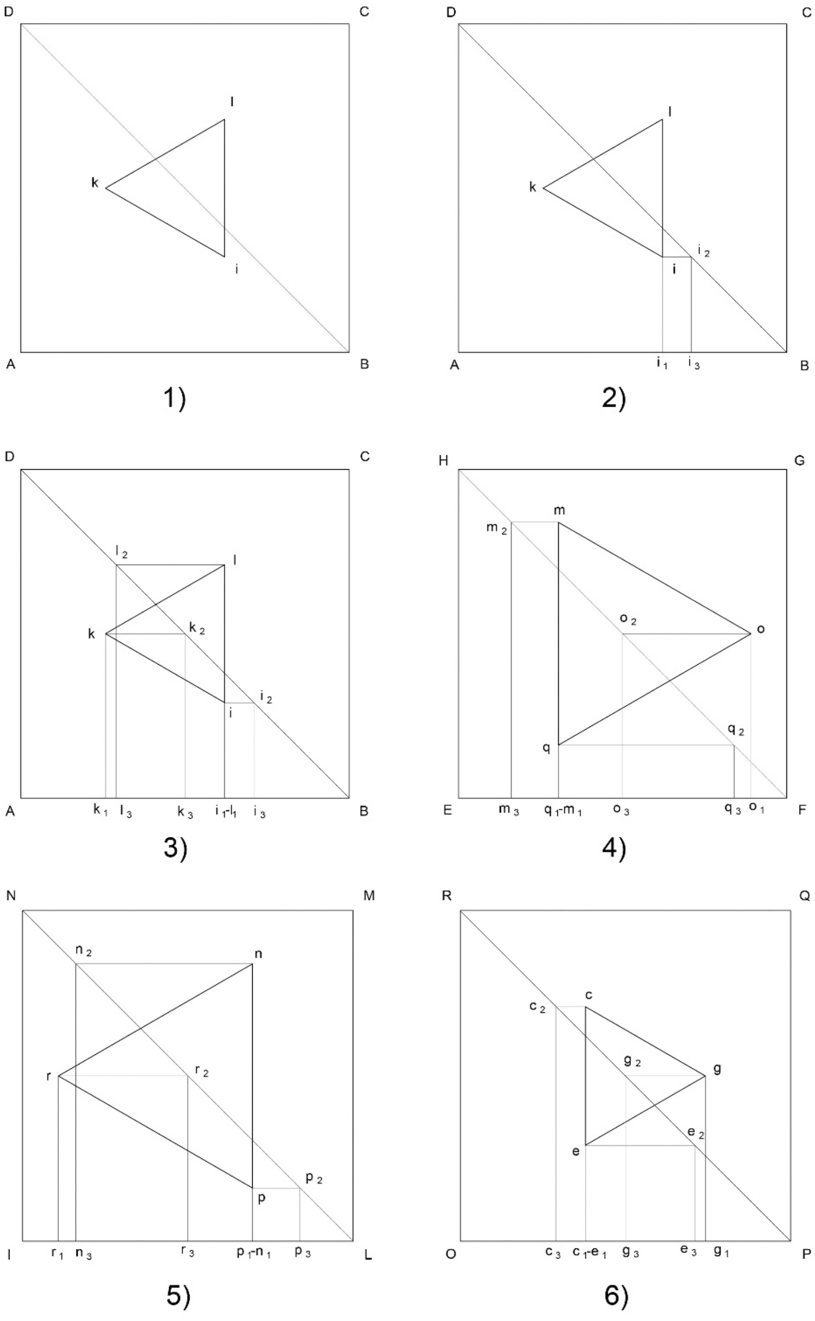
[13] Barbaro (1568, p. 38): *Pianta del triangulo, et come dal perfetto si fa il digradato*.

[14] Piero della Francesca (Mss. Regg. A 41/2: fols. 9r-12v).

[15] Dürer (1538, p. 90r).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6)

Fig. 18. The *perfetti* of *Figura X* relative to the vertices of the icosahedron that are located at the same quota

1) Place triangle *ikl*, on which the icosahedron rests (in the *perfetto* of *Figure X*, quota *r* equal to zero), in a square *ABCD*.
2) Draw the diagonal *BD* and, starting from vertex *i*, draw a vertical line that intersects side *AB* to locate point i_1, and a horizontal line that intersects diagonal *BD* to locate point i_2; finally, from i_2 draw a second vertical line that intersects *AB* at point i_3.
3) Continue in the same manner for the other vertices *k* and *l*.
4) Apply the same procedure for the vertices *m*, *o*, and *q* of the icosahedron (quota *u* obtained by rotating the height of a triangle of the icosahedron).
5) Apply the same procedure for the vertices *n*, *p*, and *r* of the icosahedron (quota *x* obtained by rotating the side of a triangle of the icosahedron).
6) Apply the same procedure for the vertices of triangle *ceg* (the uppermost triangle of the icosahedron located at quota &).

After having derived all the useful information from the *perfetti* thus constructed, it is possible to go on to the representation of their *digradati*. Let us start from triangle *ikl*, but before we do it is necessary to explain the meaning of Barbaro's *Figura C* of Part III (Fig. 19).[16] This image superimposes three elements – plan, side view and perspective – of two planes at different quotas. In plan the planes are represented by a single square *abcd*. The side view of these planes coincides with the two segments *cd* and *ie*. The segment *bd* indicates the square in profile, and point *o* the position of the eye of the observer. In consequence, the lines *oi* and *oc*, visual rays that intersect the square in two points (one marked *k*, the other not marked), make it possible to establish the depth of the planes. Finally, in perspective we note that the two planes show the sides *en* and *dh* in true size, while those parallel to the square converge in point ✠, the orthogonal projection of the eye of the observer onto the square.

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[16] Barbaro (1568, p. 44).

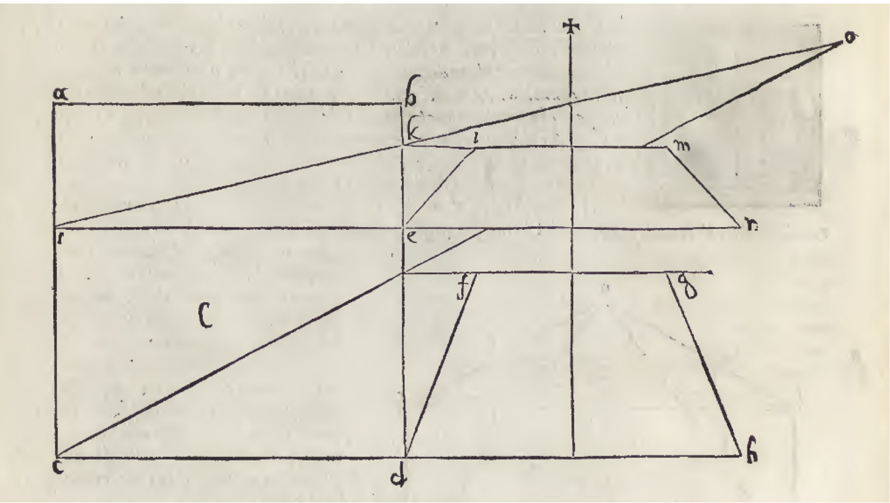

Fig. 19. Barbaro's *Figura C*: planes in perspective (Barbaro 1568, p. 44)

Given the manner of foreshortening perspectivally a square placed at any quota, with the method of the diagonals it is easy to go from the *perfetto* (plan) of a plane figure to its *digradato* (perspective). This construction was carried out for the icosahedron in Fig. 20, taking account of the quotas identified in the *perfetto* of *Figure X* and relative to the vertices of the solid, which form 4 triangles: *ikl* (quota *r*), *omq* (quota *u*), *npr* (quota *x*), and *ceg* (quota &). Finally, once the perspectival positions have been fixed for each vertex, it is only necessary to draw the edges that connect them to complete the exercise (Fig. 21).

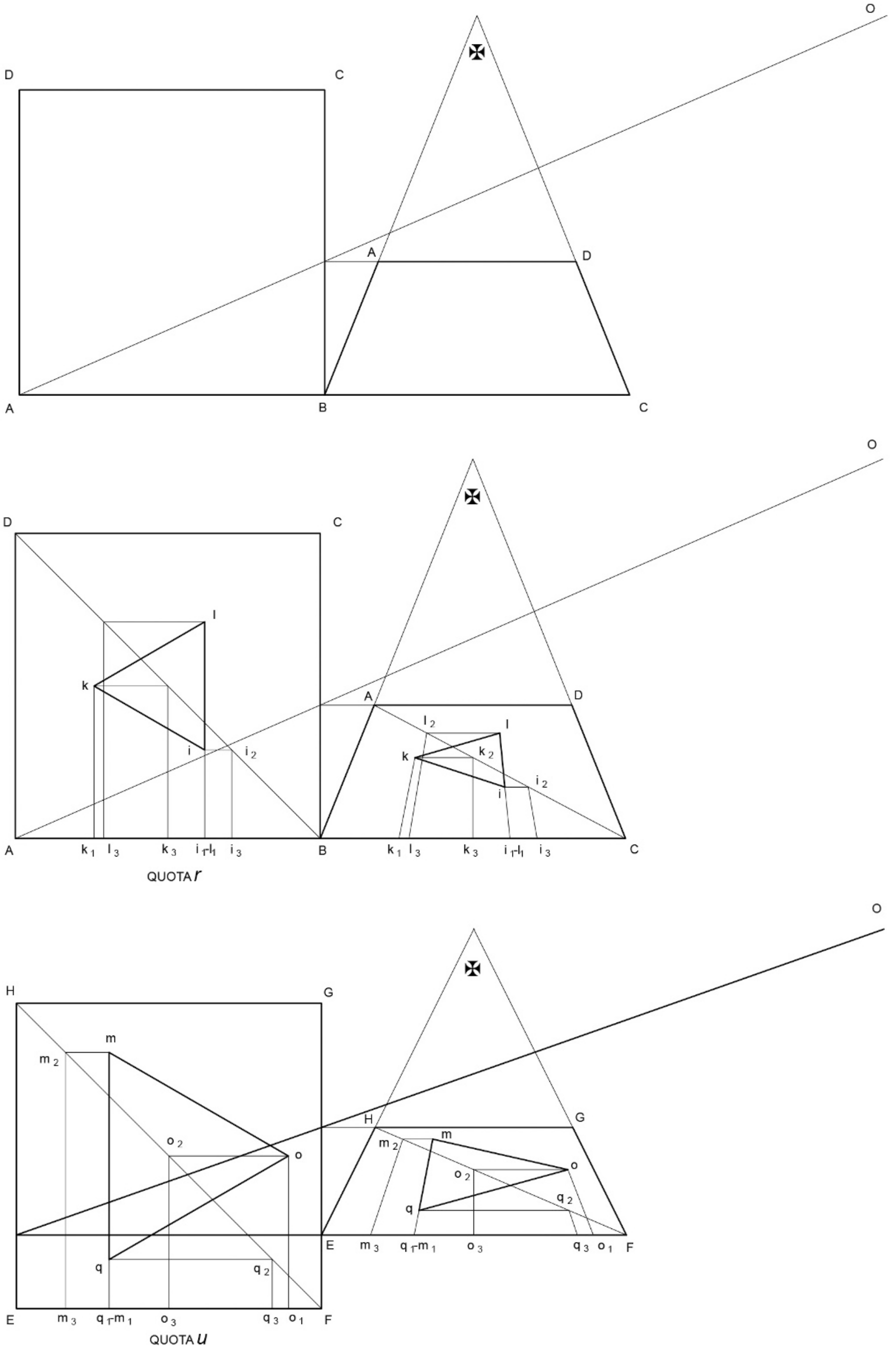
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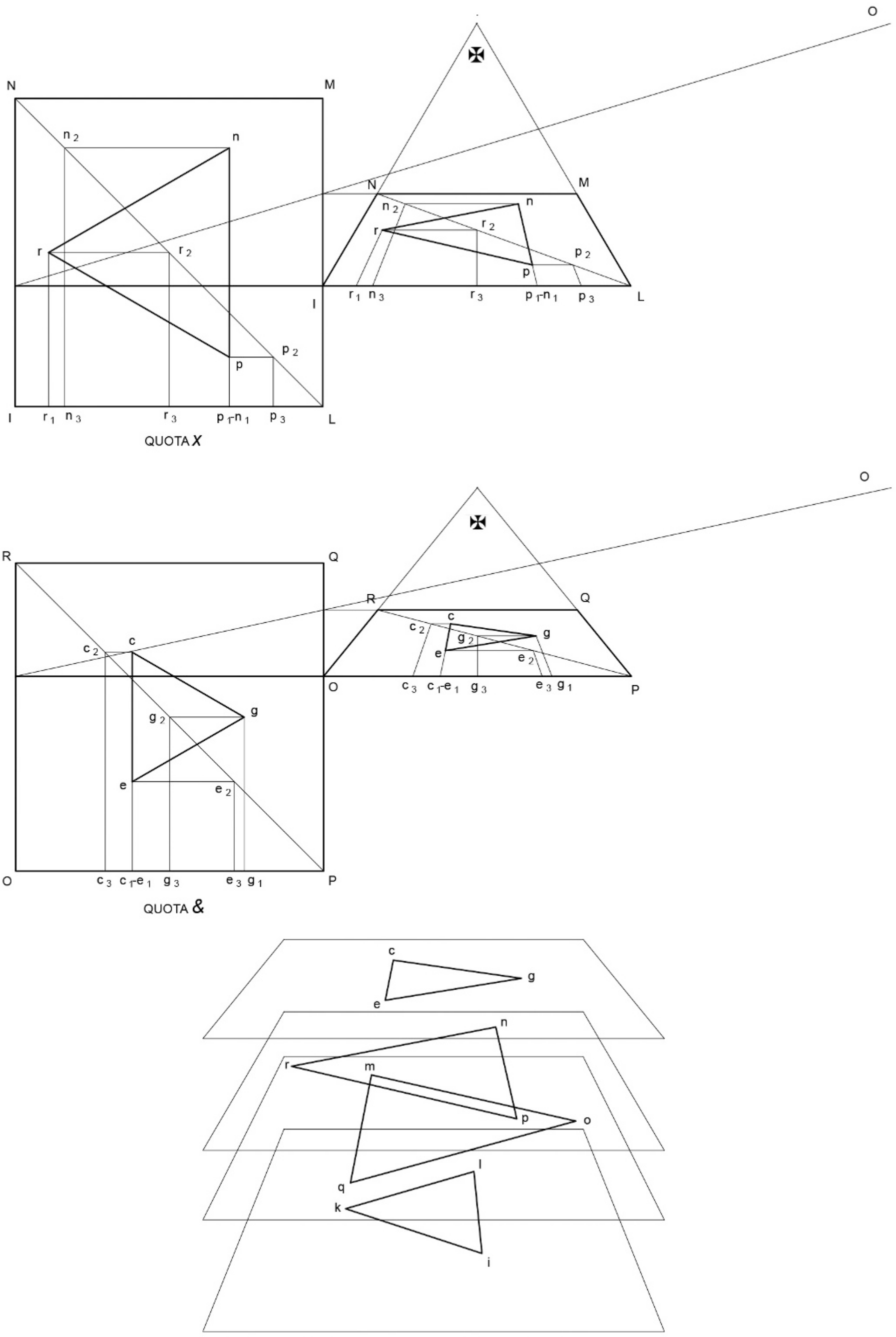
Fig. 20. The *digradati* of the vertices of the icosahedron that are located in the same plane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p 1-n 1
p 3
L
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c 1-e 1
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Fig. 20. The *digradati* of the vertices of the icosahedron that are located in the same plane

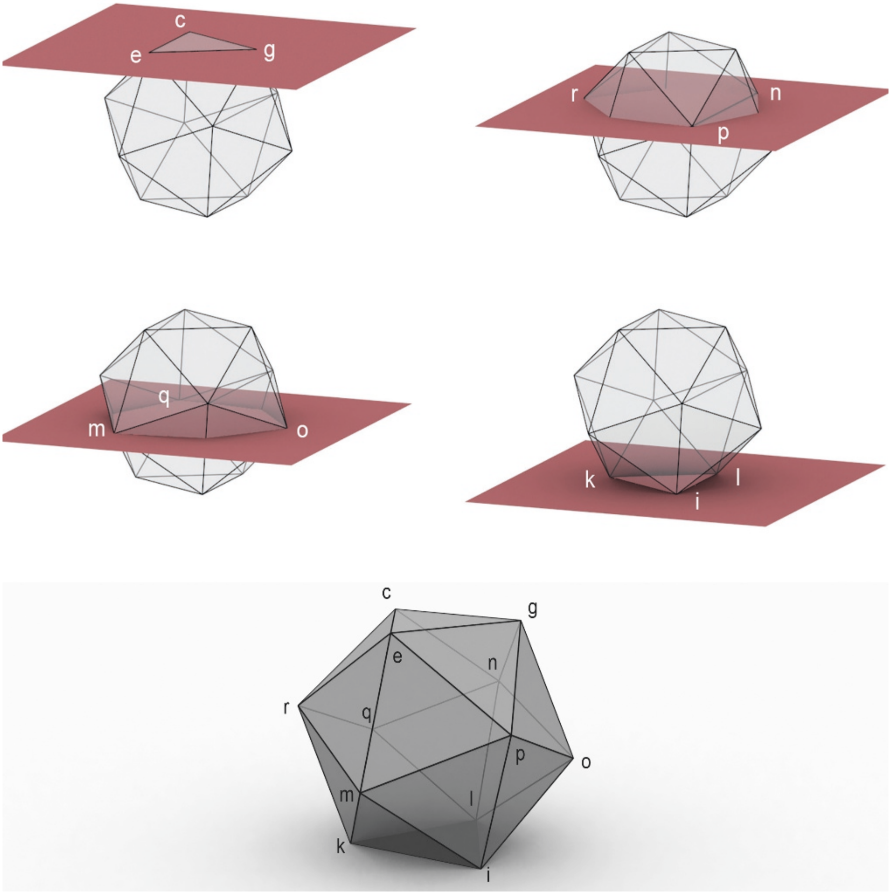

Fig. 21. The perspective of the icosahedron, making evident the vertices that lie in the same planes

It shouldn't seem strange that, after lengthy reflections about the construction of the *perfetto* and the *dritto*, Barbaro does not extend himself to provide his readers with detailed instructions useful for the the perspective delineation of the *digradati*. His educational task will be seen to be fully accomplished only when it is situated in the context of Aristotelean gnoseology, which Barbaro had had the means of studying during his years at university in Padova, but also while preparing the publication of the works of his great-uncle Hermolao Barbaro.[17] In fact, the concepts of order, brevity and simplicity entirely permeate the didactic structure of *The Practice of Perspective*: order opposes confusion and facilitates clarity; brevity opposes prolixity and encourages study; simplicity fosters memory.[18] Therefore, what may appear to be a didactic lacuna in the exercises with the polyhedra responds in reality to a precise strategy, because Barbaro has already addressed the transformation of the *perfetto* into the *digradato* in the previous pages and does not consider it opportune to go over the same arguments again. Indeed, the order of the arguments set forth in *The Practice of Perspective* is so carefully considered that for the first time a treatise opens with the basic precepts: how to set up the canvas, and where to locate the principal point and distance at which the observer must position himself. Unfortunately, Barbaro's method of perspective, completely original and alternative to Piero's *Terzo Metodo*, was not adopted by artists in the years that followed the publication of the treatise. However, Kim Williams has demonstrated that, at least in Venice, Barbaro's method was studied, comprehended and disseminated by scientists of the like of Federico Commandino.[19]

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[17] Williams (2020, pp. VIII-IX).

The creation of theatrical scenes according to the method of Pompeo Pedemonte

Part IV, dedicated to the creation of theatrical scenes, opens with a vexatious question regarding terminology used by Vitruvius regarding drawing. Briefly stated, Barbaro prefers to translate the term *scaenographia* as *sciografia*, or profile, due to the Aristotelean assonance of species that the section has with plan and elevation. However, in so doing he denies perspective a specific role in the ideation of a design and relegates its use to only the creation of scenes for the theatre. Referring to this specific context, Barbaro quickly reviews the scenes of the ancients and the basic proportions to be assigned to the base, shaft and capital of the column and entablature, then describes the appearance of the Tragic, Comic and Satiric scenes, basing the discussion on the words of Vitruvius and, as has been noted, reproducing the illustrations of Sebastiano Serlio. The only specific reference to artistic practice is found in the 'Description of the Tragic scene',[20] and this arrives after some reflections of a general nature regarding the distance of the viewing point, considered with regard to making sure the spectators can hear properly. However, Barbaro's main concern is noting that:

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[18] Williams (2020, pp. XII-XIII).

[19] In this present volume, pp. 35-41.

[20] Barbaro (1568, p. 155-156): *Descrittione della scena Tragica*.

> … those who paint the scenes place the point with great circumspection so that the things that are drawn do not appear distorted, and so that they serve the site and natural appearance.[21]

To explain how to realise this architectonic scene design, Barbaro calls into cause Pompeo Pedemonte, citing his invention aimed at foreshortening theatre scenes by means of perspective painting. In speaking in this way Barbaro introduces an innovation, since up to this point artists had been unable to consult any books on geometric theory or advice on this point. The only information at their disposal was contained in the pages of *Il Secondo libro* of Sebastiano Serlio, who had been exhaustive in the description of the 'manners' that were suitable for the three scenes, but extremely elusive regarding their effective execution. Serlio in fact describes the design details of the temporary wooden theatre set up in the courtyard of the Palazzo Porto-Breganze in Vicenza in 1539,[22] and quickly dismisses the reader with regard to the creation of the scenes, simply stating that it is best 'to exert oneself in making models and experiments, so that, studying, the way will be found'.[23] Pedemonte had entered into competition in the creation of scenic apparatuses[24] against Serlio and Egnazio Danti. Danti and Serlio preferred to mix distorted wooden architecture with pictorial illusions, while Pedemonte found it easier to create ephemeral architectural scenes at true scale, painting their extensions in perspective. Barbaro explains the method invented by Pedemonte with two illustrations (Fig. 22), both of which represent the *frons scaenae* in elevation, the stage in perspective, and the cavea in plan. Their aim is to explain to the reader that all of the orthogonal lines of the *frons scaenae* must appear to converge in the principal point *e*, even when they are broken on surfaces that are variously inclined. The representation on the right in Fig. 22 shows how to proceed with the joints of a pavement, while that on the left refers to the heights of edifices, windows and doors.

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[21] Barbaro (1568, p. 155): … *quelli, i quali dipingono le Scene, che con grande circonspettione pongino il punto, accioche le cose dissegnate non parino ruinare,* & *accioche servino lo sito,* & *l'aparenza naturale …*

[22] Marotti (1974, pp. 257-270).

[23] Serlio (1545, p. 65v): *faticarsi nel far de modelli et esperientie, che studiando trovara la via.*

[24] L'Occaso (2012, p. 73).

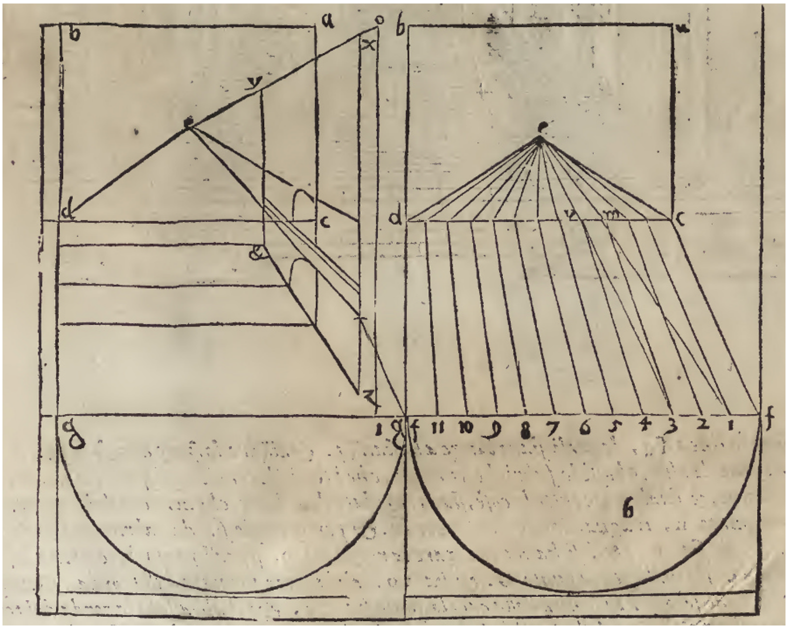

Fig. 22. How to construct a scene according to the method of Pompeo Pedemonte (Barbaro 1568, p. 155)

To draw lines that appear to be continuous and convergent to the principal point, even when in reality they are broken on different planes, the method is as follows. We divide the front of the stage into equal parts and connect each of these points with the principal point *e* by means of a cord (the red lines in Fig. 23). A collaborator, standing in the centre of the cavea *h*, can indicate where the cords appear to intersect the edge between the stage and the front of the scene. These points must be conjoined with the division made at the front of the stage *ad* with the principal point *e* (the grey lines in Fig. 23). If we remove the cords and look at the result obtained from the centre of the cavea *h*, the broken grey lines appear perspectivally continuous and convergent at the principal point *e*, even if in reality they are not at all. The same method is used to establish the apparent height of roofs, cornices, projections, windows, door and columns. The solution offered by Pedemonte, illustrated for the first time by Barbaro, responds to the need to foreshorten the buildings of theatrical scenes, resulting in an expeditious method that is particularly convenient. In fact, to maintain the illusion the artists simply had to foreshorten their appearances, with the aid of a cord, after having set up the ephemeral structures that emulate the facades of real buildings (Fig. 24).

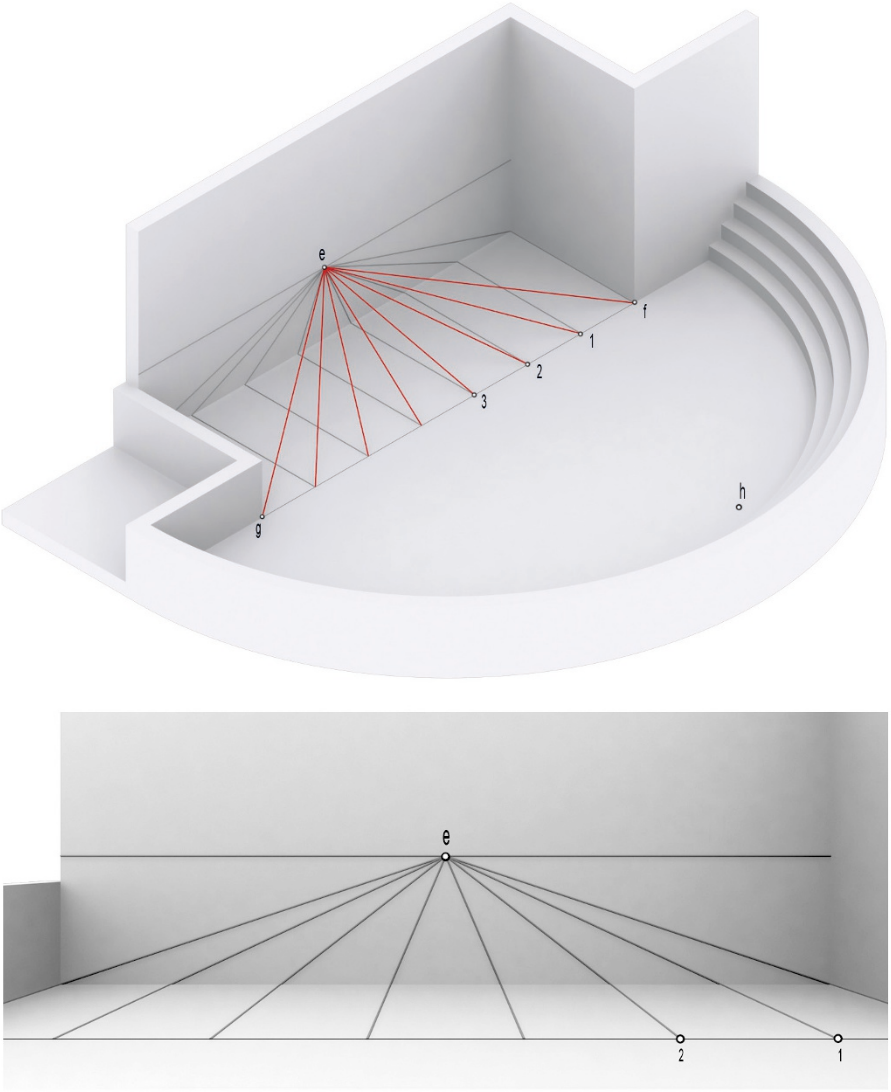

Fig. 23. Reconstruction of a stage scene according to the method of Pompeo Pedemonte: the floor

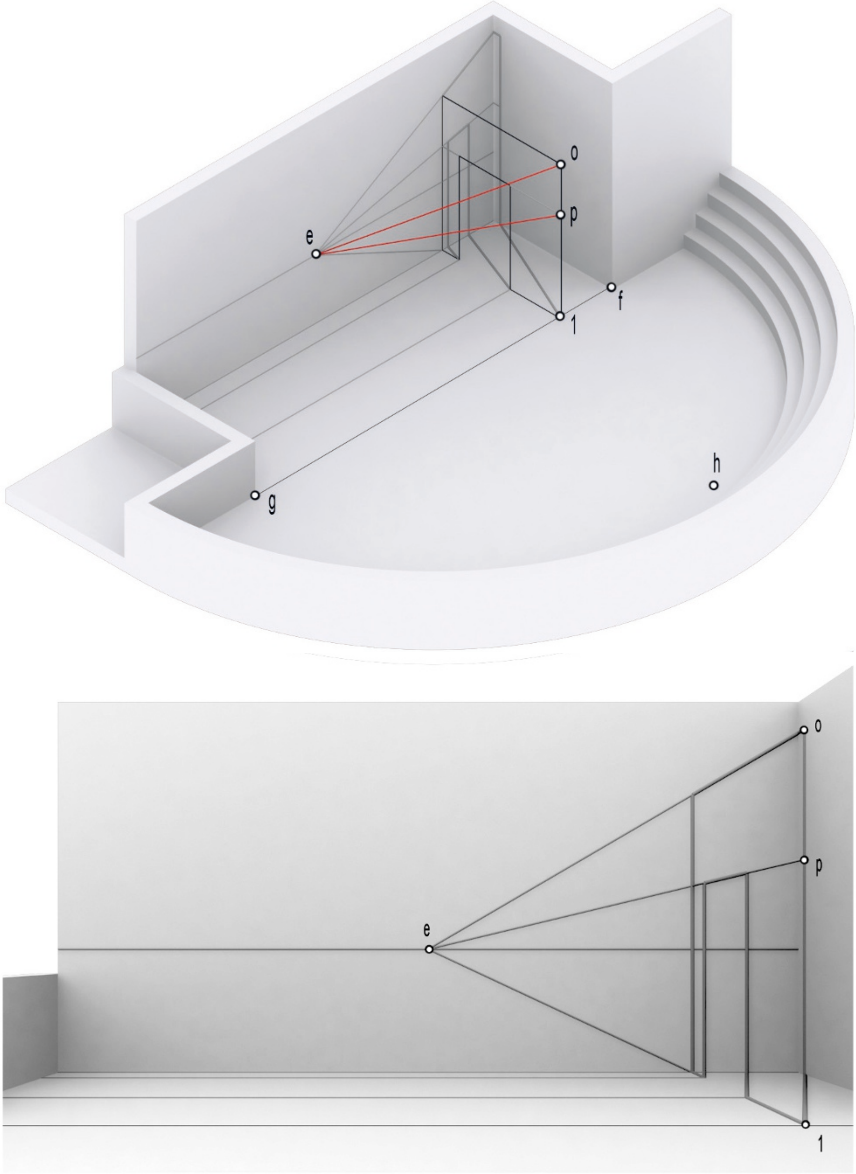

Fig. 23. Reconstruction of a stage scene according to the method of Pompeo Pedemonte: the heights of a building

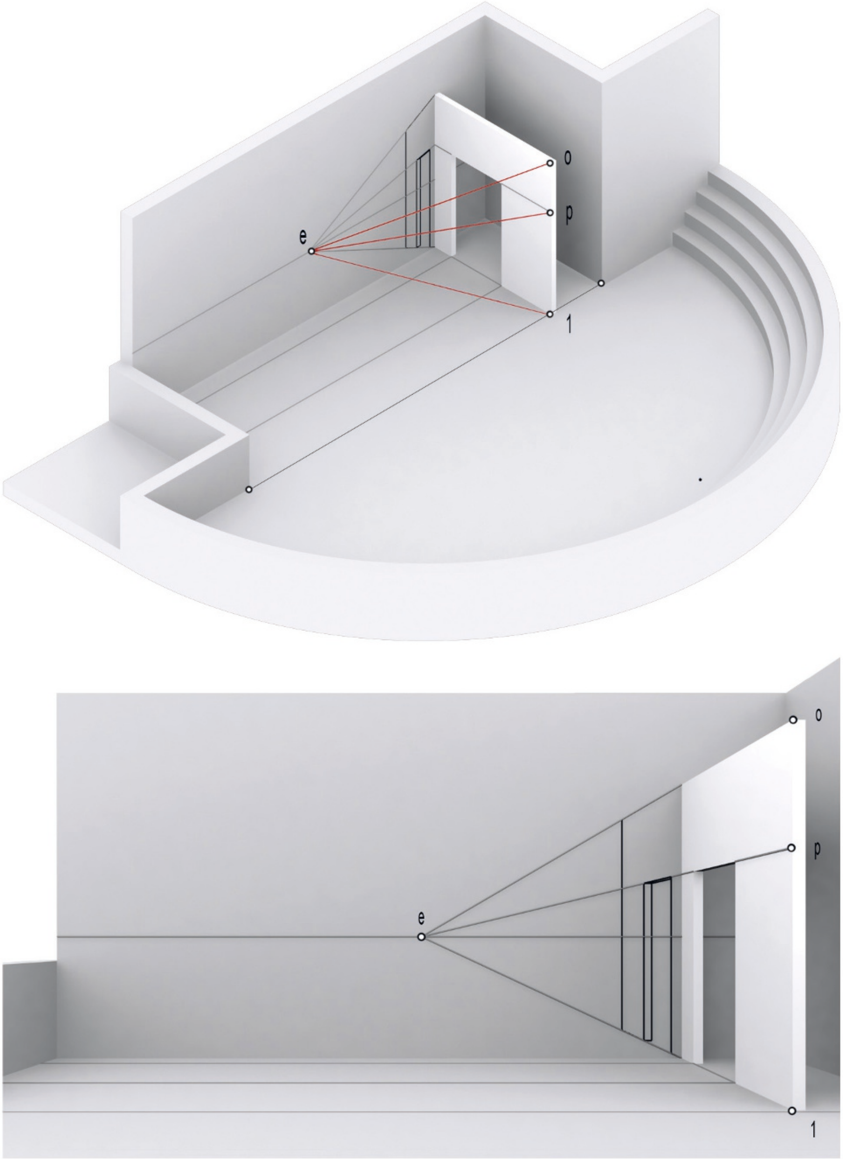

Fig. 24. Perspective continuation of the ephemeral architecture

Anamorphosis and the *tabula scalata*

In Part V Barbaro characterises as 'fine and secret'[25] that part of perspective concerning distorted delineations:

> in which, when the viewer's eye is not placed in a determined point, what appears is something other than that which is depicted, but then, when viewed from its point, there appears that which was actually made according to the intention of the painter.[26]

With these words, anamorphosis makes for the first time its official entrance into treatises on perspective. Barbaro describes artistic subjects that have not yet been named in precise terms, as the term 'anamorphosis' will appear only in the middle of the 1600s in the treatise *Magia universalis naturae et arti* by Gaspar Schott.[27] There is a simple way to create these images: after having drawn a figure on a sheet of paper and pricked holes along the outlines 'as though you wished to make a pouncing, but with rather large holes',[28] the sheet is positioned at right angles on a surface 'as though the panel were one wall and the paper another'.[29] The sheet is then exposed directly to the sun to permit the rays to go through the holes and project the points of light onto the surface. Taking this description into account, presented by Barbaro without figures, I have performed an experiment virtually using a drawing made after the celebrated portrait of Barbaro by Paolo Veronese (Fig. 25). Barbaro does not describe the geometrical rules of anamorphosis, but his mention of the topic constitutes a first that will condition future developments.[30]

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[25] Barbaro (1568, p. 159): *bella, & secreta*.

[26] Barbaro (1568, p. 159): *nelle quali se non è posto l'occhio di chi le mira nel punto determinato, ci appare ogni altra cosa, che quella, che è dipinta, che poi dal suo punto veduta dimostra quello, che è veramente fatto secondo la intentione del pittore*.

[27] Schott (1657, p. 100).

[28] Barbaro (1568, p. 159): *queste punteggierai come se ne volesti fare uno spolvero, ma con i punti alquanto großi*… .

[29] Barbaro (1568, p. 159): *come la tavola fusse un parete, & la carta un altro*.

[30] Monteleone (2020, pp. 117-128).

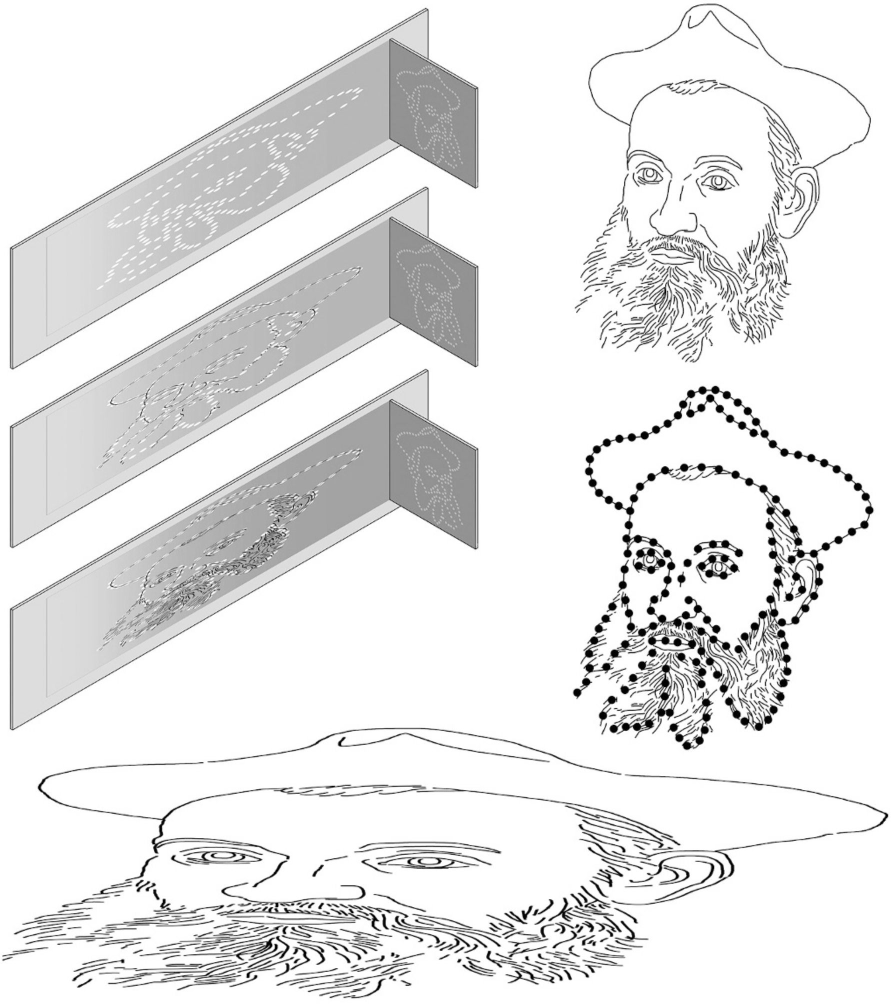

Fig. 25. Parallel deformation of the portrait of Daniele Barbaro

In the only reference to possible geometric rules Barbaro refers the reader to Part II of his treatise,[31] which is entirely derived from the manuscript of Piero, who was the first to intuit the correct characteristics of an anamorphosis, as shown by his *rinfrescatoio* (Fig. 26).[32]

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[31] Barbaro (1568, p. 161).

[32] Mancini (2015, pp. 45-54); Camerota et al. (2015, pp. 280-289, 393); Camerota (2006, pp. 94-95); Sgrosso (2001, pp. 50-73).

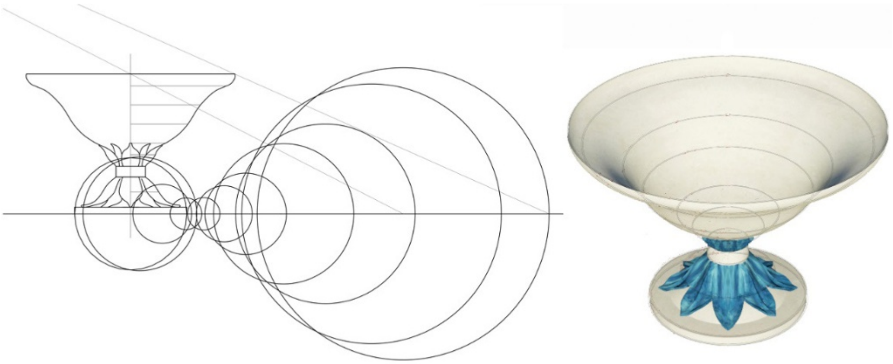

Fig. 26. Reconstruction of Piero's *rinfrescatoio*. Left) Centrally deformed perspective; right) View from the correct point

However, to deform a drawing of a head with the rays of the sun (parallel projection) is conceptually incorrect, because from the point of view of projection it behaves like the light of a candle (central projection). It is thus legitimate to ask ourselves if Barbaro fully understood the projective mechanics of light. It is not easy to respond to this question. On one hand, we have proof that Barbaro was able to distinguish the difference between projective phenomena in nature.[33] However, like many others before and after him,[34] he erroneously considered equivalent the effects of the projection of the sun and that of a candle. On the other hand, a passage in the manuscript It. IV, 39=5446 shows that he had studied anamorphosis in depth from the point of view of geometry, carefully evaluating the deforming effects that the plane of projection generates according to the place it occupies in space with respect to the eye and the object:

> This plane is now imagined to be between the thing seen and the eye, now on the other side of the thing seen, so that between the eye and the plane there is the thing seen, sometimes the plane is in the middle, partly on this side, partly on the other, of the thing seen, and of all these locations there is the same or similar reasoning and the whole with occult proportions appears to us,

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[33] In treating shadows, Barbaro distinguishes clearly between two types of sources of light: the *fuochi eterni del cielo* (eternal fires of the heaven), which produces shadows of the same size as the objects, and the *fuochi terreni* (earthly fires), which project shadows that are much larger; see Barbaro (1568, pp. 177).

[34] The misunderstanding, which began with Dürer, would continue to appear in treatises of perspective until the late 1600s; see Monteleone (2020, pp. 120-122).

> because, the terms mutating, the proportion mutates … but it is necessary to demonstrate them with lines in the degraded surfaces, and that they have, with respect to the eye, similar but not of the same species as those of the perfect surfaces, which are known to us.[35]

The question therefore remains open, but Barbaro's error might be explained with the custom of astronomers of considering the sun not as an infinitely distant source of light but as a precise point in the celestial vault, hence a point that it is possible to fix in space, exactly like the flame of a candle. This hypothesis would justify a statement that Barbaro makes in Part V, where he suggests to the reader that, in addition to using the sun, anamorphic deformations could be obtained also 'with a lantern, setting it as the view leads you to do'.[36] However it may be, the experiment with the rays of the sun, a practical and quick method to create images capable of exciting wonder on the part of the observer, constituted only one part of the curious production of Renaissance artists who, according to what Barbaro tells us, had gone well beyond direct anamorphosis: they had in fact extended their production to the catoptric and dioptric, applying:

> letters on panels that cannot be read except with mirrors, and almost seem to reverberate. Some with reflections of light have drawn clocks. Some, using the medium of water for the refraction of rays, have done marvellous experiments.[37]

Where could Barbaro have seen these perspective curiosities? We recall that he had been in London from August 1549 to February 1551 as a diplomat at the request of the Serenissima. But before his arrival, the English court had developed a predilection for anamorphic images thanks to a celebrated painting: the *Ambassadors* by Hans Holbein.[38] The cultural world that

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[35] Barbaro (Ms. It. IV, 39=5446, fol. 264r); Monteleone (2020, p. 406): *Questo piano hora imagina essere tra la cosa veduta et l'occhio, hora oltra la cosa veduta, si che tra l'occhio, et il piano ci sia la cosa veduta, alcuna fiata, il piano è di mezzo, parte di qua, parte di là della cosa veduta, et di tutti questi siti è una istessa, overo simigliante ragione et il tutto con occulte proportioni ci pare, perche mutandosi termine si muta proportione … ma bisogna con linee dimostrarle nelle soperficie digradate, et che hanno rispetto all'occhio, simili, ma non di quella specie, che sono le soperficie perfette, che ci sono note.*

[36] Barbaro (1568, p. 160): *anche alla lucerna, accomodandola come ti porterà la veduta.*

[37] Barbaro (1568, p. 159): … *lettere nelle tavole, che non si possono leggere se non con i specchi, & quasi di rinverbero, Altri con refleſsi di lumi hanno dissegnato horloggi, Altri usando il mezzo dell'acqua per la refrattione de i raggi hanno fatto prove meravigliose …*

[38] De Rosa and D'Acunto (2002, pp. 39-72).

Barbaro encountered on the other side of the Channel had already accepted with a certain pleasure the optical-figurative creations that deceived the eye and aroused wonder in the observer. In fact, anamorphic images in the field of art anticipated the exploit of wit in English literature, which aimed at amazing the reader with cleverness, ingeniousness and mystery.[39] When Barbaro mentions 'the effigies of princes'[40] in the list of the subjects most commonly depicted in anamorphic drawings, it is licit to think that he might have in mind precisely the distorted portrait painted by William Scrots in 1546 of Edward VI,[41] the young sovereign whose court Barbaro attended in London (Fig. 27).[42]

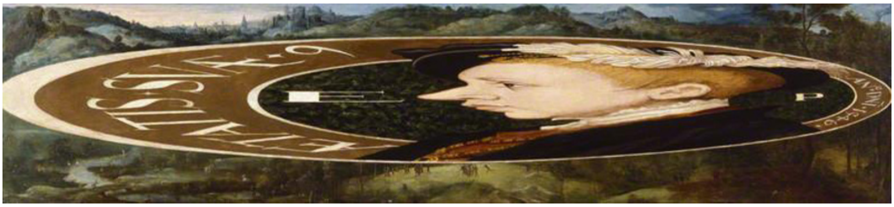

Fig. 27. William Scrots, *Anamorphic portrait of Edward VI* (1546)

Due to the scarcity of documents that have come down to us, we are not able to say for sure whether or not Scrots followed the geometric rules aimed at altering the visage of the young king, but given the substantial symmetry of the circular border that surrounds the portrait, we have to conclude that he used a parallel deformation and not a central one. In essence, the result that the artist achieved can be perfectly superposed on what could be obtained by exposing to the sun the drawing of Edward VI whose edges had been pricked as Barbaro describes in his treatise. In spite of the fact that in Part V the approach to anamorphosis is quite far from receiving the scientific rigour that the subject merits, it is in the enumeration of the examples that the value of Barbaro's work lies, because the descriptions presented would inspire artists and scholars to perfect and study in depth new lines of research in perspective, in the search for the projective rationales that underlie such curious images.

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[39] Gilman (1978).

[40] Barbaro (1568, p. 159): *effigie di principi*.

[41] De Rosa and D'Acunto (2002, pp. 21-22).

[42] The payments to the Dutch William Scrots, as a court painter, ceased in 1553, immediately after the death of Edward VI. It can be deduced that the artist was in London during the time in which Barbaro carried out his duties as ambassador. See Strong (1969, pp. 93-94).

Another example of how Barbaro inaugurated a new line of study in the field of curious images is relative to an image (Fig. 28) and an enigmatic phrase claiming that the artist 'can and even must deceive, cutting and separating the lines'.[43] Again in this case Part V does not provide a geometric rule useful for contructing such a deception, but only illustrates the optic functioning of the device, known as a *tabula scalata*.

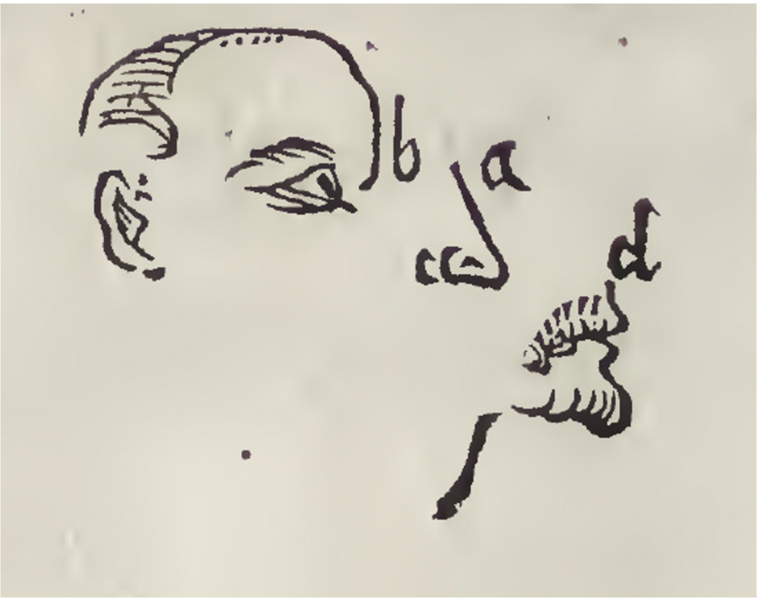

Fig. 28. Optical explanation of the *tabula scalata* (Barbaro 1568, p. 161)

This face of a man divided into pieces is certainly not an anamorphosis, but can still inspire surprise and wonder in the observer: 'if the eye is placed in the same plane, the two will conjoin, because the space between them is not seen'.[44] While Barbaro's comment about this is too brief to be clear, the illustration must have in any case attracted the attention of his contemporaries, given that in a treatise published some 15 years after that of Barbaro, *Le due regole* di Danti and Vignola (1583), we find a much clearer image of the same optical game, in which the reflection of a mirror, suitable inclined, restores unity to the fragmented portrait of Pope Gregory XIII (Fig. 29).

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[43] Barbaro (1568, pp. 161): *puo e deve anche ingannare, tagliando, & separando le linee.*

[44] Barbaro (1568, pp. 161): … *se l'occhio serà posto nel piano istesso, amendue si uniranno, percioche non si vedera lo spacio di mezzo.*

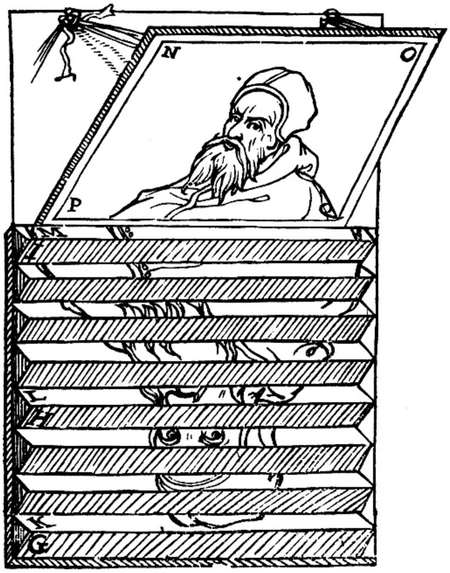

Fig. 29. Portrait of Pope Gregory XIII on a *tabula scalata*
(Danti and Vignola 1583, p. 95)

Perspective machines

In many situations applying the rules of perspective with geometric rigour can turn out to be long and arduous. Barbaro was aware of this, but he nevertheless denounced the fact that the artists who were his contemporaries, in contrast to the previous generation, preferred not to make the effort:

> We can reasonably censure our age, which has produced excellent painters but few perspectivists. I see the effort disdained; they admire what is well done, but flee from the study required to do it; they want to have the fine things of others, but do not take care to know how to make it for themselves. But [my effort regarding the practice of perspective is] to exhort those who, intimidated by the effort, do not dare to set themselves to the task of learning this very beautiful art.[45]

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[45] Barbaro (Ms. It. IV, 39=5446, fol. 298v); Monteleone (2020, p. 440): *Questa è la mia fatica circa la Perspettiva prattica… potemo ragionevolmente biasmare l'età nostra, che habbi prodotto eccellenti pittori, ma pochi Perspettivi. Vedo esser sprezzata la fatica, ma lodata l'opera della perspettiva, ammirano il ben fatto, fuggono lo studio di fare; vogliono haver le cose belle d'altri, ma non si curano di saper farle da loro; ma per essortare chiunque dalla fatica sbigottito non ardisce porsi*

The reluctance of the painters was not, however, entirely unjustified, since it is effectively easier to copy the more complex objects from life. Nevertheless, there do exist means that could have guided the hand of the artist in the faithful and rapid reproduction of what is seen by the eye. The prototype of these devices, aimed at automating drawing and reducing imprecisions, is the ‘veil’ described by Leon Battista Alberti in *De pictura*.[46] Credit must be given to Albrecht Dürer for the first developments of this, attested to by four illustrations for machines for representations in perspective, published together in the second edition of the *Underweysung der Messung* (1538). Barbaro continues these investigations in his Part IX, ‘in which are described many instruments and ways of rendering and transporting things in perspective’.[47] Hence a study that aims at tracing the history of perspective machines must recognise Barbaro’s most original contributions in this field, not least because some of them would become widespread in the centuries that followed.

Leafing through the final part of *The Practice of Perspective* we find an image of Dürer’s machine (Fig. 30). This well-known instrument substitutes a ring fixed in the wall for the eye of the observer, and considers stretched strings in place of the visual rays. Barbaro, who declares that he himself has used the device, contrives a way to improve the adherence of the crossing strings to the frame and advises keeping the ring fixed in the wall far from the instrument in order to avoid the drawing turning out too small.[48]

Barbaro follows his passage on Dürer’s device, already well known to his contemporaries, with descriptions of a certain number of perspective machines that were revolutionary and ahead of their time. This is the case of the skiagraphic instrument of Giovanni Battista Vimercati, which, thanks to the use of solar rays and two gnomons connected to a fixed assembly, made it possible to trace, by means of the shadow cast by the first gnomon, a drawing on a sheet of paper placed on a surface and transfer it, by means of the projection of the second gnomon, to another surface. Practically speaking, Vimercati’s device made it possible to copy or deform anamorphically a given drawing, thanks to the shadows cast simultaneously by two equal gnomons (Fig. 31).

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alla impresa di imparare questa si bella arte.

[46] According to Leon Battista Alberti, a painting can be likened to an open window and to a frame with a grid; see Domenichi (1547, pp. 11r-11v; 22v-23r) and Alberti (2004, p. 65).

[47] Barbaro (1568, pp. 187): *si descriveno molti instrumenti,* & *modi di ponere,* & *trapportare le cose in Perspettiva*; see Camerota (2006, pp. 155-159).

[48] Barbaro (1568, p. 191); see also Monteleone (2020, pp. 149-150).

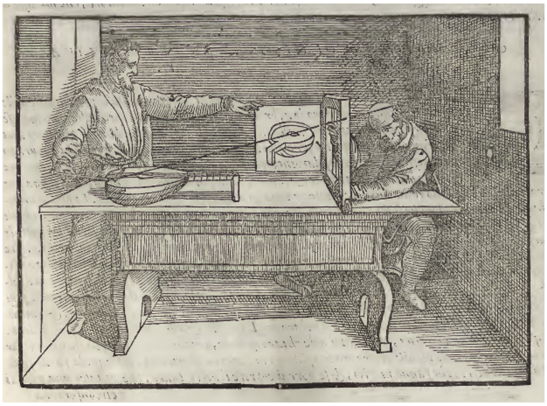

Fig. 30. Dürer’s perspective machine (Barbaro 1568, p. 191)

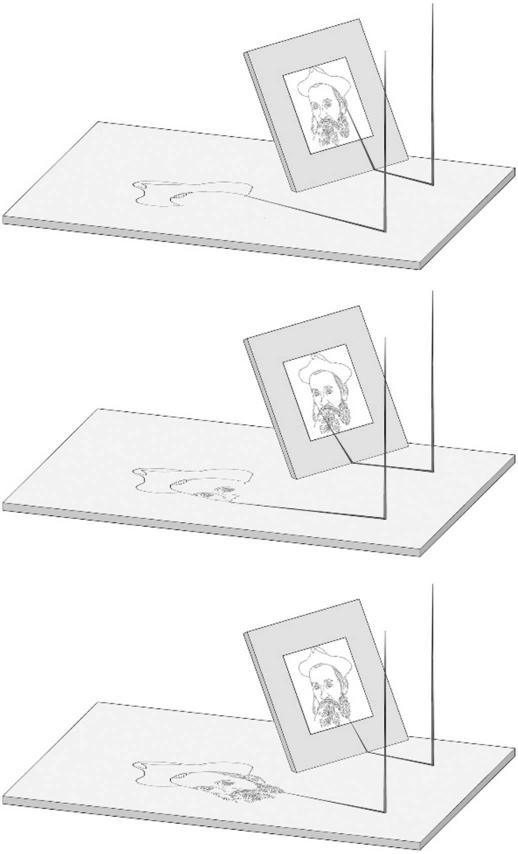

Fig. 31. Reconstruction of the skiagraphic instrument of Giovanni Battista Vimercati

The description of the *camera obscura* can also be connected back to the marvels of nature. Barbaro was the first to suggest the use of this instrument in a treatise for artists, specifying that a projection of luminous rays is naturally generated in a dark setting. Such a projection must be intercepted by moving a sheet of paper farther or nearer to a small hole in order to being the image into focus; after this it is possible to trace it with a charcoal or colour it with a brush (Fig. 32). To improve this experiment, Barbaro invites the reader to follow two simple suggestions. First he advises fitting into the hole a lens similar to 'an eyeglass of an old man – that is, one that has a bit of thickness in the middle',[49] in order to facilitate the convergence of the solar rays. Second, he advises darkening the perimeter of the lens, leaving 'just a little circumference in the middle',[50] thus improving the sharpness of the image. These two expedients are recognisably significant innovations with respect to his predecessors, who had never mentioned either the biconvex form of the lens to boost the phenomena of the rays or the reduction of the diameter of the crystal to avoid the blurring of the projected image at its edges.

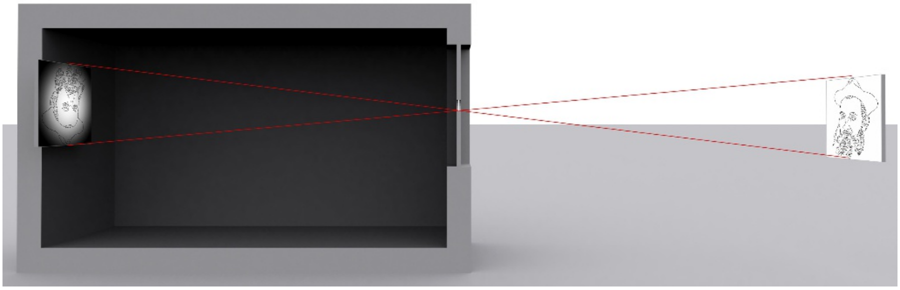

Fig. 32. Reconstruction of a *camera obscura*

The use of the camera obscura to study eclipses was widespread among the astronomers of the Middle Ages and Renaissance,[51] and in fact the sources that Barbaro consulted were scientific and not artistic: the *De subtilitate* by Gerolamo Cardano, published in 1550,[52] and the *Magiae naturalis* by Giovanni Battista della Porta, published in 1558.[53] Cardano advised inserting a lens in the hole, but without saying why, and without specifying what shape it was to have. For his part, della Porta does not mention a lens at all, but does refer to the usefulness that the instrument could have for artists. Thus to Barbaro must go the credit of having collected the few sources available and having explained in Italian, the language of artists (as opposed to Latin, the language of the erudite), how to construct an instrument, and indeed improving upon it, allowing artists to bend nature to their needs.

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[49] Barbaro (1568, p. 192): *un'occhiale da vecchio, cioè che habbia alquanto di corpo nel mezzo.*

[50] Barbaro (1568, pp. 192-193): … *che vi lasci una poca di circonferenza nel mezzo.*

[51] Lindberg (1976); Bubb (2010).

[52] Cardano (1550, p. 107).

[53] Della Porta (1558, p. 119v).

Among the instruments listed in Part IX we find one used by Barbaro to copy, enlarge or reduce a given drawing. He conjectures: 'So let us say that I wish to transport one geographical table to another'.[54] To do so it is necessary to construct two graduated semi-circles and two rulers that are divided into 'as many equal parts as I want'.[55] After having hinged the rulers in the centres of the respective circles, he proceeds by transporting the points from the original to the blank sheet, measuring an angle on the circle and the distance from the centre on the ruler (Fig. 33).

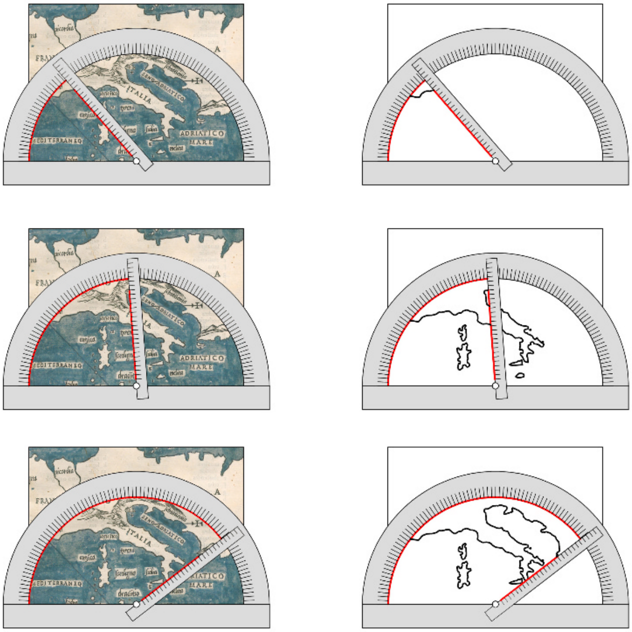

Fig. 33. Reconstruction of the instrument for copying, enlarging and reducing an image

Instead, the instrument that Barbaro attributes to Baldassarre Lanci was inspired by topographical surveying. This must be used in the following way:

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[54] Barbaro (1568, p. 193): *…poniamo caso che io voglia trapportare una tavola di Geografia ad'altro in un piano.*

[55] Barbaro (1568, p. 193): … *in quante parti mi piace.*

> Placing the instrument on some plane, you looked through the upper tube at the outlines of something point by point, and where the vision stopped in the upper tube, a wire was passed through the channel of the lower tube, which went to strike the plane opposite and mark a point there.[56]

This instrument was created to draw views of a city as seen from above. It was probably shown to Barbaro by its inventor when Barbaro, on his way to Rome, passed through Siena, where Lanci was overseeing the fortifications at the behest of Cosimo de' Medici.[57] In essence, the device is constituted of a cylindrical surface on whose intrados is applied a sheet of paper. The drawing is derived point by point thanks to a fixed assembly for sighting and tracing, positioned at the centre of the disc and anchored to a vertical support. Barbaro does not provide an image for the reader, but we have an illustration published by Egnazio Danti in *Le due regole* (Fig. 34).

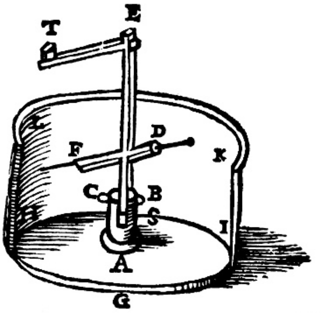

Fig. 34. The perspective machine of Baldassarre Lanci (Danti and Vignola 1583, p. 61)

We should not be surprised that a military man would invent an instrument for drawing. In the Renaissance perspective was used for military as well as artistic purposes, since by applying the rules of geometry inversely, it was possible to go from a foreshortened representation of reality to its actual dimensions. Therefore a perspective reconstruction made it possible to derive the plan of a fortress or the outline of city walls after their appearances had been represented by any kind of perspective machine.

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[56] Barbaro (1568, p. 192): *[P]osto lo instrument sopra qualche piano, si guarda per la canna di sopra i contorni d'alcuna cosa a punto per punto, & dove si ferma il vedere nella canna di sopra, si fa trappassare il ferruccio per la canna di sotto, il quale và a ferire nella piana opposta, & ivi si fa punto.*

[57] Camerota (2006, p. 155).

The *horario universale*

Part IX, dedicated to perspective machines, opens with the description of the *horario universale*, a device that Barbaro himself invented. It functioned not only as 'clock in itself, but is also an instrument for making clocks in every place, at every elevation, and with every sort of hour'.[58] Critical studies have essentially neglected the importance of this instrument, whose theoretical results would be particularly fecund in the field of perspective. The device can be likened to an armillary sphere, inasmuch as it is a simplified representation of the celestial sphere, and contains the poles, the polar circles, the two colures,[59] the horizon, and the tropics of Cancer and Capricorn. Thus composed, the instrument corresponds to a simplified interpretation of the Ptolemaic universe.[60] Barbaro, however, transforms this into a solar clock thanks to the insertion of 24 portions of meridians,[61] thus indicating the astronomical hours (Fig. 35).

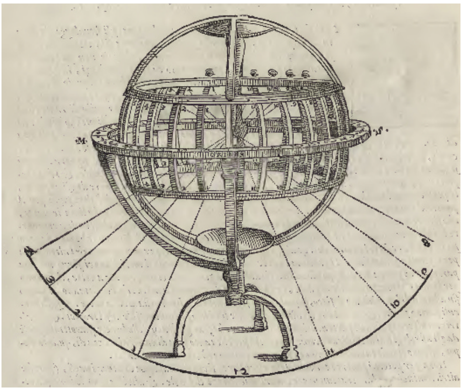

Fig. 35. Barbaro's *horario universale* (Barbaro 1568, p. 188)

But a fixed hour system would not have satisfied the needs of the readers, who in the Renaissance had different temporal systems according to the places where they lived. For this reason Barbaro suggests superimposed two further mobile circles, one for each tropic, such that they can also be divided into 24 equal parts, and to make a small hole in correspondence with each subdivision through which to tie threads of silk. It was these threads that made it possible to modify the system of hours. In addition to the *horario universale* being an instrument that can be adapted to the different hour systems used all over Europe, by making use of the pin on the axis that goes from east to west, it can be 'turned and raised and lowered according to the elevation of the poles'.[62] This motion allows the user to take into account the geographical position in which it is used, and to adjust it to suit any latitude. It is also easily transportable, with a diameter of 'a little less than half a foot'.[63]

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[58] Barbaro (1568, p. 187): *uno instrumento, i quale non solamente è horaloggio in se, ma è instrumento di fare horaloggi, in ogni piano, in ogni elevatione, & con ogni sorte di hore.*

[59] The two colures are: equinoctial colure, the meridian of the celestial vault that passes through the poles and the points in which the equinoxes occur (Aries-Libra); solstitial colure, the meridian passing through the points of the solstices (Cancer-Capricorn).

[60] Armillary spheres may have inspired Barbaro in the creation of his *horario universale*. Examples dating to the sixteen century are conserved at the Istituto e Museo di Storia della Scienza di Firenze (an exemplar by Girolamo della Volpaia of 1564, inv. 2711) and at the Museo di Storia della Fisica di Padova (the work of an unknown maker, inv. 257).

[61] Barbaro (1568, p. 187) speaks of 12 meridians, intending them to be ideally whole.

After having described how it is made, Barbaro explains how to trace a meridian, using expressions that must have appeared revolutionary to those who studied perspective:

> Begin to look where the meridian cuts the tropic of Cancer, and direct your gaze through the hole in the tip of the stylus, and through the point opposite on the edge of the meridian and on the tropic of Cancer all the way out to the plane; where your gaze falls on the plane mark the point (Fig. 36).[64]

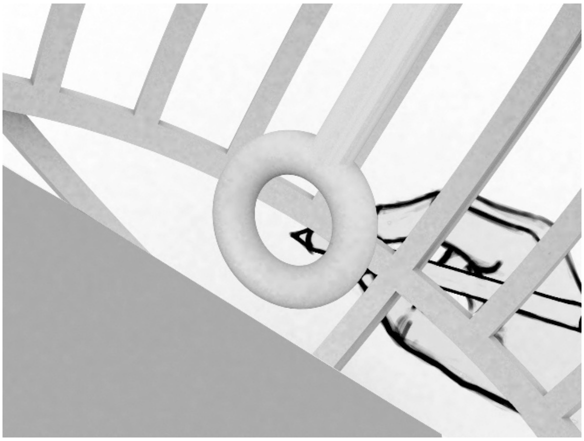

Fig. 36. Drawing clocks with the *Horario universale*.

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[62] Barbaro (1568, p. 188): *volgere, & alzare, & abbassare secondo la elevatione de i poli.*

[63] Barbaro (1568, p. 187): *poco meno di mezzo piede.*

[64] Barbaro (1568, p. 189): *comincia a guardare, dove il meridiano taglia il tropico del Cancro, & passa con la vista per lo foro della punto dello stile, & per lo punto opposto al taglio del meridiano, & del tropico del Cancro fin al piano, & la dove ti porta la vista sopra'l piano segne il punto.*

In these few lines, the perfect coincidence between the ray of light/shadow and the visual ray arrives at an absolute clarity, except that these words raise a problem regarding the position normally occupied by the elements – the eye, the surface being drawn on, the object – of a perspective. In order to understand what I mean, consider the definition given by Alberti,[65] who considered the surface of a painting to be like a window open onto the world and, as such, like a plane interposed between the eye and the object to be drawn. In contrast, Barbaro, supporting himself on the rules of gnomonics, shows that it is possible to draw while changing the position of those elements. In the art of tracing the clocks of the *horario universale*, the eye corresponds to the point of the gnomon, and the plane on which the meridian is drawn is found on the other side of the object that is being represented, in this specific case, a point of sighting. In other words, the usual spatial sequence of eye-surface-object has been rearranged into a new order: eye-object-surface. Repercussions of Barbaro's *horario universale* in the years that followed the publication of *The Practice of Perspective* are not to be looked for in the world of gnomonics, since, in spite of how innovative it was, the instrument did not break with the traditional schemes of that discipline. Instead, we do best to direct our attention to developments in perspective theory.

Some 75 years after the appearance of Barbaro's *horario universale*, the Minim friar Jean François Niceron published in Paris a work entitled *Thaumaturgus opticus*.[66] In his *Figura XXXVIII* Niceron shows the elements of a perspective ordered in the way described by Barbaro (Fig. 37).[67] The object of the representation is placed between the surface and the observer, and assumes a diaphanous aspect in order to allow the visual rays to penetrate through it and project onto a plane that is strongly angled with respect to the visual pyramid. The perspective image thus produced is an anamorphosis, more precisely a very stretched isosceles trapezoid, a plane figure that is geometrically different from the original square shape but optically identical to it when perceived from the *punctum optimum*.

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[65] Alberti (2004, p. 54).

[66] Niceron (1646).

[67] See Monteleone (2013, p. 326).

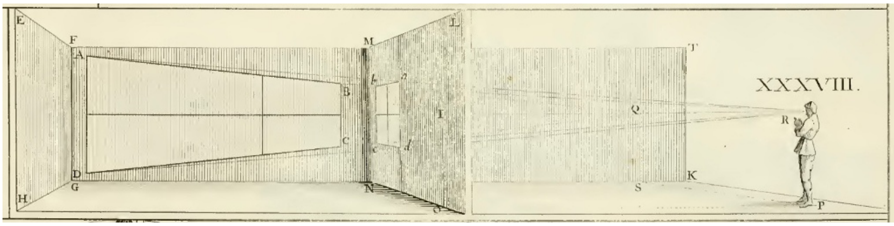
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Fig. 37. The mixture of elements (eye-object-picture plane) of a perspective (Niceron 1546, fig. XXXVIII)

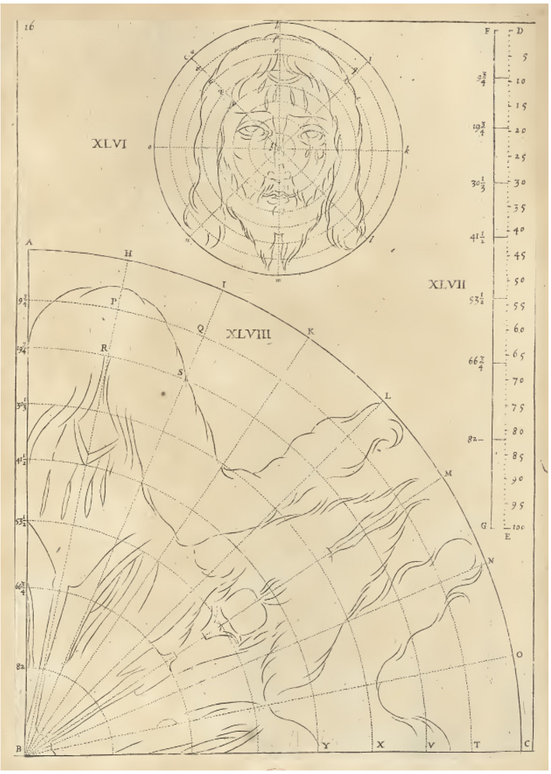
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Fig. 38. Anomorphosis on a cone (Niceron 1646, Figs. XLVI and XLVIII)

Inspired by the practice of the astronomers, who with instruments similar to those described by Barbaro were able to draw meridians on surfaces of all kinds, the inversion of object-picture plane will free the artists of the 1600s from the obligation to delineate perspectives exclusively on plane surfaces, since:

> There is no doubt that if you have well understood the way of making clocks in the plane of the horizon then you will know how to draw the clocks of the other planes, whether flat, concave, curved or whatever.[68]

Thus it is possible to trace back to the projections of Barbaro's *horario universale* the celebrated anamorphic illustrations produced by Niceron on cones and pyramids, which amaze the observer when, having placed his eye in the privileged viewing point, he is able to perceive the true sense of an image, previously perceived as a jumble of lines (Fig. 38).[69]

Conclusions

In light of what has been said here, we can only agree in part with the encomiastic judgement of Giovanni Grimani quoted in the introduction to this essay, because many of the innovations in *The Practice of Perspective* were not addressed with the scientific rigour that they merited. However, if on one hand Barbaro did not provide mathematical solutions to the problems, on the other he did exert himself to connect the practice of geometrical principles and direct experiment, anticipating the empirical method that would be adopted by the scientists of the next century. This attitude permits us to position Barbaro's treatise on perspective in the context of a profound cultural change that was being delineated in the course of the Cinquecento. Knowledge had only recently set off on the road to specialisation and scientists began to believe that it was possible to situate all disciplines under the aegis of a utopian *mathesis universalis*.[70] In this context of renewal Barbaro's book can be considered as an early formed but still immature fruit of the scientific revolution, inasmuch as it is the first text to collect, analyse and broaden all of the information then known about perspective yet lacks the rigour that will characterise the treatises of the 1600s.

After these final considerations, it is no longer of importance to establish whether or not Barbaro copied the procedures and illustrations of Piero della Francesca, Albrecht Dürer and Sebastiano Serlio, because the originality of *The Practice of Perspective* lies in having assembled, for the first time, theory, practical application, and experimental verification. It is emblematic that in the next century this approach led Kepler to establish that the eyes assume an indifferent function in the process of cognisance, registering exclusively appearances, without playing any role at all in the formation of judgments.[71] While science would compare the function of the eyes to that of a camera obscura, at the same time philosophy was moving towards the definitive separation of intellect and reality, a Cartesian detachment of *res cogitans* from *res extensa* that can be mediated solely by the sense, running the risk of formulating judgments that are erroneous or incomplete. This Socratic awareness of not being able to arrive at the truth, if not by subjecting the world that surrounds us to the doubt of reason, coincides with the conviction arrived at, according to Paruta, by Daniele Barbaro, who after having devoted a lifetime to theory and practice, is said to have stated, 'having taken great care, and much study, I find that I have learned to know that I know little.[72]

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[68] Barbaro (1568, p. 190): *Non ha dubbio, che se haverai bene inteso il modo di formare gli horaloggi nel piano dell'orizonte, che tu non sappia disegnare gli horaloggi ne gli altri piani dritti, cavi, piegato o quali si sieno…* .

[69] Monteleone (2013, p. 339-343).

[70] Crapulli (1969, pp. 21-21); Gavagna (2009, pp. 105-106).

References

Alberti (2004)

Alberti, Leon Battista. 2004. *On Painting*. Cecil Grayson, trans. Introduction and notes by Martin Kemp. Repr. with revised Further Reading. London: Penguin Classics (1st ed., London: Phaidon Press, 1991).

Barbaro (1568)

Barbaro, Daniele. 1568. *La pratica della perspettiva*. Venice: Camillo e Rutilio Borgominieri.

Barbaro (Ms. It IV, 39=5446)

Barbaro, Daniele. n.d. [La pratica della perspettiva, manuscript in Italian]. Ms. It IV, 39=5446. Biblioteca Nazionale Marciana, Venice.

Barbaro (Ms. It. IV, 40=5447)

Barbaro, Daniele. n.d. [La pratica della perspettiva, menabò]. Ms. It. IV, 40=5447. Biblioteca Nazionale Marciana, Venice.

Barbaro (Ms. Lat. VIII, 41=3069)

Barbaro, Daniele. n.d. Scenographia [manuscript in Latin]. Ms. Lat. VIII, 41=3069. Biblioteca Nazionale Marciana, Venice.

Bubb (2010)

Bubb, Martine. 2010. *La Camera obscura. Philosophie d'un appareil*. Paris: L'Harmattan.

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[71] Summers (1987, p. 45).

[72] Paruta (1579, p. 24): *havendovi molta cura posta, et molto studio, mi trovo d'haver imparato a conoscere, che poco ne sappia*.

Camerota (2006)

Camerota, Filippo. 2006. *La prospettiva del Rinascimento. Arte, architettura, scienza*. Milano: Electa.

Camerota et al. (2015)

Camerota, Filippo, Di Teodoro, Francesco P., Grasselli, Luigi. 2015. *Piero della Francesca. Il disegno tra arte e scienza*. Milano: Skira.

Cardano (1550)

Cardano, Gerolamo. 1550. *Hieronymi Cardani Medici Mediolanensis De subtilitate Libri XXI*, Nuremberg: Johann Petrejus.

Crapulli (1969)

Crapulli, Giovanni. 1969. *Mathesis universalis: genesi di un'idea nel XVI secolo*. Rome: Edizioni dell'Ateneo.

Danti and Vignola (1583)

Danti, Egnazio, Barozzi, Jacopo [Vignola]. 1583. *Le due regole della prospettiva pratica*. Roma: Francesco Zanetti.

Della Porta (1558)

Della Porta, Giovanni Battista. 1558. *Magiae naturalis, siue De miraculis rerum naturalium libri IIII-XX*. Naples: M. Cancer.

De Rosa and D'Acunto (2002)

De Rosa, Agostino, D'Acunto, Giuseppe. 2002. *La vertigine dello sguardo. Tre saggi sulla rappresentazione anamorfica*. Venice: Cafoscarina.

Domenichi (1547)

Domenichi, Lodovico. 1547. *La pittura di Leonbattista Alberti tradotta per M. Lodovico Domenichi*. Venice: Gabriel Giolito de Ferrari.

Dürer (1538)

Dürer, Albrecht. 1538. *Underweysung der Messung*. Nürnberg: H. Formschneyder (1st ed. 1525).

Gavagna (2009)

Gavagna. Veronica. 2009. Euclide a Venezia. In: *Pacioli 500 anni dopo*, Enrico Giusti, Matteo Martelli, eds., pp. 97-123. Sansepolcro: Centro Studi Mario Pancrazi.

Gilman (1978)

Gilman, Ernest B. 1978. *Curious Perspective. Literary and Pictorial Wit in the Seventeenth Century*. New Haven: Yale University Press.

Lindberg (1976)

Lindberg, David C. 1976. *Theories of Vision from Al-Kindi to Kepler*. Chicago: University of Chicago Press.

L'Occaso (2012)

L'Occaso, Stefano. 2012. *Anselmo Guazzi, un allievo di Giulio Romano*. Mantova: Il Rio.

Mancini (2015)

Mancini, Matteo F. 2015. *Le anamorfosi del De prospectiva pingendi*. In: *Le teorie, le tecniche, i repertori figurativi nella prospettiva d'architettura tra il '400 e il '700*, Maria Teresa Bartoli, Monica Lusoli, eds., pp. 45-54. Florence: Firenze: University Press.

Mariotti (1974)

Marotti, Ferruccio. 1974. Teoria e tecnica dello spazio scenico dal Serlio al Palladio nella trattatistica rinascimentale. *Bollettino del Centro Internazionale di Studi di Architettura Andrea Palladio* 16, pp. 257-270.

Monteleone (2013)

Monteleone, Cosimo. 2013. De proiectionis in planis obliquis: il Secondo Libro de La perspective Curieuse e del Thaumaturgus opticus, tra vocazione pratica e regole scientifiche. In: *Jean François Niceron. Prospettiva, catottrica e magia artificiale*, Agostino De Rosa, ed., 2013, pp. 323-349. Rome: Aracne.

Monteleone (2020)

Monteleone, Cosimo. 2020. *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto it. IV, 39=5446*. Canterano (Rome): Aracne editrice.

Niceron (1646)

Niceron, Jean François. 1646. *Thaumaturgus opticus seu Admiranda Optices*. Paris: Francisci Langlois.

Paruta (1579)

Paruta, Paolo. 1579. *Della perfezione della vita politica*. Venice: Domenico Nicolini.

Piero della Francesca (Mss. Regg. A 41/2)

Piero della Francesca. ca. 1472-1492. *De prospectiva pingendi*. Mss. Regg. A 41/2. Biblioteca Panizzi, Reggio Emilia.

Rose (1969)

Rose, Paul L. 1969. The Accademia Venetiana: Science and Culture in Renaissance Venice. *Studi veneziani* XI, pp. 191-242.

Rose (1975)

Rose, Paul L. 1975. *The Italian Renaissance of Mathematics*. Geneva: Droz.

Schott (1657)

Schott, Gaspar. 1657. *Magia universalis naturae et arti*. Würzburg: Henricus Pigrin.

Serlio (1545)

Serlio, Sebastiano. 1545. *Il primo[-secondo] libro dell'architettura di Sebastiano Serlio Bolognese*. Paris: J. Barbé.

Sgrosso (2001)

Sgrosso, Anna. 2001. *La geometria nell'immagine. Storia dei metodi di rappresentazione. Vol. 2: Rinascimento e Barocco*. Torino: Utet.

Strong (1969)

Strong, Roy. 1969. *Tudor and Jacobean Portraits*. London: Her Majesty's Stationery Office.

Summers (1987)

Summers, David. 1987. *The Judgment of Sense. Renaissance Naturalism and the Rise of Aesthetics*. Cambridge (Mass.): Cambridge University Press.

Williams (2020)

Williams, Kim. 2020. *Prefazione*. In: Cosimo Monteleone, *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto it. IV, 39=5446*, pp. VII-XIX. Canterano (Rome): Aracne editrice.

The Practice of Perspective

Daniele Barbaro
Patriarch Elect of Aquileia

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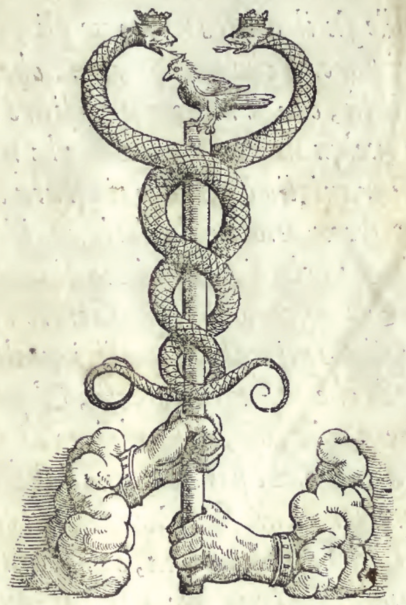
https://doi.org/10.1007/978-3-030-76687-0_3

LA PRATICA
DELLA PERSPETTIVA
DI MONSIGNOR
DANIEL BARBARO
ELETTO PATRIARCA D'AQVILEIA,

Opera molto vtile a Pittori, a Scultori, & ad Architetti.

Con due tauole, vna de' capitoli principali, l'altra delle coſe piu notabili contenute nella preſente opera.

CON PRIVILEGIO.



IN VENETIA,
Appreſſo Camillo, & Rutilio Borgominieri fratelli, al Segno di S. Giorgio.
M D LXVIIII.

To the Very Magnificent and Excellent Mr. Matheo Macigni[1]

Daniel Barbaro, Patriarch Elect of Aquileia

[p. 2] Magnificent and excellent brother, our studies having been shared for many of our years under the care of good preceptors, and the knot of friendship between us having been tied by the virtue of honest and thus firm connections, it comes to me to publish our closeness under part of those titles (whatever they may be) that with worthy efforts we have made sure to acquire. So you will look well upon the fact that under your correction I bring to light a treatise on the practice of perspective, which a great deal of time ago I arranged for my own pleasure, and then for the utility of all brought to such perfection as well as I knew how to and could. Great was our delight in mathematical studies from the early years, and it appeared to us that the pleasure that we got from them was the greatest utility that could be gotten. But going farther, that pleasure on which our diligence had already lingered brought us a marvellous fruit, because it paved the way for us to the highest and most subtle speculation. Of this we have a thousand times thanked divine goodness, which has conducted us from light to light to attain, through some reasoned discourse, to the most secret cognitions, the splendour of which blinds the eyes of those who do not distinguish what they know and what they can do, from those who truly know and do. They attribute much to the few things that it seems to them to know. But since it is not easy to believe that which appears far away from what is possible because of the weakness of the inner eye, I will be satisfied if we avail ourselves of those disciplines as aids to a most noble and most useful cognition, and that more of mankind avails itself of the arts to which they are devoted to sustain their lives or their names, as long as it pleases God for us to see the triangle of an infinite straight line, and the centre larger than the circumference. Be well.[2]

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[1] The Dedication in the printed edition presents the following differences from that of the Italian menabò manuscript It. IV, 40=5447, n-fol. 12: 1) 'Mattheo' in the manuscript, 'Matheo' in print; 2) 'lo eletto D'Aquileggia' (I the elect of Aquileia) in the manuscript and 'Daniel Barbaro eletto Patriarca d'Aquileia' in print. The dedication itself is practically identical, even as regards punctuation. The very small differences are erasures that eliminate a few words, some of which are then substituted by the author with similar terms.

[2] Cfr. Cusanus, *De docta ignorantia*, I.14 (An infinite line is a triangle) and I.21 (The likening of an infinite circle to oneness).

Foreword[3]

[p. 3] Among the many and illustrious parts of perspective, there is one that the Greeks call *scenographia*, which in my comments on Vitruvius I recall having promised to treat.[4] Although this has many marvellous rationales in its use and exercise that are very useful for painters, sculptors, and architects, it is also very abandoned, not to say disdained and avoided, by many to whom it is more necessary than sought. Already in Athens, when Aeschylus was teaching, Agatharchus created the Tragic scene. About this he left in writing a most beautiful commentary; this having been noticed by Democritus and Anaxagoras, they also wanted to write on the same thing.[5] We read that the ornaments and the apparatuses of the scenes were greatly esteemed by the Romans. Before our own times the painters who found themselves in those days left many fine remembrances of excellent works in this art, in which not only the towns, mountains, woods and buildings are seen to be handsomely drawn and shaded, but also human bodies themselves and the other animals are put into perspective with lines traced to the eye, as to a centre, in a very refined way. But none of them in their writings (as far as I know) has left a memory of how and according to what precepts this was done, unless we want to call precepts and rules some light practices set down without order and foundation and roughly explained. Of these there are also some by Pietro [Piero della Francesca] of Borgo S. Stefano [*recte* of San Sepolcro[6]], and others which might serve for the uneducated. Only a few things have been left by Albrecht Dürer, although they are ingenious and subtle.[7] Serlio has contributed more sketchily, but both of these (I would say) stopped at the threshold of the portal. The painters of our times, otherwise celebrated and of great reputation, let themselves be led by simple practice, and in their panels do not demonstrate anything of this part [of perspective] that is worthy of much commendation, and in written works we find no precepts that are given by them. Federico Commandino, in the *Planisphaerium* of Ptolemy, has given some learned demonstrations, as is always his custom to do, pertaining to perspective. As principles of that, they are useful for exciting the minds of scholars, but with regard to the exercise and introduction to persons who are new to operating with their hands, they are obscure and difficult. So, because the rationale and use of this art is no less pleasant and delightful than necessary and beneficial, I have sought in this part [of perspective] as well, as far as I am able, to benefit many. So, with great study and diligence I have acquired teachers and masters, and all that I could imagine that might help me to this end. Chance led me to one Giovanni Zamberti, a citizen of Venice, who I have used as a guide in the practice of perspective. From him I acquired many things that have been useful and **[p. 4]** pleasing. It is true that the delight of knowing, and of being of benefit, made me add to his rules not just a little study and effort so that I could, with mathematical precepts and rationales, prove those things that he, operating simply with the set square and compass, showed me. Thus with the laws of science I gave strength to the experiments of art. Because I feel I have achieved this, I feel that I can confidently exhort those who would evade effort, and with this promise alone reawaken them to the study of perspective: that among the arts that they want to demonstrate with some exercise and work, there is none that has terms that are more certain and sure, nor is there one that can be more readily reduced to perfection than the part of perspective that will be treated here. Having thus situated its rationale (as Vitruvius says[8]) in knowing how from a certain and determined point the rays of vision must be extended so that they correspond to natural lines,[9] so that from an uncertain thing certain images of the buildings in the paintings of the scenes correspond to what is truly seen, and so that those things that are drawn in the front and plane surfaces appear in relief. To be sure, all of this practice resides in only three terms and in the knowledge of them, that is, eye, rays, and distance. Of these things, the most certain teachings are given, and the most delightful practices are experienced. And because what I have said is sooner had with doing than with words, I will commence, with divine help, to keep my promise.

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[3] The Foreword of the Italian menabò manuscript It. IV, 40=5447, fols. 1r-1v is practically identical to the printed edition, while the Foreword in the Latin manuscript Lat. VIII, 41=3069, fols. 1r-1v, corresponds to a faithful translation of the printed edition, with the sole exception that Piero della Francesca is not mentioned. The extended Italian manuscript It. IV, 39=5446 is without a foreword as such, but a summary of the arguments that will be developed successively for the printed text is found on fol. 298r (*Parte quarta che tratta della scenografia. Proemio*). These few lines are important in that they add information about Barbaro's intentions regarding the composition of a treatise on perspective and the role of Giovanni Zamberti as a preceptor.

[4] In his commentary on Vitruvius, Barbaro refers to his the unpublished treatise twice: Barbaro (1567, pp. 257 and 399); see Williams (2019, pp. 421 and 664).

[5] This is related in the Preface to Book VII in Vitruvius (VII.Pref.11).

[6] Cor*recte*d in the Errata on an unnumbered page at the end of the book.

[7] Barbaro drew on Dürer's *Underweysung der Messung* (1525) (see, for example, Part VI) and his *Vier Bucher von menschlicher Proportion* (1528) (see Part VIII).

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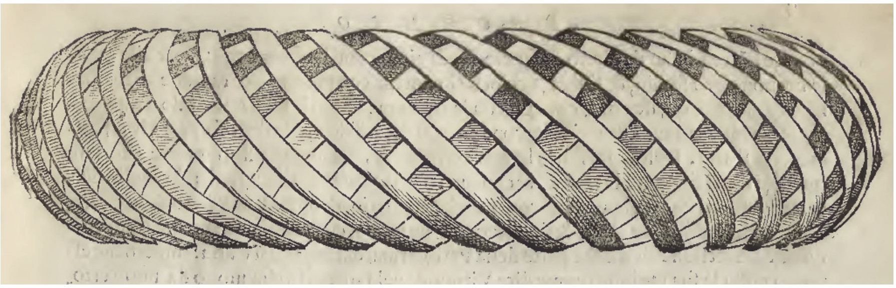
[8] Cfr. Vitruvius (VII.Pref.11).

[9] In Part I, Chap. V, Barbaro distinguishes between 'natural lines' and 'mathematical lines'.



[Part I, setting up a perspective]

The Division of the Treatment of Perspective[1]

[p. 5] I will treat that part of perspective that the Greeks call *scenographia*, that is, the description of the scenes, which, with admirable rationale (*ragione*) of lines regulated by a point according to the distances, shows the surfaces of bodies, the reliefs, the moving away and coming back, the projections of the fabrics of all the bodies, and even more the profiles and parts of the inside and those that are in the faces opposite those that are seen, perceived only by the imagination with marvellous and delightful deception of the eye. This is something very useful to many makers, as will become clear to those who wish to consider it diligently.

The present treatment has nine principal parts.

In the first will be ordered the principles and the foundations of perspective and the things that it is necessary to know, or to presuppose, when one begins to work.

In the second will be treated the way in which must be drawn the plans of the bodies in the planes, which is called *ichnographia*, that is, description of the plan, in both perfect and degraded form.

The third will demonstrate how to elevate (*levare*) the bodies from the plan and form the elevations, which is *orthographia*, that is, description of the thing seen straight on (*dritto*), as they say.

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[1] The Italian menabò manuscript It. IV, 40=5447, fol. 2r, is practically identical with the printed edition, while this part is missing in the Latin manuscript Lat. VIII, 41=3069. In the extended Italian manuscript It. IV, 39=5446 on fol. 85v is found a draft of a list of the topics.

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In the fourth will be described the manners of scenes, with the parts of the buildings pertinent to architecture. Thus the present work is called *scenographia*, that is, the description of scenes.

The fifth will set forth a fine and secret practice of perspective that does not allow the thing painted to be seen except from a certain and determined point, beyond which what is painted cannot be distinguished.

In the sixth will be practiced the way to locate the points and circles of the sphere in the plane, according to the ancients.

The seventh will treat light and shadows.

The eighth will touch on some things pertinent to the measures of the human body, in order to both foreshorten it and to transport it from one scale (*quantità*) to another.

In the ninth will be formed many instruments to put things in perspective, for the convenience of many who only wish to practice.

Chap. I. The ordering of the principles and foundations of perspective

To proceed in an orderly fashion, it is necessary to know what the intention of perspectivist is, so that from this cognition can be determined which parts are appropriate for him and find the principles and foundations of perspective. So I say that the intention of the perspectivist is none other than to draw in planes or on panels subjected [to sight] all the forms or figures that are visible, and make them appear in the way that their resting position, location, and distance requires them to be. It being thus, there is no doubt that we must consider the eye that sees; the way in which it sees; the thing which is seen; the distance from which it is seen; and the plane on which the perspectivist must draw the things that are to be seen. I will treat these things clearly in the ordering of the principles, and first the eye.

[p. 6]

Chap. II. On the eye

The eye, called the centre, mark and point by the perspectivists, is the starting point and foundation of the entire investigation and demonstration of perspective, because in that is the point and tip of that pyramid that is usually made in the manner of seeing. I will speak of this briefly. We must imagine that the thing seen is like the base of a pyramid that is formed by the rays of vision, which depart from the point of vision and arrive to the surface and edges of the thing seen. So, by the concourse of the rays of vision into the centre of the eye are made angles, under the ratio of which the things seen are variously represented, as will be said below. The eye is that to which refers the general denomination of perspective, which by the Greeks is called *optica*, by the Latins *prospetto*. By this name is not meant a simple seeing, but an aware and considered seeing. Simple seeing is none other than naturally receiving the form in virtue of vision and the similitude of the thing seen, but aware and considered seeing, in addition to the simple and natural reception of the form, includes the consideration and investigation of the way of seeing. So the simple perception is an operation of nature, and *prospetto* is the office of reason. Now this is not the place for us to go into the question, as is usually done, of whether vision is done by sending the rays out from the eye to the thing seen, or by receiving them in the eye from the thing seen. Whatever way this may be has no place in our rules and precepts, since however it may be, the eye is placed as the centre and point in which all the rays concur and make the tip of said pyramid.

Chap. III. On the way of seeing

The eye cannot see except by way of a straight line, so it is necessary that from all points that are on the surface of the thing seen can be drawn some straight lines to all the points of the surface of the eye. This is none other than sending or receiving the rays so that the similitude of the things seen, called by naturalists the 'visible species', can arrive to the eye. From this it proceeds that the species of the thing opposite vision, called the object, are ordered and arranged on the surface of the eye and received into the soul in the way in which they are ordered and arranged in the plane or surface of the object. Thus consequently it happens that seeing is made in the shape of a pyramid whose tip is in the eye, and whose base is in the surface of the thing seen, and that seeing is done by straight lines. This can be declared by Apollonius in the fourth theorem of the first book of the elements of conics, and by the *versa* of the fourteenth proposition of the eleventh book of Euclid.[2] In our Latin treatment we have demonstrated this at length in reasoning about perspective. Here we are satisfied with the example marked F (Fig. 1.1), in which the pyramid of vision is understood. The eye is at letter *a*;[3] the thing seen at *bcde*; the rays are *ab*, *ac*, *ad*, and *ae*, which concur in the eye and form the pyramid of seeing, whose base is *bcde*. It is then necessary to imagine that from every point of that base come **[p. 7]** the rays of seeing to the eye, and likewise the edges, so that everything is seen.

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[2] 'Planes at right angles to the same straight line are parallel'.

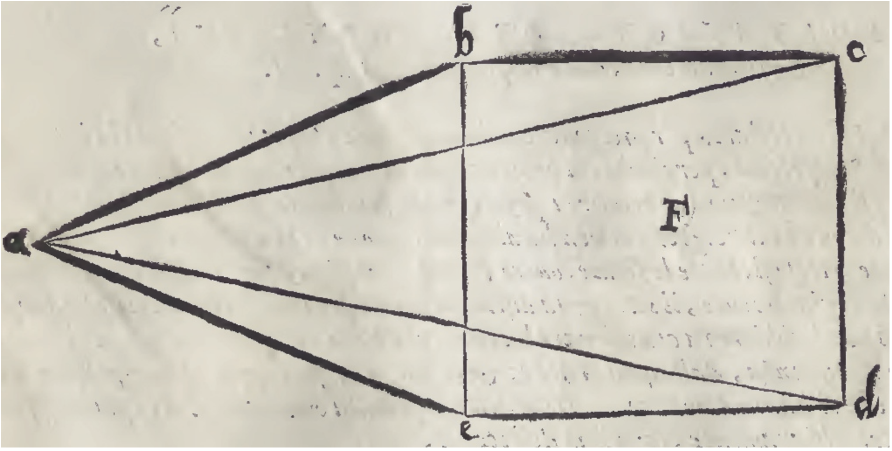

Fig. 1.1 **[p. 6]**

Chap. IV. On the thing seen

By means of light are seen the shapes and magnitudes, colours and forms of things. But for our purposes we will take the magnitudes, continuing the declaration of the things said above. I say that it is necessary that the thing that is to be seen be at some sensible quantity with respect to the surface of seeing. By surface I mean that superficial part of the eye opposite the surface of the hole in the black of the eye, which is called the 'uvea'; some say that it is so called from its colour. If the thing seen were not of a sensible magnitude, the eye would not receive that effect that the form of the thing seen must make in it. How this is understood will be declared below in the treatment of distance from which is seen, where everything is considered minutely, according to the rationale of the naturalists, since that does much for our understanding.

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[3] Remarked in the Errata on an unnumbered page at the end of the book.

Chap. V. On distance

In addition it is necessary that between the thing seen and the eye there be a certain distance, since it is said by the philosophers that sensible things placed on top of the instrument of sense take away the power to sense them. While this is true universally, it is most clearly demonstrated in the sense of sight, because vision cannot receive any object without light, since it is light that makes it so that the bodily form becomes spiritual, and by which the visible species multiply and are made apt for being comprehended by the sense of sight, as the naturalists say. This is because (and this is to our purpose) in seeing are sought some angles, which would not be there if the visible things lay on top of the eye, which is the instrument of seeing, or if they were larger or smaller than they would need to be. It is thus necessary that the distances correspond to the eye with certain proportionate ratios of rays and angles so that how many and what they are, seeing them, can be more certainly considered. So that what I have said can be more clearly understood I will reprise from natural and mathematical principles some useful and delightful things to state the ways of seeing that are apt and appropriate for our purpose.

So, I say that you can't see under any angle whatsoever, since there are natural angles and mathematical angles. The mathematical angle, because it is made by non-direct concourse of lines in a point (because lines have no width), is infinitely divisible, since the mathematical point, likewise considered immaterial, has no parts. The natural angle, [natural] line and natural point, because they are things conjoined to material and have forms that are subject to the sense, are divisible and terminated. This is said in natural things to be the minimum which, in the smallest quantity that can be found, can conserve its form and perform the operations consequent to that form. Since the natural forms are conserved in a determinate magnitude and each performs its operations, if they were divided into smaller parts, they would lose their name and rationale and could not operate, and would almost go out of natural terms and pass into mathematical terms. Therefore, in perspective, which we are treating, we reason about the natural signs, lines and angles. Thus perspective (like many other branches of knowledge) is subject to two sciences, that is, to natural science and to geometry. From geometry it receives the line and its rationale; from natural science it receives seeing. And from the two sciences and rationales put together it receives its name, and the rationale of its subject, as do those sciences that are called subaltern.[4] So from the line and seeing, the visual line (as I would call it) takes its name; this is comprised in a single voice used by the perspectivists: ray. The ray, insofar as it is straight and its middle parts do not overshadow its ends [i.e., it is not curved], when considered mathematically, loses its width, like the simple line, but insofar as it serves the office of seeing, it becomes sensible and is a natural line. Therefore, by rays that concur in a point are made natural angles, that is, sensible, under the ratios **[p. 8]** of which things are seen.

But it is necessary to note that while it is true that you cannot see any magnitude unless it is placed under some angle, it cannot be seen under just any angle. So, it cannot be seen under the least angle [0°], nor under the right angle [90°], nor under the greatest [180°], nor under the angle of contact, as shown by the perspectivists. It cannot be seen under the least angle (as is manifest) because it happens that the continuous quantity can be divided[5] to infinity; nor does its division proceed under a sensible form, until it is possible by the divided part to come into cognition of the species. The continuous quantity is infinitely indivisible in potential and not in act, as the mathematicians say. This is the reason why nothing can be seen under the angle of contact, because, with respect to rays, it is indivisible, and the angle under which is seen must be divisible by the axis of the visual pyramid, as the perspectivists say, and that cannot be done in contact.

Nor can any quantity be seen under the right angle. This comes from the constitution of the eye, as the speculators[6] say. Since that part of the black of the eye, which we said was called the uvea, which is missing the rotundity of the sphere, is less than the fourth part of the largest circle, and since the right angle is comprised by the fourth part of the circle, the rotundity of the uvea not arriving at the necessary fourth, then necessarily it is not possible to see under the right angle. And if it is not possible to see under the right angle because of the rotundity that is lacking in the uvea, then necessarily it is not possible to see under the obtuse angle. If it seems to the inexpert to be otherwise, this proceeds from the extremely fast movement of the eye, which runs from one side to the other thanks to its mobility, and deceives those who do not avail themselves of reason. So many painters make things that are unseemly, because they don't understand this rationale, as I will say below. It therefore remains that things are seen under the acute and natural angle, from the ratio of which comes the appearance of visible things and the principles of perspective, which are the following:

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[4] A subaltern (or subordinate) science takes its principles from a higher science; the concept was delineated by Aristotle in *Posterior Analytics* I.13.

[5] In the Errata on an unnumbered page at the end of the book, the original *vedere in infinito* is cor*rect*ed to *dividere in infinito*.

[6] For Barbaro, a speculator (*speculatore*) is an investigator of philosophical questions.

Those things that are seen under equal angles appear equal.

The things that are seen under larger angles appear larger.

Those things that are seen under smaller angles appear smaller.

Those things that are seen under more angles are seen more distinctly.[7]

In sum, the ratio of the angles makes the things appear right, left, high, and low, as occurs when seeing is done under right, left, high and low angles.

These things, though they are short and few in number, are however many and large in virtue, and admirably serve our purpose.

And so that I may satisfy those who are not exercised in mathematics, I will say clearly what I mean. It must be known that the measure of the angles is taken from the parts of the circumference that are embraced by those lines that make the angles. Here is the example in the figure G (Fig. 1.2).

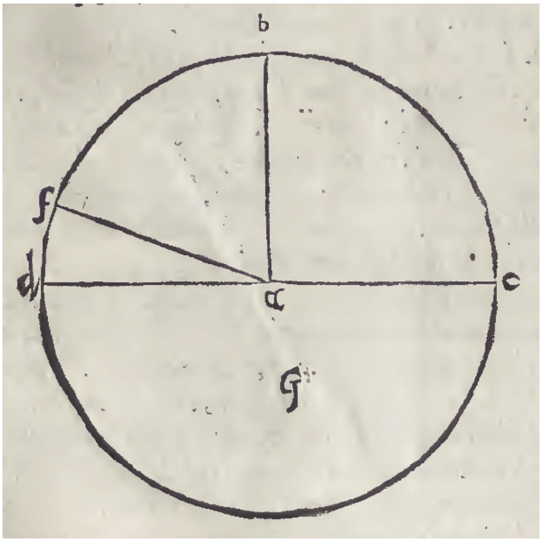

Fig. 1.2 **[p. 8]**

The lines that make the angle *bac*, which is a right angle, embrace a larger part of the circumference of the circle *dbc* than is embraced by the lines that make the acute angle *baf*. So angle *bac* is larger than angle *baf*. Similarly, this

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[7] Here he says *distintamente*; below he says *certamente*.

[*baf*] is larger than angle *fad*, and both are acute angles. Angle *fac*, which is an obtuse angle, is greater than all of the previous angles, because it embraces a greater part of the circumference than the others.

[p. 9] Now I will speak about those things which, seen under equal angles, appear equal. It is a wonderful thing that under the equality of angles many things that are unequal appear equal. This can be seen in the figure M (Fig. 1.3), where the eye is *a*, the rays are *ab* and *ac*, which make the angle *bac*, and the different magnitudes are *de*, *fg*, *hi*, and *kl*, which are different and unequal.

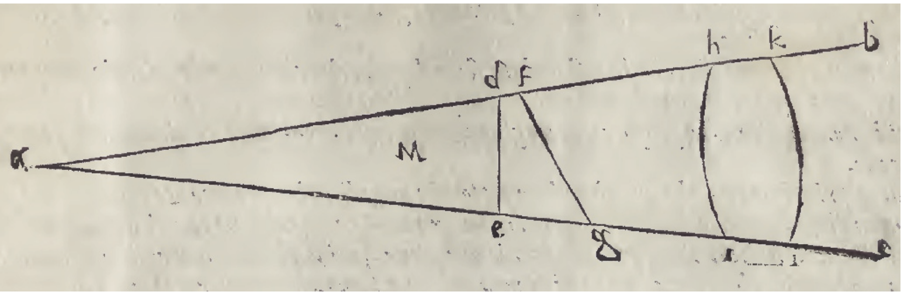

Fig. 1.3 **[p. 9]**

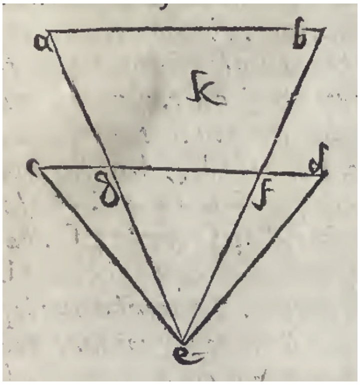

Fig. 1.4 **[p. 9]**

Because these are seen under the same angle, which serves equally for all, they appear equal. In my Latin [treatise on] perspective I put the proofs of all the foundations of this art, but for now I leave them out so as not to slow down the scholars of the practice of perspective, who must take these principles as most certain.

The second principle was that those things that are seen under larger angles appear larger. This can be seen in the same way by means of the example in figure K (Fig. 1.4), in which there are 2 equal magnitudes *ab* and *cd*, seen under different angles, from which one will appear larger than the other, that is, *cd* will appear larger than *ab*, because the angle under which *cd* is seen, which is *ced*, is larger than the angle under which *ab* is seen, which is *aeb*.

Likewise from the same figure is declared the third principle, which is that the things seen under a smaller angle appear smaller. So magnitude *ab* appears smaller than magnitude *cd*, since *ab* is seen under the angle *aeb*, which is smaller than the angle *ced* under which is seen magnitude *cd*. By the things said above, *gf* will appear equal to *ab*, because the two of them are seen under equal angles.

From these things can be understood the rule and form of Albrecht Dürer's quadrant, with which he proportions the letters or figures that are on the height of some column or wall.[8] From this the painters and the architects will divide the heights into proportional parts.

Let, for example, the height of the column be *ab*, on which are to be placed the letters or figures in proportion such that the upper ones appear to be equal in height to the lower ones. Make a quadrant, or portion of a circle that is, for example, *ef*, from the centre *c*, and divide that portion into equal parts. Let them be placed such that with due distance the point *c* directly corresponds [on a horizontal line] to the point *b*, where the feet of the letters or figures are to begin. Then through the point of view you look on the divisions made in the portion of the circle, and come to the view of the column *ab*, and there mark **[p. 10]** *g*, *h*, *i*, *k*. Thus having placed the letters where the rays terminate on the column, there will be also terminated the magnitudes of the letters or figures, which will appear equal as long as the division of the quadrant, or portion of the circle, is made equally. This is shown in Fig. I (Fig. 1.5), since things that are seen under equal angles appear equal.[9] The converse is also true, that if the arcs are equal, then the angles are also equal, by the twenty-sixth and twenty-seventh propositions of the third book of Euclid.

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[8] Cfr. Dürer (1525, figure following fig. 27, pages not numbered). Barbaro returns to this point at the end of this chapter.

[9] This is also illustrated in Serlio (1566, p. 9).

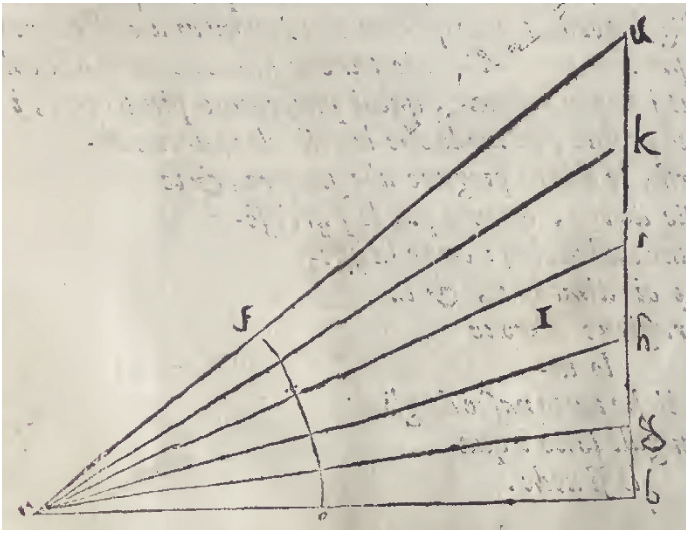

Fig. 1.5 **[p. 9]**

From this it is manifest that if the things are under either double or triple angles, they will appear to be the double or the triple. I say 'appear' because in truth they are not the double or the triple. The ratio at the bases of the triangles do not follow the proportions of the angles opposite them, as Ptolemy says in the first book of the *Almagest*.[10]

The fourth principle is that things seen under more angles will appear more certainly. This is manifest, because if we take 2 equal magnitudes that are parallel to each other, of which one is closer to the eye than the other, that which is closer will be seen under a larger angle than the one that is farther. But the larger angle can be divided into more parts than the smaller angle. Thus the closer magnitude will be seen under more angles than the one farther away. Since the axis of the visual pyramid that arrives at the surface of the thing seen is shorter in the things closer to the eye than the axis of the pyramid that arrives at the thing seen that is farther away, it follows that the things seen under more angles are more distinctly and clearly seen.

With regard to these things it must be noted that lines or other parallel quantities, be they high or low or on the sides, will appear to the eye as though wanting to run together and conjoin the farther away they are from the eye. Here is the example in the figure L (Fig. 1.6).

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[10] Cfr. Ptolemy's table of chords, in Toomer (1984, I.10 'On the size of chords' and I.11 'Table of chords'). See also Maor (2011, Chap. 2, 'Chords').

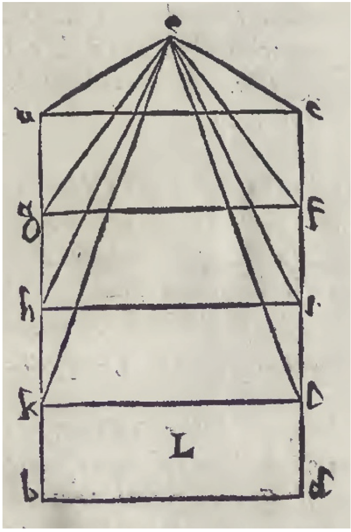

Fig. 1.6 **[p. 10]**

Not only will sides *ab* and *cd* appear to come closer to each other with the parts farther from the eye *e*, but also the lines *ac*, *gf*, *hi*, *kl* and *bd* will do the same, so that *bd* will appear closer to *kl* than *kl* to *hi*, and *hi* closer to *gf* than *gf* is to *ac*, since *bd* is seen under a smaller angle than *kl*, and *kl* [is seen under a smaller angle] than *hi*, and so too with the others. Similarly, the parts of the lines *ab* and *cd* that are farther from the eye (as I have said) will appear to come closer together than those that are closer [to the eye], since the spaces that are between the parts that are farther away will appear closer, because they are seen under smaller angles. From this it comes that if it is desired to place in perspective a covered and colonnaded portico, when we stand with the eye in the middle on one end, it will appear to us that the ceiling lowers and the pavement rises little by little as it moves away from the eye. Similarly parts of the wall on the right that are farther from the eye will appear to draw closer to the columns on the left. So too the spaces between the columns, which will appear smaller the farther away they are from the eye, such that high things appear to lower, low things to rise, the right to bend towards the left, and the left to the right, the farther things are from the eye, as experience shows us.[11] The rationale I teach elsewhere.[12] All of this is born from the angles under which things are seen. **[p. 11]** These things will be manifest when there is a square, over which stands the eye, and a ray from the eye falls in the middle of the square, where the two diagonals cross. The diagonals between themselves, and the sides will appear to be equal. This is seen in figure Q [Fig. 1.7], in which the square is *cdef*, the eye *a*, the diagonals *cf* and *de*, the ray that falls from the eye to the middle *ag*, and the rays that go from the eye to the corners of the square *ac*, *ad*, *ae*, *af*.

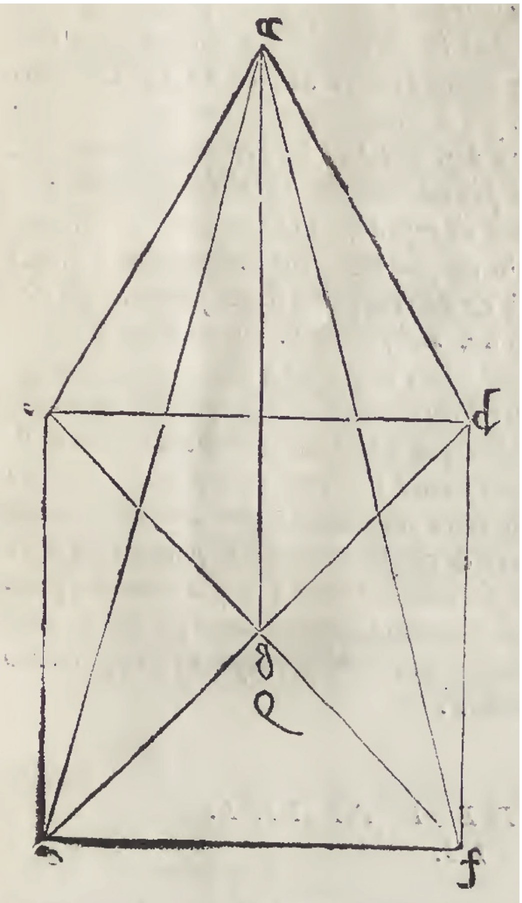

Fig. 1.7 **[p. 11]**

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[11] In his second book Serlio shows an arched colonnade in single point perspective that illustrates this principle. See Serlio (1566, p. 19).

[12] See below in Part IV, esp. Fig. 4.21 **[p. 153]**.

Where the sides are seen under equal angles, they appear equal, as they are. Likewise the diagonals appear equal, as they are, because they are seen under equal angles, as Euclid proves in his *Prospettiva*,[13] and we have proven in our Latin treatise. With the same ways and procedures we will proceed when we wish to demonstrate the same of all figures with straight lines and many angles that are regular, such as those of 5, 6, 7, and more angles, and also of the circular figure, where it is manifest that such figures are seen as they are if from the eye over them falls a line at right angles to their centres.

But if the eye is set in another place, the figures change their appearances. This is the reason why the circle will sometimes appear oval, sometimes it shows an appearance of other curved lines that are not portions of circles. Further, because of the mutation [of the position] of the eye, each figure that is set in the same plane as the eye, it happens that what is seen is not the base of a figure such as a pyramid, but rather the base of a triangle, of which only the base is seen, as is manifest below and also in Part V.

In addition to the things said above (so that perspective can be practiced with greater surety), it must be known that if the eye from above looks at the surfaces below set in the same plane that are farther from the eye, they will appear higher, as in figure S (Fig. 1.8), where the surfaces seen are *be*, *ed*, and *dg* lower than eye *a*, of which *ed* and *dg* are equal. I say that *dg* will appear higher than *ed*.

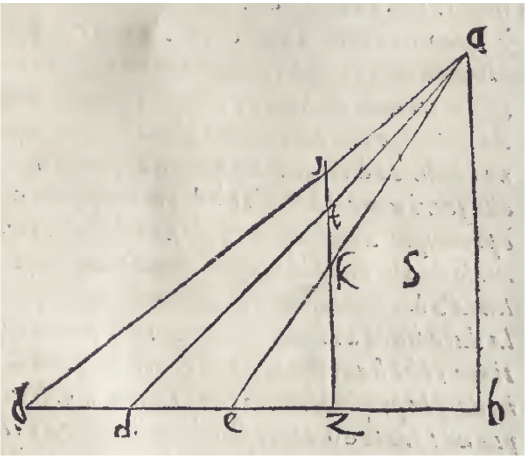

Fig. 1.8 **[p. 11]**

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[13] The first Italian translation of Euclid's *Optics* was that of Egnazio Danti, *La prospettiva di Euclide* (1573), five years after the publication of Barbaro's treatise on perspective. Meanwhile the first English translation appeared centuries later, Harry Edwin H. Burton's 'The Optics of Euclid' (1945).

Line *ab* falls from the eye *a* vertically onto line *bg*, on which is set point *z*. On *z* is drawn at right angles line *zi*. Then rays are sent from the eye *a* to the sides of the surfaces set in plane *bg* to points *e*, *d*, and *g*; these cut line *zi* at point *i*, *t*, and *k*. Because the eye sees those surfaces through the points that are in *zi*, point *i* is higher than point *t*; point *i* corresponds to point *g*; point *t* corresponds to point *d*. So, since *i* (as I have said) is higher than point *t*, and *t* higher than *k*, it follows that *g* will appear higher than *d* and *d* higher than *e*. **[p. 12]** A similar thing is declared of the surfaces in the same plane set above the eye: those surfaces that are farthest from the eye will appear to be lower. I say they will appear to be lower as shown in figure T (Fig. 1.9).

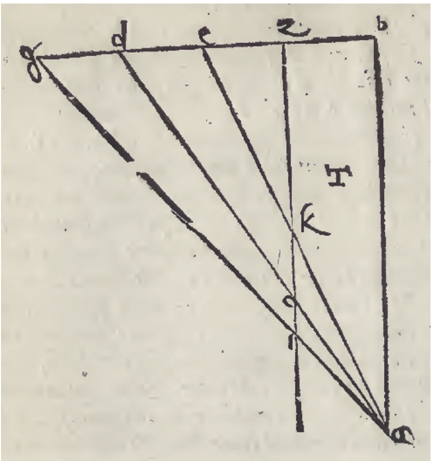

Fig. 1.9 **[p. 12]**

Let the point of seeing be in *a*, set below the plane *bg*, in which the surfaces to be seen are *be*, *ed*, *dg*, the rays are *ab*, *ae*, *ad*, and *ag*, and line *ab* falls at right angles to *bg*. I say that line *dg*, set as one of the surfaces, will appear to be lower than *de*, and *de* will appear to be lower than *eb*. As above, let *zi* be parallel to *ab*, and be cut by ray *ag* in point *i*, by ray *ad* in point *c*, and by ray *ae* in point *k*. Thus by the things said above, the visible species of point *g* will appear lower than the species of point *d*, and this will appear lower than that of point *e*. This is because the point *i*, through which is seen the point *g*, is lower than point *c*, through which is seen point *d*, and similarly the point *k*, through which is seen the point *e*, is higher than point *c*. According to the arrangement and location of these lines, the species of these marks are impressed upon the eye.

Chap. VI. On the division of planes

Before I come to the division and compartition (*compartimento*[14]) of planes, I will declare with examples that a line is to be divided according to a given ratio (*proportione*[15]), and I will give other manners of proportional compartition to make the things that follow easier. So, I say that if there is a given line that we wish to divide according to the ratio that there is between two given lines, it must be done in this way. Let there be line *ab* to be compartitioned according to the ratio that there is between two lines, *d* and *c*, and let *c* be the double of *d*. Put these two lines together in a single length, and make one line of the two [this will be line *ad*]. You will draw it [*ad*] in such a way that it makes an angle with line *ab*. At the other end, where the line *d* is, you will draw a line to point *b*, such that it will make triangle *abd*. Then from point *c*, which is the beginning of line *d* and the end of *c*, you will draw, towards *ab*, a line parallel to line *bd*. I say that this line will have divided the given line *ab* in point *e* into that ratio that line *c* had to line *d*, that is, the ratio of the double [2:1], so that the part *ae* is the double of part *eb*, as seen in figure P (Fig. 1.10).

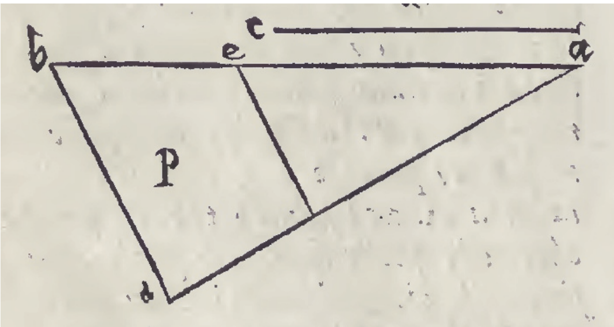

Fig. 1.10 **[p. 12]**

A similar thing will happen to you in whatever ratio you want to divide the line *ab*, as you will discover through practice. We have given the proof elsewhere.

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[14] In the term 'compartition', as opposed to 'division' or simply 'partition', the prefix 'com-' implies division of a whole into parts that relate both among themselves and to the whole. For a discussion of the term, see Williams (2019, p. lxxxix).

[15] Barbaro refers to *proportione*, which in modern terms would refer to ratio (that is, a comparison of quantities). This is consistent with the vocabulary of the day; see Silvio Belli, *On Ratio and Proportion* (2003, p. 15). The related term, *proportionalità*, refer to what we today call proportion (that is, a comparison of ratios). In what follows, for the sake of clarity the modern terms have been adopted.

[p. 13] In addition to this it must be known that if between two outer rays of a given magnitude, a line is drawn parallel to that magnitude, the other rays that are in the middle will cut the drawn line in the same ratios in which the given magnitude is cut. Let the given magnitude be *bc*, the eye *a*, the outer rays *ab* and *ac*, and the intermediate rays *ad*, *ae*, *af*, *ag*, and *ah*, which divide the given magnitude however you wish. Let line *pi* be drawn parallel to the given magnitude *bc*, and be comprised between the outer rays *ab* and *ac*. I say that the line *pi* will be divided by the intermediate rays into those ratios in which those same rays cut the magnitude *bc*.

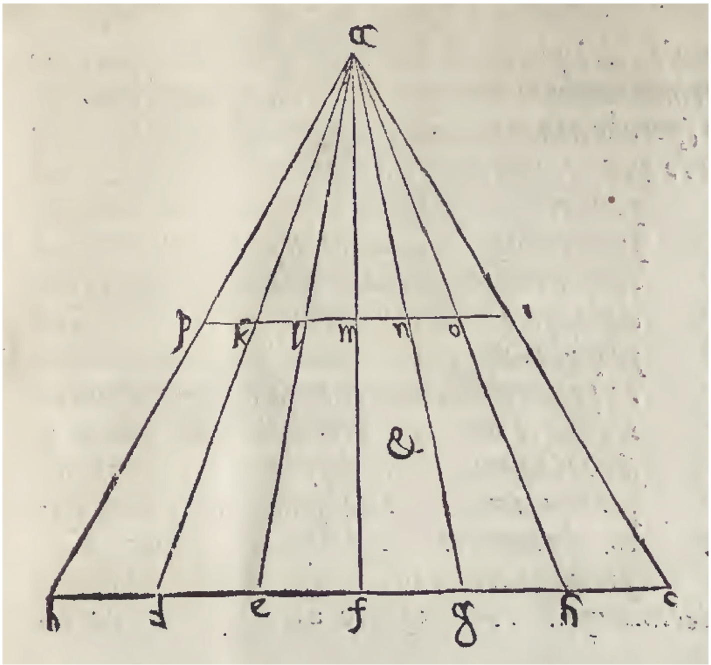

Fig. 1.11 **[p. 13]**

So, the part *pk* will have the same ratio to line *pi* that the part *bd* has to the whole line *bc*; since *bd* is the sixth part of *bc*, then *pk* will be the sixth part of *pi*, and the same is found in the rest, as shown in figure & (Fig. 1.11).

Further, it must be manifest that if two rays are sent from the eye to a line to which is drawn another line parallel to it, that line will be divided and will also divide the rays in the same ratio, as seen for example in figure O (Fig. 1.12), in which the eye is in point *a*, the rays are *ac* and *ab*, which go to line *cb*. Then let there be drawn a line equal and parallel to line *cb*, which will be *fd*. I say that line *fd* will be cut by ray *ac* in point *g*, which will cut the ray *ac* such that *gd* will be the double of the part *gf* of line *fd*, and the part *ga* will also be the double of part *cg* of ray *ac*. This example will be very useful for many divisions of lines and planes in the practice of perspective, and will make many things easy that appear to be difficult to those who do not have the use of compass and set square.

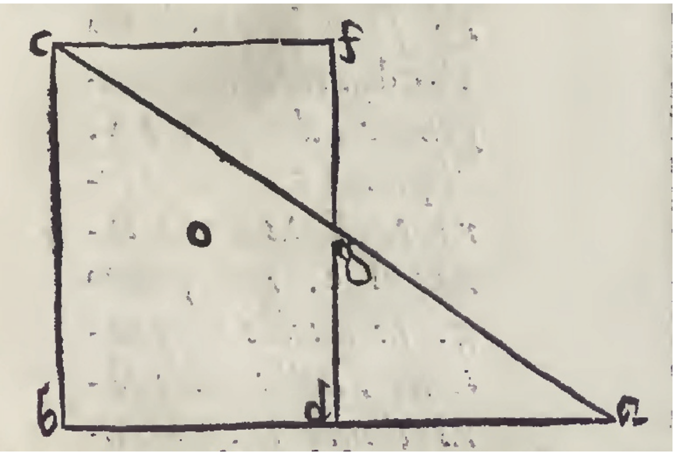

Fig. 1.12 **[p. 13]**

From the things stated above some have found an easy way to compartition lines in however many parts they want, and make as needed a plane divided into many equal parts. They draw parallel lines from one side to the other, and carry the length of the line that they want to divide onto that plane, keeping one end of the line in an angle fixed in a corner of the divided plane, and raising and lowering the other so that it touches that line of the divided plane where they have marked the number of parts into which they want to divide and compartition the proposed line, and thus they find the line divided. So, let the line to be divided be *ab*, and the plane divided beforehand *abdc*. We take, with the compass, the length of line *ab*, and placing the foot of the compass at corner *a* of the divided plane, we place the other on that line of division marked with the number of points into which the proposed line is to be divided. If the line is to be divided into 4 parts, then it will be where the number 4 is marked; **[p. 14]** if into 5, then where the number 5 is; if in 6, over the number 6, and so on according to the intention, as shown in the figure B (Fig. 1.13).

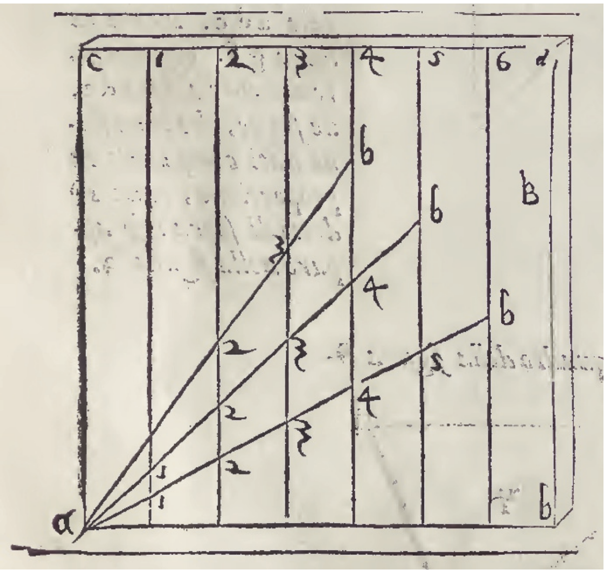

Fig. 1.13 **[p. 13]**

Now I will demonstrate the way to divide the panels and planes on which are to be placed the plans and elevations of bodies in perspective. So, first of all it must be shown how a given part – be it a third, or a fourth, or a fifth, or whatever – can be cut from a given triangle or square or quadrilateral (because it doesn't matter what figure is desired) and how to divide and compartition it according to a given ratio. So, let there be a plane triangle *abc*, which must either be compartitioned into a given ratio, or from which must be taken a proposed part. Let side *bc* be divided according to the given ratio; for now let it be compartitioned into 3 [*recte* 2] parts in double ratio [2:1] at point *e*, such that the part *ec* is double the part *eb*. Then let a line be drawn from point *e* to point *a*. I say that the part *ace* of the triangle *abc* is the double of the part *aeb* of said triangle, as shown in figure Y (Fig. 1.14).[16] If, therefore, the part *abe* is taken away from triangle *abc*, a third will have been taken away.[17]

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[16] The gist is 'as the sides, so the areas': *be*:*eb*=1:2; area *bae*: area *eac*=1:2.

[17] In the Errata on an unnumbered page at the end of the book is the following remark regarding the match of figure Y to its description: The figure Y should be made so that the base *bec* is three times as long as the space *be*. Or make the words at the letter [paragraph] H, to say, 'For now let it be compartitioned into 4 parts in quadruple ratio at point *e*, such that the part *bc* is quadruple of the part *eb*, and under that, 'I say that the part *ace* of the triangle *abc* is the quadruple of triangle *abe*', and under that, 'a quarter will have been taken away'.

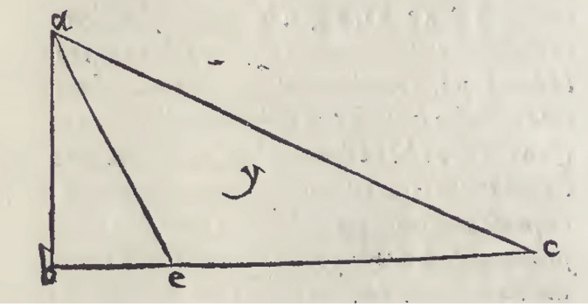

Fig. 1.14 **[p. 14]**

A similar thing is declared in the square *abcd*, with its part *abef*. If in the square *abcd* is drawn the diagonal from point *a* to point *c*, that will be cut by line *fe* in point *g* in that ratio into which is divided the square and its sides. Line *gf* being drawn in triangle *adc* parallel to side *dc*, it follows that the sides are compartitioned in proportion, as said above, and as shown in the figure 2 (Fig. 1.15).[18]

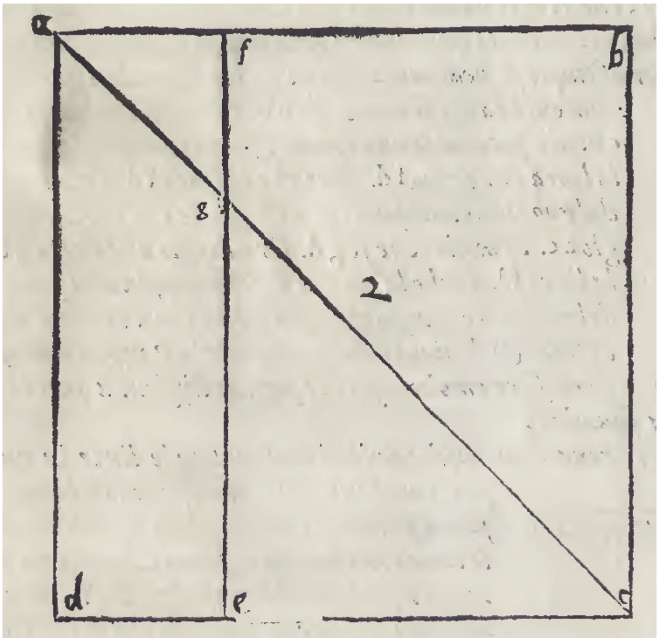

Fig. 1.15 **[p. 14]**

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[18] The gist is 'as the sides, so the sides': *de*:*ec* :: *ag*:*gc*.

The same will be done in the quadrilateral of figure ƶ (Fig. 1.16).[19]

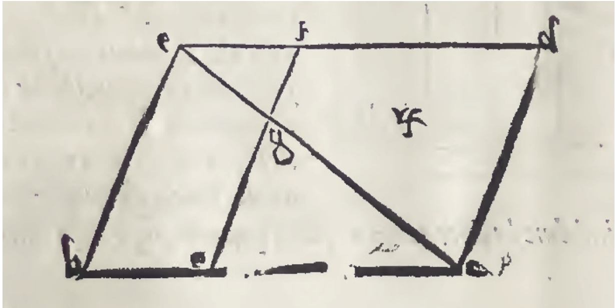

Fig. 1.16 **[p. 14]**

[p. 15] Similarly, it is declared that if the sides of a square or quadrilateral are compartitioned into equal parts, and from the points of the division are drawn lines parallel to the sides, the lines will likewise divide the spaces of the square or quadrilateral into parts that are equal and square, as seen in the square *abcd* in figure I (Fig. 1.17).

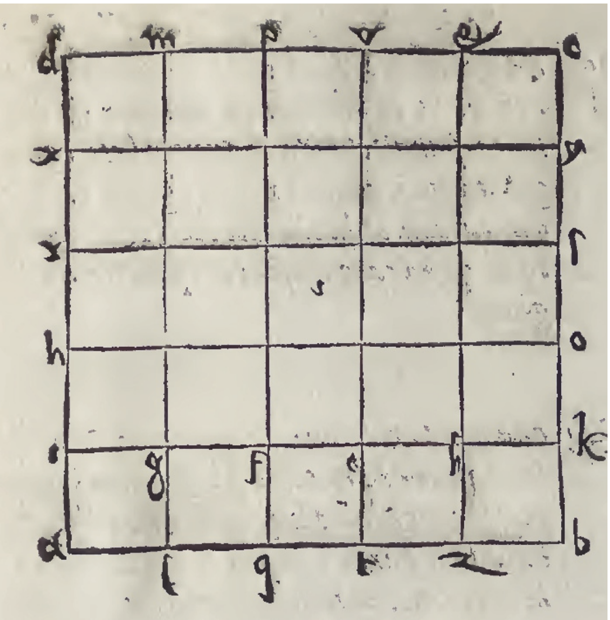

Fig. 1.17 **[p. 15]**

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[19] *be*:*ec* :: *ag*:*gc*.

I say further that if the diagonal of the square is divided into equal parts, and through the points of the division are drawn lines parallel to the sides of the square, those lines will divide the area of the square into spaces that are square and equal. Let the square be *abcd*, whose diagonal *db* is divided into 4 parts in points *e*, *f* and *g*. Let lines be drawn parallel to the sides through the points of division of the diagonal, and for one side let be *hei*, *kfl*, and *mgn* be parallel to sides *ab* and *dc*, and *oep*, *qfr*, *sgt* parallel to the other sides [*da* and *cb*]. I say that those lines will divide the area of the square into square and equal spaces, as can be seen in figure O (Fig. 1.18), and as we have proven elsewhere.

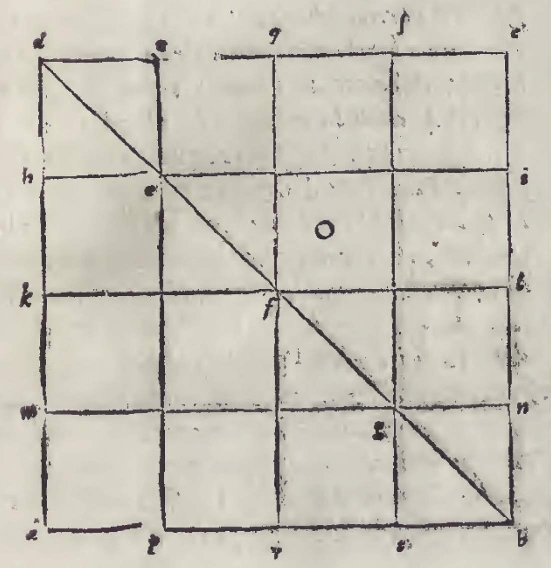

Fig. 1.18 **[p. 15]**

To the things said above I would add that if the side of the square is not divided into equal parts, with that ratio and proportion the square will be divided by the parallel lines that divides the side. If the diagonal is divided, and the lines drawn parallel to the sides of the square through the points of the division of the diagonal, the area of the square will be similarly compartitioned with the same lines and the same ratio into which the side is divided.

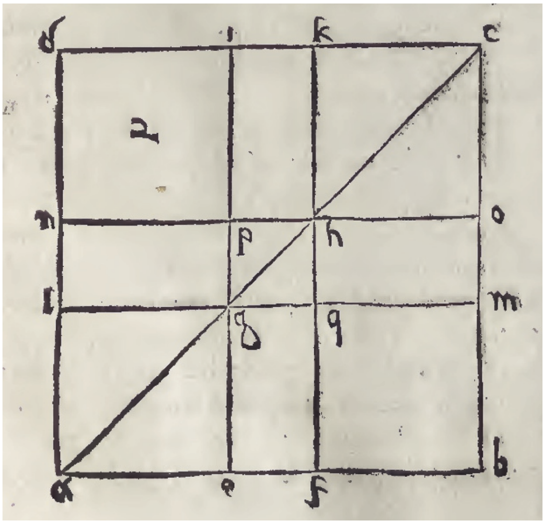

Fig. 1.19 **[p. 15]**

In figure 2 (Fig. 1.19), we see the square *abcd*, whose sides and diagonal are divided into unequal parts, in one direction by lines *npho* and *lgqm*, and in the other by the lines *ipge* and *khqf*. The diagonal is also divided into 3 unequal parts in the points *h* and *g*.

[p. 16] The same happens in those figures that the geometers call 'parallelograms', which for now I will call 'quadrilaterals'. These are made with parallel lines and they have equal sides but not equal angles, as shown in figure 3 (Fig. 1.20).[20] This figure is marked with the same letters as the square above (Fig. 1.19), because the rationale is the same.

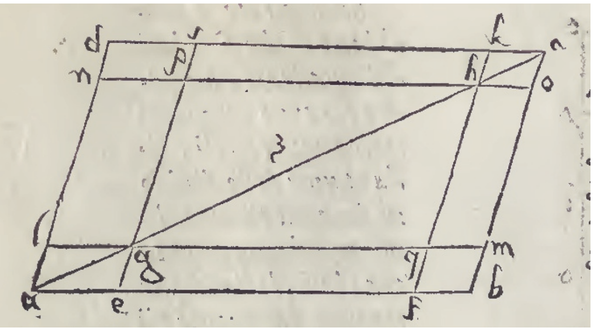

Fig. 1.20 **[p. 16]**

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[20] *ae*:*ef* :: *ag*:*gh*.

In the figures that the geometers call 'trapezoids', which for now we will call 'corbels' (*mensole*), which are quadrilaterals but with unequal sides and angles, although they enclose the same area as above (whether their bases are divided into equal or unequal parts), they still need the following conditions: the sides that are their bases meet in a point; the lines drawn from the compartitioned base arrive to that same point; and finally, that the fourth line of the figure is parallel to the base, as can be seen in figure 4 (Fig. 1.21).

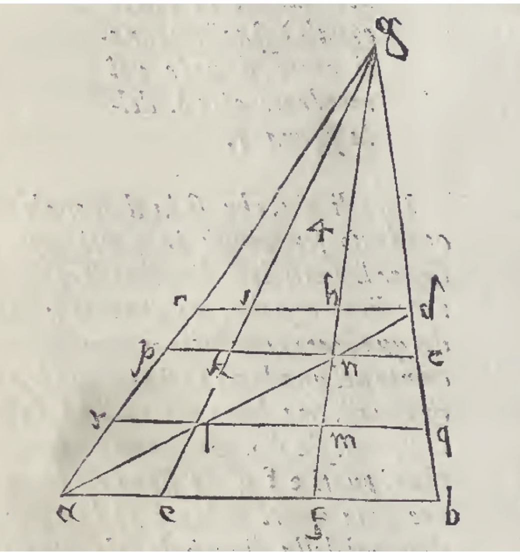

Fig. 1.21 **[p. 16]**

The corbel is *abcd*; its base *ab* is divided into 3 parts, equal or unequal as they may be. The 2 sides *db* and *ac* meet at point *g* drawn from the ends of the base *ab*. The other side, which is *cd*, is parallel to the base [*ab*]. Then the rays of the division of the base made at points *e* and *f* also meet at point *g*; these rays are *eg* and *fg*. Then let the diagonal *da* be drawn; it will be cut by ray *eg* in *l*, by ray *fg* in *n*. From the divisions of the diagonal are drawn *rq* through *l* and *po*[21] through *n* parallel to the base *ab* and to side *cd*. I say that the areas of the corbel *al*, *cm* and *fg*[22] will have the same proportions among themselves that *af*, *ef*, and *fb* have among themselves. I say further that the proportion that is between *ae*, *ef* and *fb* will be the same as that between *rk*, *ln* and *nd*. And likewise *pi*, *kh* and *nd* will be like *al*, *ln*, and *nd*, which are parts of the diagonal, as has been proven elsewhere.

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[21] The *o* looks like an *e* in the figure in the printed edition, but the logic of the drawing in fact identifies it as an *o* (it is a vertex of the quadrilateral *opqr*). In the Italian menabò manuscript It. IV, 40 =5447, fol. 9v it is clearly seen to be an *o*, while in the Latin manuscript Lat. VIII, 41=3069, fol. 10r it is clearly an *e*. The figure does not appear in the extended Italian manuscript It. IV, 39=5446.

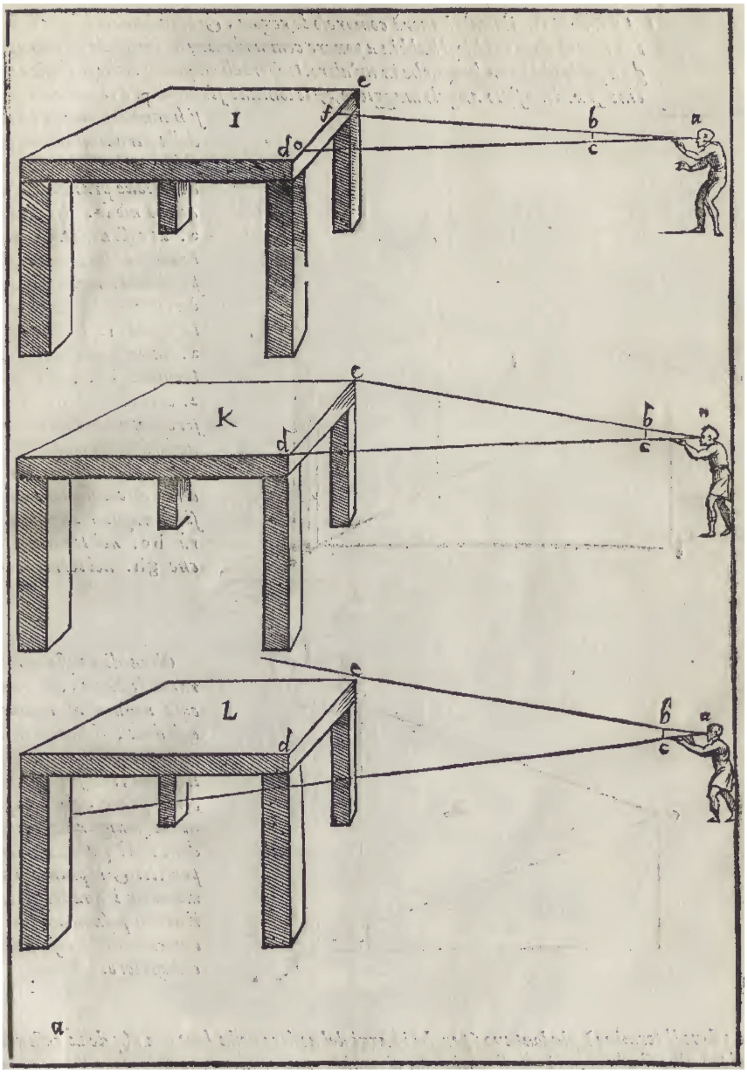

Fig. 1.22 **[p. 17]**

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[22] In the Errata on an unnumbered page at the end of the book these three segments are cor*recte*d to: *al*, *em*, *fq*.

For the use of the practitioners, I want with a single delightful example to declare many things said above, and make an experiment in the following figures I, K and L, which will demonstrate clearly the power of the angles under which thing are seen (Fig. 1.22). Take the measure of a palm as an example, and have a table before you that is 15 or 20 palms long. I say that placing that measure at some distance from your eye, then moving it to a greater or lesser distance away, the measure will seem now greater, now equal to, now smaller than the length of the table. So, if you want to see the measure as less than the length of the table, let, for example, the eye be *a*, and the measure *bc*, and the length of the table *dc*, as in figure I. Make it so that the rays of vision pass through the ends of the measure *bc* and arrive at the points *f* and *o*. This you will do by moving the measure away from the eye. In this experiment it is necessary to note that the space that is between the measure and the length of the table is not seen, but both lengths – that is, *bc* and *de* – are as if in the same plane, as will be recognised with delight through practice, as we mentioned above when we said that the figures change in appearance when the eye changes.[23] But if the measure is placed at another location, as in figure K, then when you have set it up, it will appear to be the same size as the length of the table, since the ends of the measure *bc* and the ends of the length [of the table] *de* are seen under the same angle. But when the measure *bc* is closer to the eye (as seen in figure L), the rays that pass through the ends of the measure *bc* will widen so much that they will go outside the length of the table, and measure *bc* will appear longer than length *de*.

[p. 18]

Chap. VII. Where the eye must be placed

Everyone who has understood the rules and foundations given above can know where the eye must be appropriately placed, and what effect will be had by placing it in one location or another. Therefore, knowing that things close by are seen under a larger angle and the same things far away are seen under a smaller, and that the termination (*termine*) of the pyramid of vision must be placed in the plane, as soon as this consideration is made, it is put into practice in the following way.

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[23] See **[p. 10]**: 'But if the eye is set in another place, the figures change their appearances'.

Let the eye be *a*, the thing seen *bc*, and the panel and termination where the visual rays finish *de*, in a drawing like figure 1 (Fig. 1.23, top), and let the eye be *a*, the thing seen *gh*, the termination *ik* in figure 2 (Fig. 1.23, bottom).

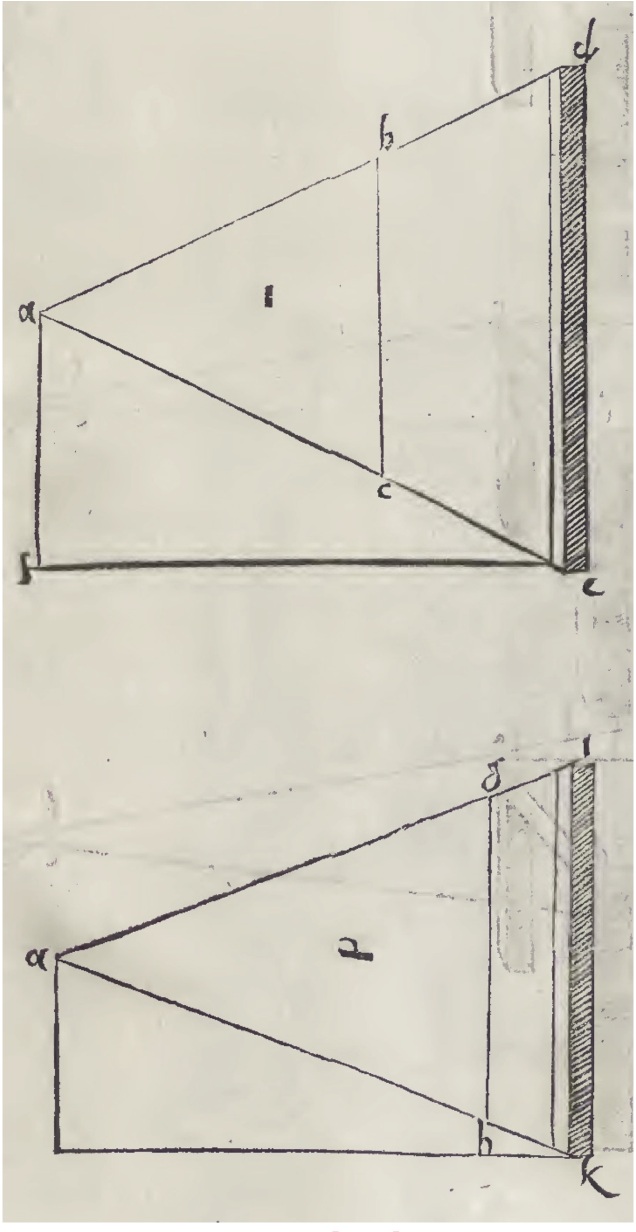

Fig. 1.23 **[p. 18]**

It is certain that *bc*, because it is closer to the eye, will appear larger than *gh*, which is farther from the eye. Consequently *bc* will appear under a larger angle at the termination *de* than *gh* will at termination *ik*.

In addition to this, considering the space that is from the thing seen to the termination, it will be said that if the thing seen is in the termination, that is, on the plane, it will be drawn as it lies and will occupy those similar places where it is set, and so the elevations (*perfetti*) and the plans are formed in the paintings according to the perpendicular placement and touching of the thing seen with them.

But if the termination is farther away (so that the rays of vision widen across the distance from the thing seen to the termination), the ends of the pyramid will comprise a larger space than if the termination, or plane as we want to call it, were close to the thing seen. This is seen in figure 3 (Fig. 1.24), where *a* is the eye, *de* the termination closer to the thing seen, and *fg* the termination farther away. The rays *abd* and *ace* take up less space on the termination, or plane, closer to *bc*, than the rays *abf* and *acg* on the termination *fg*.

[p. 19]

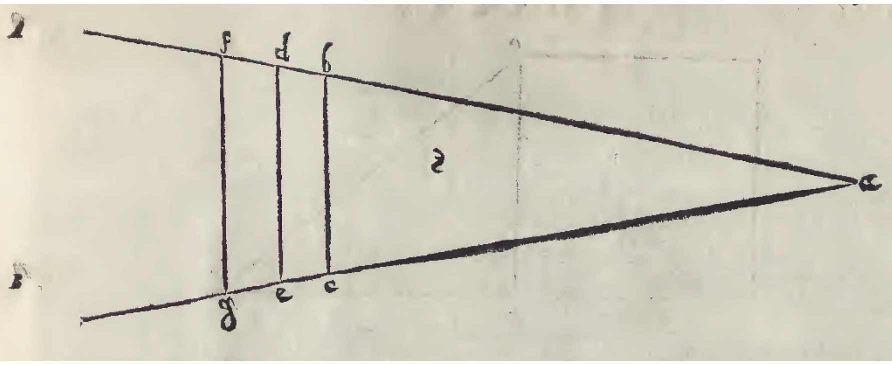

Fig. 1.24 **[p. 19]**

Similarly, wanting to see a thing that is higher, it is necessary to place the vision under higher rays, and so too in the rest. Therefore, regulating according to the rationales given above the distances from the eye to the thing seen, and from the thing seen to the plane, and considering the quality and the site of the rays of vision, the judicious man will place the eye where appropriate to make the effect of vision that he wants. This is a thing worthy of consideration in order to resolve many doubts.

Chap. VIII. On the distance

The distance is regulated by the size of the painting or plane that is drawn on, such that the larger the painting, the larger the distance must be. So, having to make things in the plane proportionate to the size of the plane, and when a man regards a large thing, he withdraws far away, so a larger plane requires a greater distance. That distance, however, must be established as neither greater nor lesser than that at which the thing seen can be seen equally. Therefore, when it is desired to represent a thing to the eye perfectly that is seen under the equality of the larger angles caused by the rays of vision, both in the base, as in the point that terminates in the eye, I want to make an example of that thing that is evident and clear. So, let the eye be *a*, the thing seen *cd*. There is no doubt that angle *a* is larger than angle *c*, as Euclid proves in the eighteenth proposition of Book I [of the *Elements*]. And not being equal (as said), the thing seen cannot be perfectly comprised by the vision, as shown in figure 4 (Fig. 1.25).

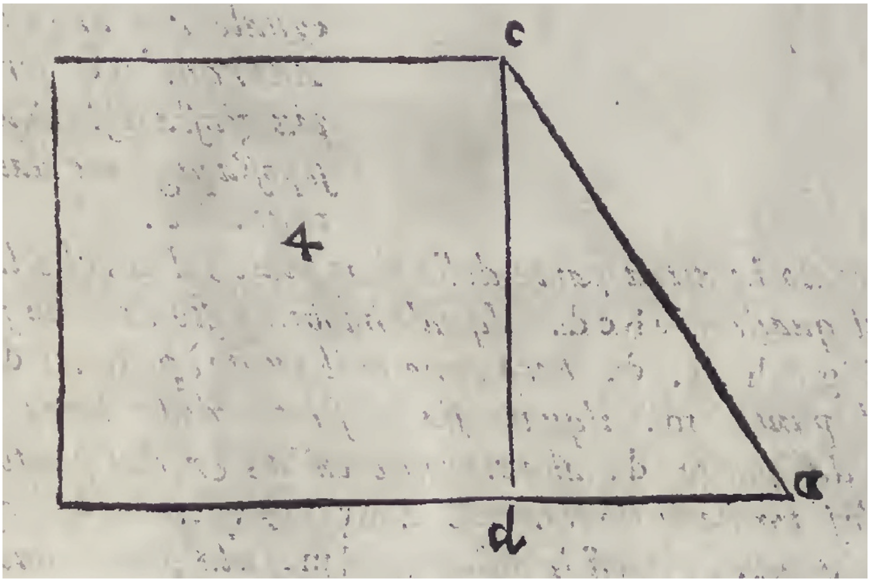

Fig. 1.25 **[p. 19]**

Similarly, if the magnitude *ef* is seen from distance *fb*, since the angle *e* is found to be greater than angle *b*, the magnitude *ef* will not be comprised perfectly, because it is the equality of the angles that makes things be seen perfectly. Here the angles are unequal, as seen in figure 5 (Fig. 1.26).

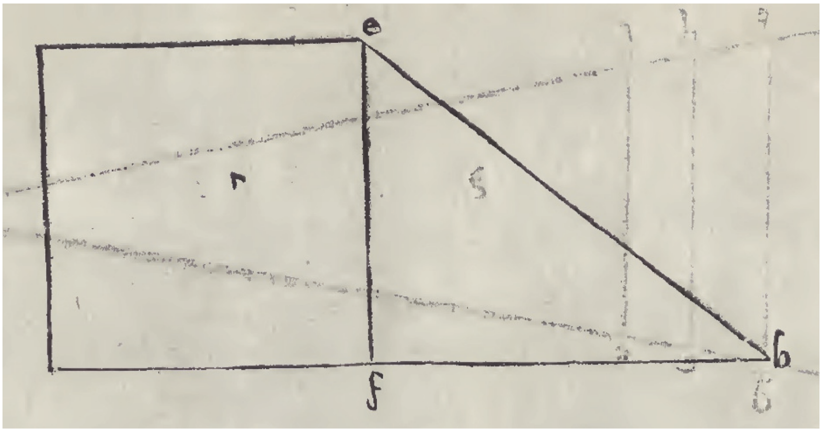

Fig. 1.26 **[p. 20]**

So making the distance so that the angles are equal will make the thing be seen perfectly. How this is done I will show in figure 6 (Fig. 1.27).

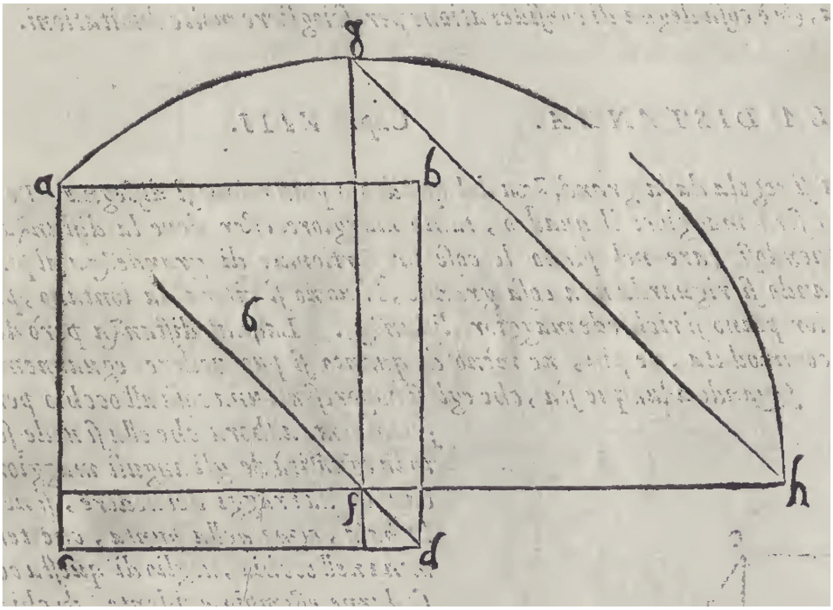

Fig. 1.27 **[p. 20]**

[p. 20] Let the painting be *abcd*, the diagonal *afd*, the height of the eye *ef*,[24] the distance *fh*, which is equal to *af*. Place the foot of the compass in *f* and draw the circle from *h* towards *a*. Draw vertical line from *f* so that *g* terminates on the circumference of the circle, and line *gf* is parallel to line *bd*, such that lines *gf* and *hf* form a right angle at point *f*. Whereby, *f* being a right angle, and the lines *fh* and *fg* equal because comprised by the circle, the angle in the distance *h* will be equal to angle *g*. Therefore distance *hf* will be the correct distance that can be taken from that painting.

If the point of the eye is fixed in another part of the diagonal, I say that the distance is found in the following way. Let the square be *abcd*, which has the eye placed on the diagonal at various points *a*, *e*, *f*, *g*, *h*, *i* and *d*.

I say that if the point [of the eye] is placed in *d*, let there be drawn with the space [radius] *ad* a circle to point *m*, which is the end of the line *cd* extended. Let the line *dbr* be drawn from point *d* to the circumference and from point *e* to point *m* draw line *rm*, which will form the triangle *dmr*. Since *dr* is equal to *dm*, the angles *m* and *r* are equal, and thus the distance *dm* is the most appropriate when the eye is fixed in point *d*.

Similarly, if the eye is fixed at point *i*, the circle over *i* will be drawn with the space [radius] *ia*, and lines *il* and *is* drawn (as was said above), and the correct distance will be found in point *l*. With a similar procedure is taken the distance if the eye is in *h*, by drawing the circle with the space [radius] *ha* and the lines *hk*, *ht* and *tk*, since when the triangle is formed angles *t* and *k* will be found to be equal; thus *k* is the point of the distance, when the eye is in *h*. But if the **[p. 21]** eye is in point *a*, let there be drawn from *a* the circle *zdq* with the space [radius] *ad*, and then let line *ac* be extended to point *z* and line *ab* extended to point *q*, which being equidistance from the centre to the circumference of that same circle are equal to each other. If then a line is drawn from *z* to *q*, the triangle *aqz* is formed, and since *az* and *aq* are equal, then angles *z* and *q* are also equal. Thus when the eye is in *a* the due distance will be in point *q*.

If the eye is in *e*, let the centre of the circle be *e* and the space [radius] *ed*, and draw circle *pdy*. Then drawing the lines as above, forming triangle *epy* with the lines *ep*, *ey* and *py*, it will be found that when the eye is in *e* the true distance is in point *p*.

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[24] In Fig. 1.27 the point *e* is not marked on line *cd*, as it is instead in all the manuscripts: Ms. It. IV, 40=5447, fol. 10v; Ms. Lat. VIII, 41=3069, fol. 12v; Ms. It. IV, 39=5446, fol. 3v and fol. 265r. This lacuna is noted in the Errata on an unnumbered page at the end of the book.

Finally, if the eye is in *f*, drawing the circle with the space [radius] *fd* on centre *f*, and drawing lines *fx*, *fo* and *ox*, forming triangle *fxo*, the true distance will be in point *o*, as is seen in figure 7 (Fig. 1.28). I have left out the lines *qz*, *py* and *ox* in order not to make the figure confusing.[25]

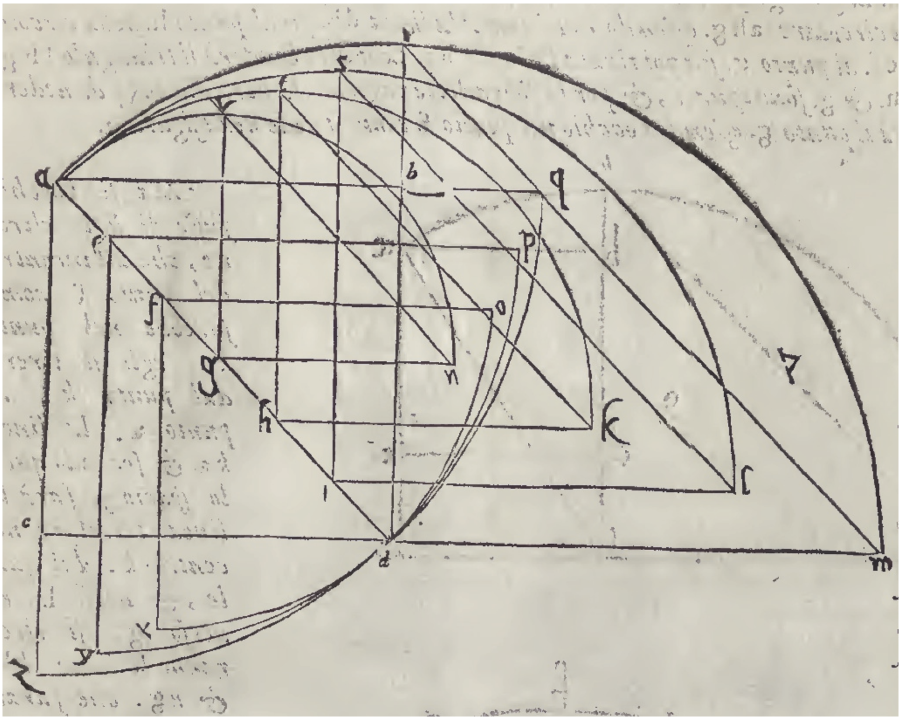

Fig. 1.28 **[p. 21]**

But it can happen that the point in not fixed on the diagonal, but elsewhere, so it is necessary to demonstrate how the correct distance is to be taken.

So, let there be square *abcd*, in which the diagonals are *ad* and *bc*, and let the eye be in point *f*. With centre at *f* and the space *fh* [*recte fb*],[26] draw the circumference *ibh*. Then draw the lines *fi* and *fh* at right angles from point *f*. I say that when the line *hi* is drawn, it will form a triangle in which angles

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[25] In Fig. 1.28, which was drawn from Barbaro's extended Italian manuscript It. IV, 39=5446, fol. 4r and fol. 265v, the eye at point *g* is missing.

[26] This same error is found in the Italian menabò manuscript It. IV, 40=5447, fol. 11r, but the passage is correct in the extended Italian manuscript It. IV, 39=5446, fol. 5r and fol. 265v. Instead, in the Latin manuscript Lat. VIII, 41=3069, the text relative to figure 8 on fol. 13r is only discursive and does not indicate the construction point by point.

h and *i* are equal. Thus *hf* will be the correct and reasoned distance when the eye is in *f*, as can be seen in figure 8 (Fig. 1.29). Similar manners can be brought into play to solve the problem of finding the correct and appropriate distances if one wishes to be diligent, which will give birth to much grace in perspectives.

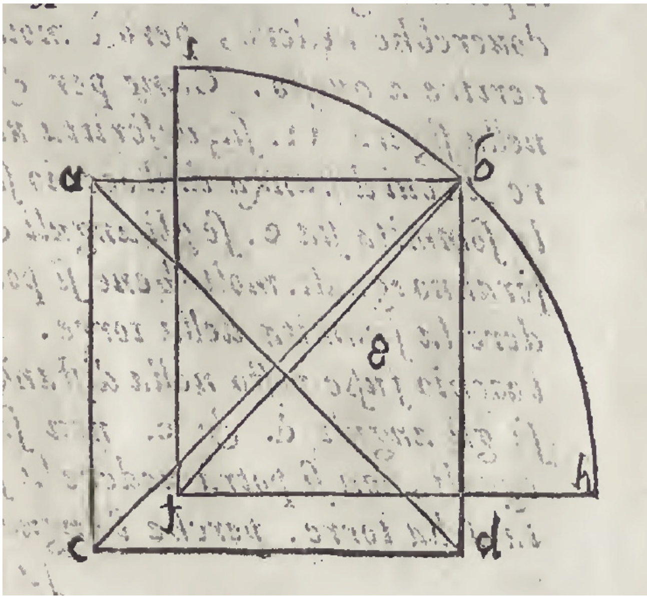

Fig. 1.29 **[p. 21]**

But if the painting is higher than the eye, as is sometimes necessary (as was done by Mantegna with much artfulness in Padua in the sacristy of the Church of the Eremitani), where because the eye of the observer is lower than the painting, there are many figures whose feet are not seen because the plane on which they are standing is hidden. So, it is necessary to do the following.

[p. 22] Let the square be *abcd*, under which is added another [area] of the same width, which is *cdef*, of which line *ef* is at right angles to the eye [*f*]. Then draw the diagonal *af*. From centre *f* with space [radius] *fa* draw the circular line *ahg*, with line *db* being extended to point *h* on the circumference, and line *ef* extended to point *g*. Then draw line *hg*, which will close triangle *fhg*, of which angles *h* and *g* are equal. By the rationales given above, the true distance of vision of square *abcd* will be in point *g*, when the eye is in point *f*, as seen in figure 9 (Fig. 1.30).

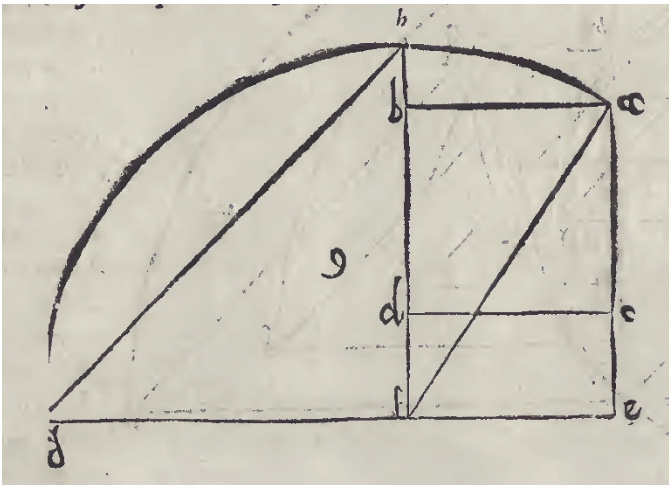

Fig. 1.30 **[p. 22]**

If the eye is lower but somewhere other than at point *f*, say for example in point *k*, then a line is drawn from point *k* to point *a*, and that line *ka* will be the space [radius] with which is drawn the circle with centre at point *k*. Lines *kh* and *kg* will be drawn to points *h* and *g*; these will meet at right angles at point *k*. Drawing then line *hg* will close the triangle *khg*, of which angles *h* and *g* are equal, and *g* will be the true point of the distance, as seen in figure 10 (Fig. 1.31).

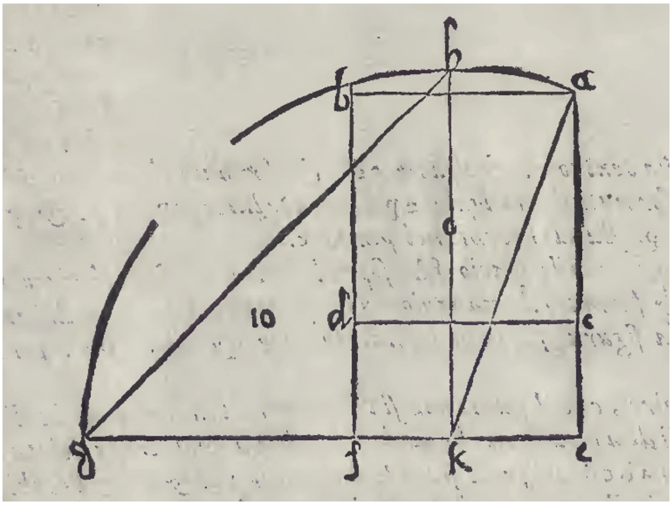

Fig. 1.31 **[p. 22]**

The distances are formed according to this rationale. If not, the painter, erring in the placement of the point of the distance, will err greatly. He might make a thing be seen that according to the rationale of the distance shouldn't be seen, and this should be carefully observed, as for example in figure 11 (Fig. 1.32). Let there be drawn a tower whose distance from the eye is *bc*, and whose top is at *e*. If the angles *c* and *e* are equal, then the top of the tower can be seen very well, but if the eye is placed at distance *bd*, and angles *d* and *e* are not equal, then the top of the tower cannot be seen, because the angle that **[p. 23]** makes the ray to *f* is blocked, as seen in figure 11 (Fig. 1.32). That is all that needs to be said regarding the regulation of the distance.

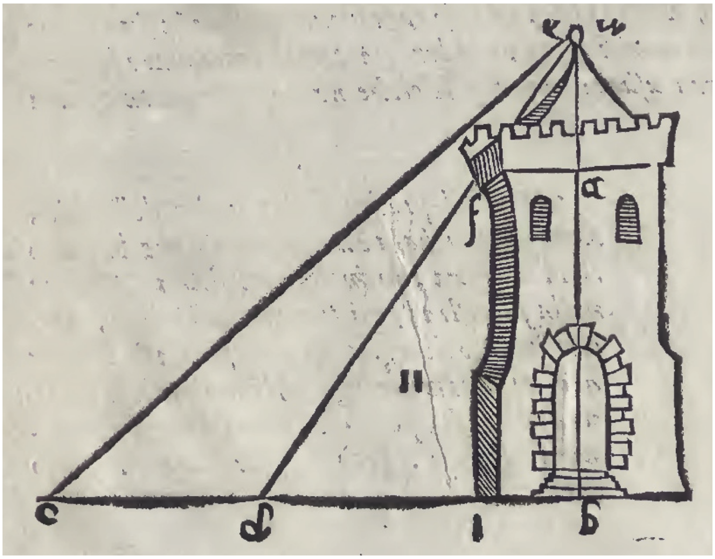

Fig. 1.32 **[p. 22]**

Chap. IX. Of the size that figures must be made in the painting

The largest figure, which is that closest to the eye – supposing that painting is large and not occupied by some edifice so that there is a reason for the figure to be smaller than life size – is set (I say) at life size. But the other figures farther away must be made as art will show us in practice. It is thus necessary to proportion the figures to the edifices when there are some, in order not to fall into that error into which many who do not understand the strength of perspective fall, making the figures larger than the doors and the columns of the buildings nearby, which is very inept and clumsy and false. It can happen that the figures must be made larger than life size, so that they are well understood, as when the painting is very far above the eye. But care must be taken not to paint according to whim and opinion, because quite often this does not lead to the praise that is desired. So, how this need can be satisfied I will say as follows (Fig. 1.33).

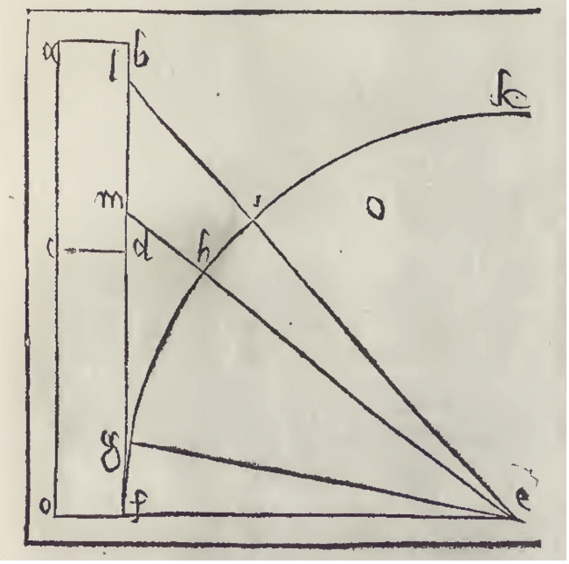

Fig. 1.33 **[p. 23]**

Let the painting be placed at height *abcd*, the distance is *fe* and *fg* is the perfect height of a figure placed under that painting on plane *fo*. Let the circular line *fgk* be drawn from centre *e* with space [radius] *eg*. The bottom of the figure must be placed in point *m*. Draw then the line *me*, which will cut the circular line at point *h,* and from point *h* towards *k* be set the magnitude *fg*, and that will be *hi*. Then draw from centre *e* through point *i* the line to the painting at point *l*. So, since *hi* is on the circular line by a distance equal to *fg*, and *hi* stopping on the painting at *ml*, then I say that the figure must be made in the painting equal to the length of *ml*, when it is desired that from point *e* it appears to be as large as figure *fg* is on the plane [at eye level]. This is worthy of notice in the gilded angel on the top of the tower in San Marco in Venice,[27] which, thanks to these rationales, it is seen distinctly and appears as large as life size. With this artifice Albrecht Dürer made the instrument for proportioning letters or figures placed on columns or at another height, as shown in figure *i* of the fifth chapter.[28]

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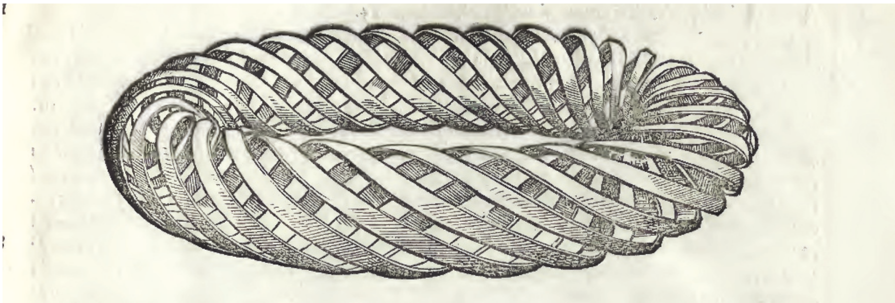
[27] This reference to the statue of the archangel Gabriel placed atop the tower in Piazza San Marco is related to an event that Daniele Barbaro experienced personally:

This is the end of Part I, which treated the principles, rules and foundations of perspective.

> *... come si vede dimostrarsi l'angelo posto sopra la pyramide de la torre nostra, che chiaramente si cognosce non solamente con che modo, a, formato: ma se li vede la fronte, gli occhi: il mento, et tutte le altre parti della forma sua, che se fosse figurato tanto grande: quanto, è, la perfetta forma humana, non se li vedrebbe cosi particolarmente le cose predette: ben che forse alcuni si credeno chel sia formato quanto, è, il naturale, o, poco più non lo havendo veduto altramente, che si ben mi aricordo essendoli accanto il giorno, che da piedi di detta torre fu portata quella altezza, elevando il braccio quanto più poteva con fatica gli aggiungeva all'ombelico: né per ciò credo esser del numero del minori: et se per ventura questo cotale essempio non mi valesse, a, sostenir la detta demostrazione, vagliami le antiquissime colonne ellevate in Roma figurate con tale regola della basse al sommo scapo: per quanto mi ha reso ferma credenza messer Giacomo sansovino fiorentino non solamen- te statuario, et architetto famosissimo: ma sotilissimo perspettivo...* (Ms. It. IV, 39=5446, fols. 7r-7v).

The information contained in these few lines make it possible to delineate a plausible interval of time, between 1548 and 1554, for the composition of the extended Italian manuscript It. IV, 39=5446 and thus of Barbaro's interest in perspective. See Monteleone (2020, pp. 41-42).

[28] Cfr. Dürer, *Underweysung der Messung* (1525, pages not numbered).



Part II, in which are treated *ichnographia*, that is, the description of the plan

Chap. I. The practice of describing figures of many angles in a circle

[p. 25] Before I come to describe plans, it is necessary to practice the way of describing the figures called 'polygons', that is, those having many equal angles and sides circumscribed by a circle. So, the description of the triangle is conjoined to the definition of the figure of 6 equal sides and angles. It is easy because having made the circle, the circumference is divided into 6 equal parts. It is for this reason that the compass is usually called a *sesta*, because it goes 6 times into the circumference of the circle that it made.[1] If you then join with lines all 6 parts of the circumference, you will have described the figure of 6 equal sides and angles, as seen in figure 13 (in Fig. 2.1). If you leave out every other point and draw the lines of that circumference, you will have drawn the triangle of equal sides and equal angles, as seen in figure 12 (in Fig. 2.1).

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[1] For the etymology of the English 'compass', see http://languagehat.com/compass/.

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https://doi.org/10.1007/978-3-030-76687-0_5

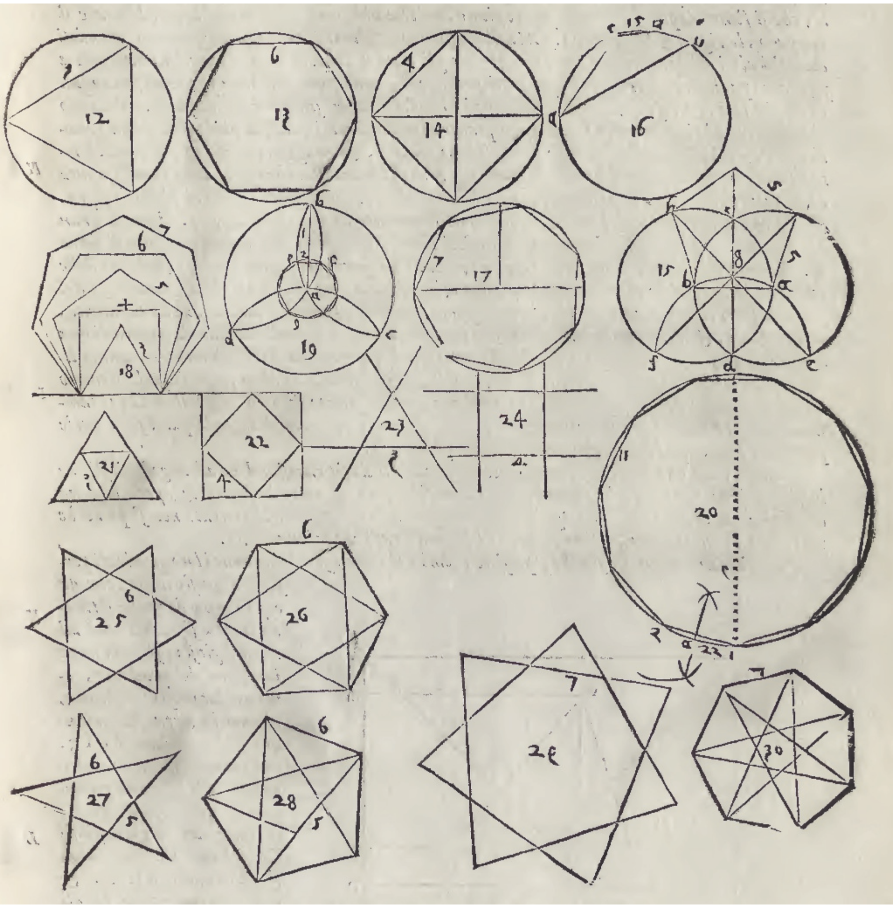

Fig. 2.1 **[p. 27]**

The quadrangular surface called 'tetragon' is made by putting 2 lines in a cross at right angles, making a centre of the crossing and opening the compass towards the end of those lines, as far as you like, and turning it around. The circle will be found to be divided into 4 equal parts, and if you join each quarter with lines, you will have the square surfaces, as seen in figure 14 (in Fig. 2.1).

The surface with 5 equal angles and sides called 'pentagon' is usually made in many ways, of which I will select an easy one that can be made with one opening of the compass.[2] So, let the line *ab* be one of the sides of the pentagon. Open the compass to that length, [draw 2 circles whose centres are in *a* and *b*], and mark the crossings of the circumferences *c* above and *d* below, then connect *cd* with a line. With the compass at the same opening, make *d* the centre and draw a circle, which by necessity must pass through the centres of the crossed circles, that is, through *a* and *b*; it will cut those circles in *e* and *f*, and it will also cut line *cd* at point *g*. Then draw a line from point *e* which, passing through *g*, arrives to the circumference of circle *bcde* [*recte bdcf*] at point *i* [*recte h*] [and draw a line from *f* which passes through *g* and arrives to the circumference of the circle *acde* at point *i*].[3] Finally draw the lines *ai* and *bh*, and you will have 3 sides of the proposed figure. The other two lines are made in this way. You won't move the compass, but placing it on the points *h* and *i* you will cut the circles made on line *dc*, which you have first extended, and placing here the end at point *k*, **[p. 26]** you will have the other two sides, as seen in figure 15 (in Fig. 2.1).[4]

---

[2] Cfr. Dürer, *Underweysung der Messung* (1525, figure 16).

If you wish to have the figure of fifteen equal sides and angles, you will make a circle and in that place the side of a triangle, which is *ab*. In the same circle place the side of a pentagon, which is *ac*, and you will divide that space between *c* and *b* into 2 equal parts. One of those, conjoined with a line, will give you the said surface of fifteen equal sides and angles, as figure 16 (in Fig. 2.1) will show you.

Of the heptagonal figure or surface, that is, the one of 7 equal sides and angles, a manner is found which is common to the description of all the surfaces of equal sides and angles. You will divide the circumference comprised by the right angle into 7 equal parts, and the chord that is taken by the arc of 4 of the 7 is the side of that surface, as seen in figure 17 (in Fig. 2.1).

This rule is universal to all the figures of equal sides and angles, as long as you divide the circumference comprised by the right angle into as many parts as the figure you want has sides. That is, if you want the triangle, you will divide the right angle in 3 parts and you will take 4 of them. If you want the square, you will divide the circumference of the right angle into 4 parts, and taking all four you will have one side of the square; if you want the one of 5, you will divide that same angle into 5, taking 4 for one side of the pentagonal surface. Similarly you will do for all the other surfaces, and this is worthy of note, as we have set out in our Latin treatment.

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[3] The Italian menabò manuscript It. IV, 40=5447 (fol. 13v) has this same omission in the construction and the same errors in the notations. The Latin manuscript Lat. VIII, 41=3069 (fols. 13v-14v) contains a section dedicated to the regular polygons with an in-depth discussion of only the heptagon. Instead the extended Italian manuscript It. IV, 39=5446 introduces the construction of the *perfetto* without explaining how to draw the regular figures. This absence in the extended Italian manuscript leads us to believe that Barbaro introduced the argument only in a second phase, in order to maintain the 'order' that he had promised the reader.

[4] Dürer's construction of the pentagon is only approximate. See Dürer, *Underweysung der Messung* (1525, figure 16).

From the things said it can be shown how with an opening of compass set on a given segment are described the surfaces of 3, 4, 5, 6 and 7 equal sides and angles. Let the given line segment be *ab*, on which, as by the first proposition of the first book of Euclid, is made a triangle of equal sides and equal angles. On the same line is formed the square surface via the crossing of the circles, as was said above; and similarly you will make the surfaces of 6 and 7 equal sides and angles. The rationale of these figures is such that the angle of the triangle of equal sides is as 2 to 3 with respect to the right angle,[5] since its 3 angles are equal to 2 right angles. The angle of the square is equal to the right angle, so its 4 angles are equal to 4 right angles. The angle of the pentagon is as 6 to 5, so its 5 angles are equal to 6 right angles. The angle of the hexagon is as 8 to 6, and the angle of the heptagon is as 10 to 7, as seen in figure 18 (in Fig. 2.1).

Albrecht Dürer makes the figure of 9 sides and angles as follows. Divide a circle whose centre is *a* into 6 equal parts, and mark on the circumference the starting points of the sides of one triangle with the letters *bcd*. From point *b* to centre *a* draw line *ba*, which is divided into 3 parts with the numbers 1 and 2, of which point 2 is closest to centre *a*. Cut line *ba* at right angles with line *ef* crossways. Then with the same opening of the compass, with one foot of it placed in the points on the circumference that were skipped over by the triangle, you will draw the circumferences of the circles out to the circumference of the first circle. These will cut line *ef*, which will remain one of the sides of the figure with 9 equal sides and angles, made around the smaller circle, where working with the straightedge, the circumference of the larger circles can be divided into 9 parts, and that will form the enneagon, that is, the figure of 9 equal sides and angles, as seen in figure 19 (in Fig. 2.1).

The surface of eleven equal sides and angles is formed by taking the fourth and eighth part of that fourth of diameter of the circle in which you want to make that surface. That is, of 32 parts of the diameter, 9 make one side of that surface that is to be described, as seen in figure 20 (in Fig. 2.1).

But if you want to multiply the sides of the figures above, doubling them (for example, from one of six making one of twelve), you can do it easily by dividing each arc, by crossings of circles, into two equal parts, and drawing lines to the mark of the divisions. This is seen in figure 20 above, where the arc set between 1 and 2 is divided into 2 parts at mark *a*; if you draw a line from point 1 to point 2, you will have one side of this doubled surface.[6]

---

[5] That is, 60°:90° :: 2:3.

This is not the place to reason about those surfaces that have neither equal sides nor equal angles, because they are irregular and are up to the painter. But I will say that in triangles and squares, if from the midpoints of their sides are drawn lines inside them, you will make similar figures, as clearly seen in the figures 21 and 22 (in Fig. 2.1).

If you extend the sides of the surfaces of 5, 6, 7 and more sides (excepting the triangle and the square, which **[p. 27]** do not make this effect, as you can see in figures 23 and 24), you will make similar figures, as seen in figures 25–30, which can be similar both outside and inside, if lines are drawn from the vertices to the vertices, from the sides to the sides, and from the sides to the vertices.

Chap. II. The way of describing plans

Continuing on, I will demonstrate the way to make the plans of those things which are to be put into perspective, because without *ichnographia*, that is, the drawing of bases and plans of things,[7] nothing can be described, since everything that is raised is born from the plan, as the tree is born from the roots. So, the plan is either of a surface such as a triangle or square or other superficial shape of many sides and many angles or even circular, or of some other body, be it **[p. 28]** regular or irregular. If it is a surface, it isn't very hard to do, because you form a perfect square in which the correct surface is drawn, as was done in the previous chapter. From this is born the occasion of compartitioning the pavements that are made in stone in many plane figures, in which case it is necessary to know very well how to use the compass, straightedge and set square. However, I don't wish to treat this here because it has been very diligently treated by others, including Dürer, an excellent man distinguished in depicting pavements in triangles, squares, pentagons, hexagons, and other figures of many sides and angles,[8] and with roses and leaves, and knots, and tiles as desired, so I refer the reader to him.

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[6] The text and figure are not quite correct. Figure 20 shows a 12-sided figure; dividing each side into two would form a figure of 24 sides. Point *a* divides segment 1-2 into two equal parts; drawing lines from points 1 and 2 to *a* forms two of the sides of the 'doubled' figure.

[7] Cfr. Barbaro (1567, p. 30), Williams (2019: 60).

But the important thing is to make the plans of many bodies, which at their base are of one size and then, rising on various sides, they have smaller and larger outlines and projections and retractions, all of which must be delineated on the plane. Since it is easy to make the plane of a cubic body and other bodies which have their bottom plane equal to their top plane, for us to set them in plan, it can be sufficient to form only the bottom surface at whatever size we like. But if a body has its bottom plane true and level, but on top it has a cap, or surbase, or cornice that projects, such as the pedestals of columns have, then it is necessary to make their plans, first making a square the size of the bottom plane, then making others that are larger or smaller according to the outlines of the cap, surbase, cornice and other little members that there may be. In sum, anyone who wants to make the plan of any body whatsoever, needs to imagine that from all the outlines, greater and smaller, fall vertical lines onto the plane that will serve as a canvas, paper, panel or something else on which we are to make the plan of the proposed body. Of all the difficult plans, the most difficult is the plan of the human body, because of the multitude and diversity of the outlines of all its parts. The next most difficult are the plans of the Ionic and Corinthian capitals, as will be seen.[9]

When all the surfaces or outlines on top correspond at right angles to the surfaces below, the plan made is the surface of all the surfaces, so it is necessary with the utmost diligence measure each part and transport it to the plan, or else nothing could be put into perspective, as will be made clear in the chapters that follow.

All the plans of surfaces, as well as those of bodies, are drawn in their perfect squares in the following way. From each of their vertices are drawn lines at square angles (that is, at right angles) to one of the sides of the square, whether the lower or the right or left side makes no difference. So, in square *defg* let there be described the triangular surface *abc*. From the vertices of that draw lines at right angles to side *fe* below; these will be lines *al*, *bi* and *ck*. Likewise from the same vertices to the right side *de* draw lines *am*, *bn* and *co* at right angles. I say that in this way will be described the triangular surface in that square, as will be seen in figure 31 (Fig. 2.2).

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[8] See Dürer, *Underweysung der Messung* (1525, figures 22-26, pages not numbered). Barbaro did, however, draw, pavement designs in the extended Italian manuscript It. IV, 39=5446, fols. 11r-12r; see Monteleone (2020, pp. 203-204).

[9] See Part IV, Chap. VIII and Chap. IX.

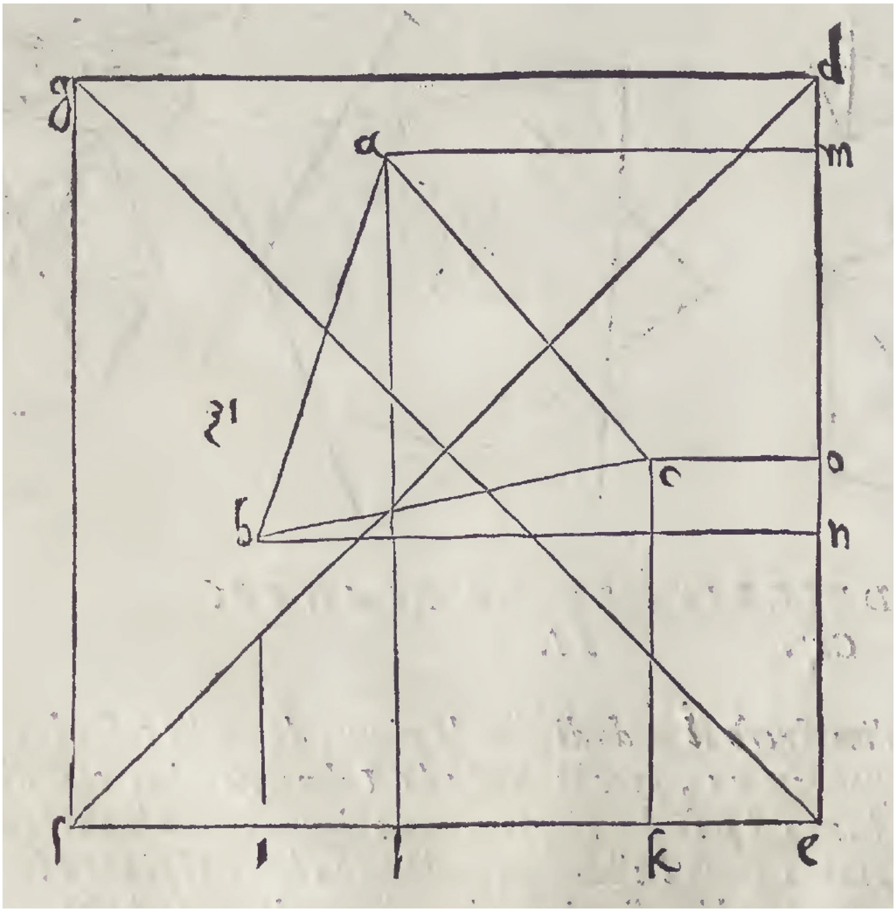

Fig. 2.2 **[p. 28]**

[p. 29] A similar way is used in the square figure, and in the others as well. Let there be a perfect square *efgh* in which is described the square surface *abcd*. Draw lines from each vertex of that surface at right angles to the bottom side *gh*; these will be *ai*, *bk*, *cl* and *dm*. Let there also be drawn the lines from the same vertices at right angles to the right side *fg* [*recte fh*]; these will be *an*, *bm*, *cp* and *dq*. I say that this will be the perfect of the square surface, as seen in figure 32 (Fig. 2.3).

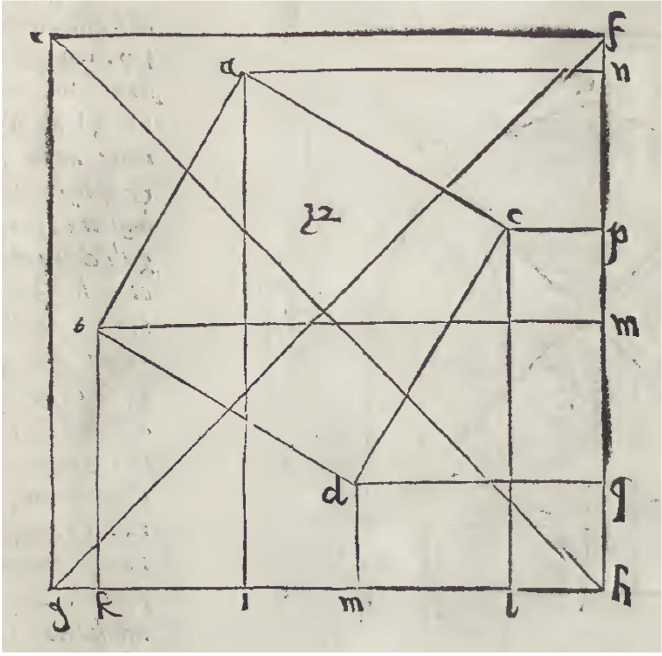
e
f
a
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Fig. 2.3 **[p. 29]**

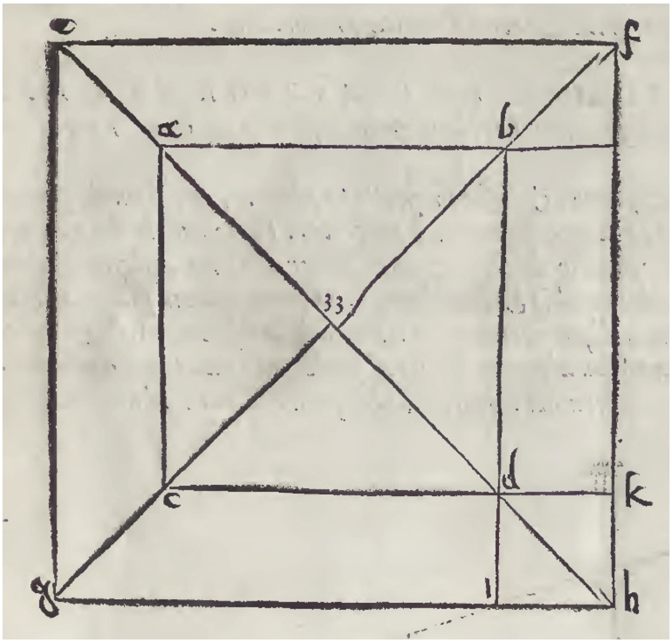
e
f
a
b
33
d
k
c
g
l
h

Fig. 2.4 **[p. 29]**

If the square surface is placed in the middle of its square such that all its vertices and sides correspond to the vertices and sides of its square, and all its sides are parallel, all that is needed is one line to the bottom side and one to the right side, as seen in figure 33 (Fig. 2.4)
In square *efgh*, from vertex *d* of the square surface *abcd*, draw the line *di* to the lower side, which also goes up to the point *b*. Similarly the line *dk* is drawn to the right side [from point *d*], and also serves for point *c*. This is because the vertices of the square surface *abcd* correspond directly and parallel to the vertices of square *efgh*. The reason why those lines are drawn from the vertices of the described squares to the sides of the square will be clearly known through practice, because they are marks to be placed in the squares in perspective. Just as the squares on the plane are called the perfects (*perfetti*), these [in perspective] are called diminished or degraded (*degradati*).[10]

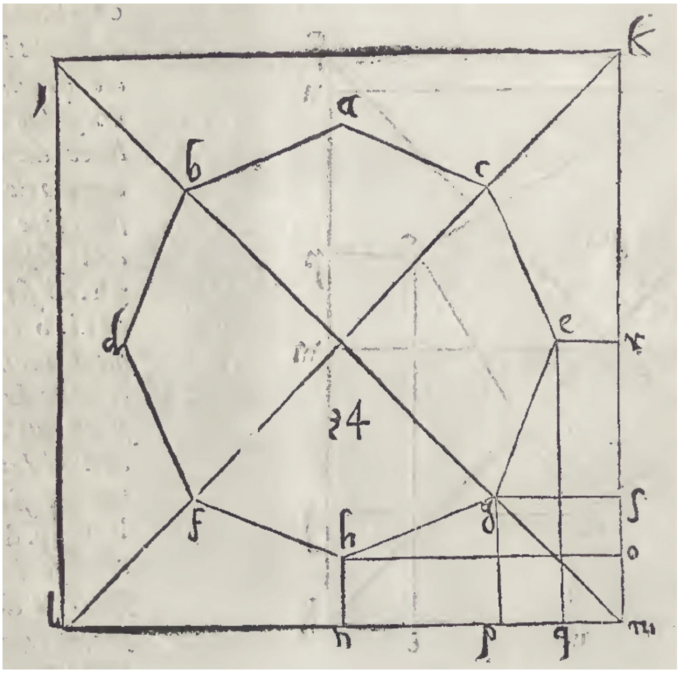

Fig. 2.5 **[p. 30]**

[p. 30] Similarly, in the perfect square *iklm* is described a surface with eight equal sides and angles *abcdefgh*, where from the vertices *g*, *h*, and *e* are drawn lines to the base *eq*, *gp*, and *hn*, and to the right side *km*, the lines *er*, *gf*, and *ho*, as seen in figure 34 (Fig. 2.5). These lines are sufficient, because the vertices *h*, *g* and *e* of that surface are a rule for all the others to which their opposites are equal and parallel to the sides of the perfect, which will be clearly known through practice. This is observed not only in angular surfaces, but also in circles and other plane figures, as will be said below. In sum, all surfaces and plans placed in the perfects, according to this way of practice, are regulated by drawing lines from their vertices or sides to the bottom and right sides, as has been shown in the preceding figures, because they are marks or signs for transporting elements from the perfect to the degraded with due proportion.

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[10] For a discussion of the terms *perfetto* and *degradato*, see Field (1997, p. 86).

Chap. III. The way to degrade a given plane in the prosposed termination according to the eye and the distance[11]

Having made the perfect, and placed within it the plan that is to be put into perspective, there follows the degradation of the perfect. So that this is more easily done, it is necessary to know how to degrade a given plane in the termination according to the eye and the distance. So, the given plane *bc* is to be degraded in the termination *b* according to the distance *db* and the eye *a*. Extend plane *bc* to point *d*, on which falls a line from the eye *a* at right angles. Similarly there falls another line from point *f* to point *b*, also at right angles, which is *fb*. Then draw a line from the eye *a* to the point *c*, which is the end of the plane *bc*; this will be *ca*. I say that line *ca* will cut line *bf* in point *e*, and that *be* will be plane *bc* degraded **[p. 31]** according to the eye and the distance. If we bear in mind the principles given above, we will confess that *be* appears to us to be as large as *bc*. This is because drawing line *ab* will make the triangle *abc*, which has two bases, *be* and *bc* opposite the same angle, and so they are represented to the eye as equal. In this way is degraded the plane *bc* in termination *b* with the line *be* according to the eye and the distance, as seen in figure P (Fig. 2.6).

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[11] This same construction is found in Piero della Francesca's *De prospectiva pingendi* (Mss. Regg. A41/2, fols. 7r-7v).

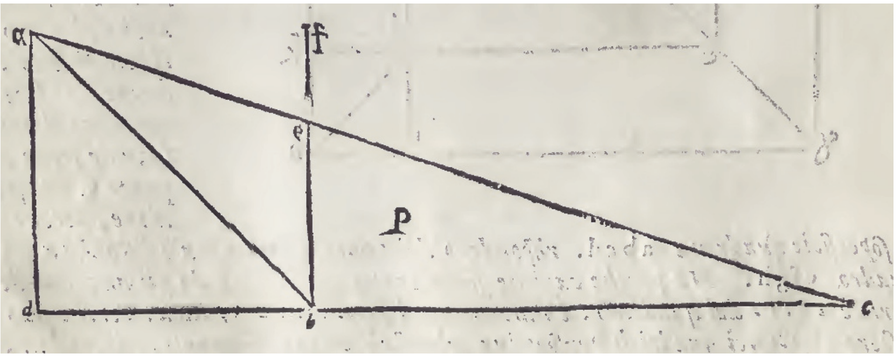

Fig. 2.6 **[p. 30]**

Chap. IV. The way to reduce the degraded plane into a square[12]

Now, if we wish to reduce the degraded plane into a square, we will do as follows. Let line *cd* (as in the previous figure) be divided at the point or termination *b* and let fall onto point *b* the line *fb* equal in length to plane *bc*. Let the eye *A* be at its distance and height at right angles over *d*. In a similar way the line *gc* falls onto *c* at right angles and is of the same length as plane *bc*. From point *g* to point *f* is drawn the line *gf* equal in length and parallel to plane *bc*. I say that up to now there the plane [*bc*] has been reduced into a perfect square, which is *bcfg*. This square must now be degraded, which is done in the following way. Let rays be drawn to the eye [*A*, corresponding to *a* in Fig. 2.6] from points *c* and *g*; these are *cA* and *gA*. These rays divide *bf* at two points, that is, ray *gA* divides *bf* at *h*, and ray *cA* in *E* [corresponding to *e* in Fig. 2.6]. I say that *E* represents itself to the eye as higher than *b* because *A* is located over *E*, and *h* represents itself as lower than *f* because *A* is lower than *f*, as can be understood from the principles set forth in Part I. I say further that in the given termination *bE* appears equal to *bc*, and that in the given termination *Eh* appears equal to *cg*, according to what was declared above. And for the same reason *hf* will appear equal to *fg*. Draw the rays *Af* and *Ab* from the eye *A*. I say that we will have 3 triangles, each of which has 2 bases: the triangle

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[12] The first construction of this chapter is found in Piero della Francesca (Mss. Regg. A 41/2, fols. 7v-8r). The second, instead, considers a case that Piero had not addressed, that is, the poisitioning of the eye of the observer in a way that is eccentric with respect to the perceived object. This is a clear example of how Barbaro, in his treatise, did not stop at reporting the works of others but sought to broaden and complete them.

Abc has 2 bases, *bc* and *bE*; the triangle *Acg* has 2 bases *gc* and *hE*; the triangle Agf has 2 bases *fg* and *fh*.

Thus by the things said above the base *bE* will appear equal to the base *bc*, since they are under the same angle; base *Eh*, for the same reason, will appear equal to base *cg*; and base *fh* will appear equal to base *fg*. Up to here we have the degradation of 3 planes in the square, since *bE* **[p. 32]** degrades the plane *bc*, *Eh* degrades *cg*, and *fh* degrades *fg*. The ratio, or comparison, of *Ab* to *bE* is the same as that of *dc* to *bc*, and the ratio of *Eh* to *cg* is the same as that of *AE* to *Ac*. The same ratio of both *bE* and *fh* to *cg* is the same as that of *Ag* to *hg*.[13] When the distances and the things are in the same ratio to the height of the eye to the thing degraded, then there is no doubt that the degradation is reasonable and proportioned.

Now to close the degraded square, you will do the following. Draw from eye *A* an infinite line [i.e., a line with no end point] parallel to the line *dc*. Then divide *bc* equally [into two] at point *i*. At *i* will fall line *ai* at right angles to the infinite line. Similarly from point *E* draw a line parallel to *bc*, which will cut *cg* at point *k*.[14] Then from point *a* to point *b* you will draw a line that cuts *Ek* at *d*. Similarly from point *a* to point *c*, you will draw a line that cuts *Ek* in point *e*. In this way you will have reduced the degraded plane into a square, which is *bcde*. The proof is that *de* is equal to *dh* [*recte Eh*], which appears exactly similar to *cg*, since the ratio of *ab* to *ad* is that of *ac* to a*E*, and the ratio of *de* to *bc* is the same as that of *eh* to *cg*. Being proportional, they are either equal or similar, but they are equal since we set *bc* equal to one and *bc* [*recte cg*] to the other (Fig. 2.7).

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[13] Barbaro is arguing that the accuracy of the construction is proved by the fact that all ratios remain the same. To prove this, he provides measurements for the lengths in Fig. 2.7, allowing us to verify that the significant segments are in the ratio 1:3, but his wish to use integers to refer to irrational values leads him into error. *AE* (21) is indicated as 7 but in fact it is the hypotenuse of a triangle whose sides are 2 and 6, equal therefore to about 6.32; *Ec*, indicated as 14 (that is, 21–7) is equally incorrect, since the hypotenuse of a triangle whose sides are 12 and 4 is equal to about 12.65, not 14.

[14] Point *k* is shown in the wrong place in Fig. 2.7.

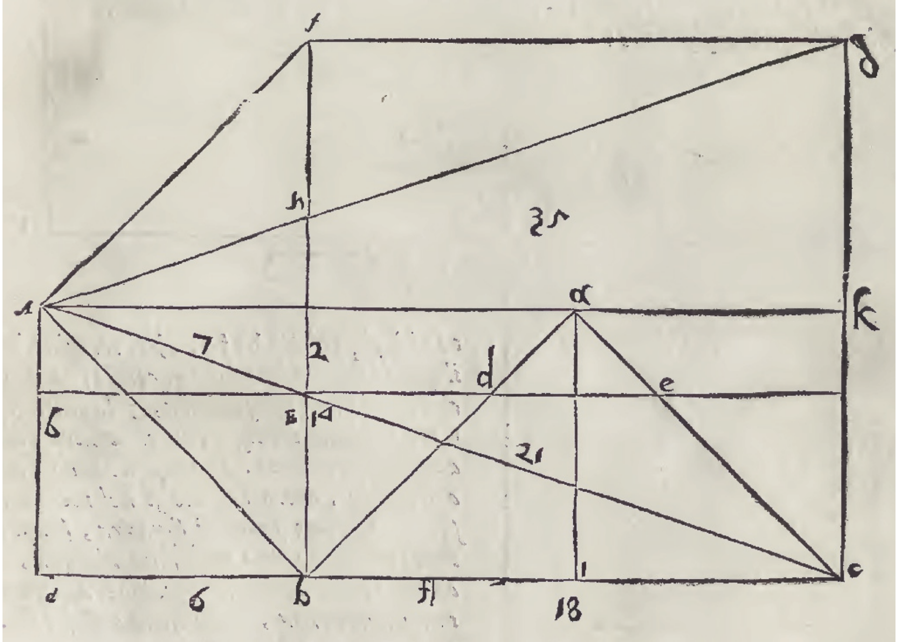
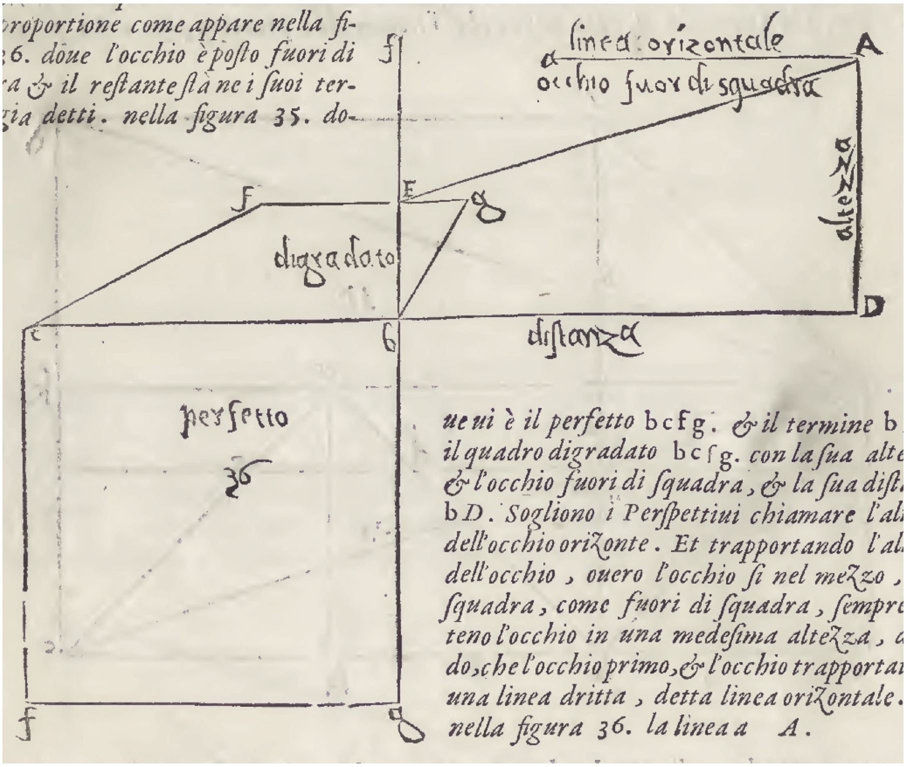


Fig. 2.7 **[p. 31]**

If you were to ask why the eye has been transported to the middle [from *A* to *a*], I would respond that it is because it is more appropriate to put it in the middle, because things are seen in perspective better when the eye is in front of the square than when it is beside it. But the eye can be placed where desired when the terminations are not surpassed, as will be said later. Wherever you place it, it will come in the same proportion as appears in figure 36 (Fig. 2.8), in which the eye is placed beside the square and the rest remains in its terminations said above, as in figure 35 (Fig. 2.7). Fig. 2.8 shows the perfect *bcfg*, and the termination *bf*, and the degraded square *bcfg* with its height and eye beside of the square, and its distance *bD*. The perspectivists call the height of the eye the 'horizon', transporting the height of the eye, or the eye, beside the square, but still putting the eye at the same height, such that the first eye [*A*] and the transported eye [*a*] are on a straight line, called the 'horizon line', as line *aA* in figure 36 (Fig. 2.8).



roportione come appare nella fi-
6. doue l'occhio è poſto fuori di
a & il reſtante ſtà ne i ſuoi ter-
gia detti. nella figura 35. do-

ue ui è il perfetto bcfg. *& il termine* b
il quadro digradato bcfg. *con la ſua alte*
& l'occhio fuori di ſquadra, & la ſua diſt
bD. *Sogliono i Perſpettiui chiamare l'al*
dell'occhio oriZonte. Et trapportando l'al
dell'occhio, ouero l'occhio ſi nel meZzo,
ſquadra, come fuori di ſquadra, ſempre
teno l'occhio in una medeſima alteZza,
do, che l'occhio primo, & l'occhio trapporta
una linea dritta, detta linea oriZontale.
nella figura 36. la linea a A.

Fig. 2.8 **[p. 32]**

Chap. V. Division of the degraded square according to the perfect square[15]

Once the plane has been degraded from the perfect, it is necessary to know how it is compartitioned according to the divisions of the perfect, and how every plan in the perfect can be degraded, and in addition to this, how it is possible to add to, take away from, and raise the degraded square. This will be easy thanks to the things in Chap. VI **[p. 33]** of the Part I. It is thus necessary to form the perfect and divide it with the diagonals into whatever parts we want to, and then form the degraded plane according to the rationales given above. So, let the perfect be *abef*, and the degraded likewise *abef*. Draw the diagonals in both squares, which are *be*. Then divide the perfect into 5 parts (for example)

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[15] As in the previous chapter, the figures 37 and 38 (Fig. 2.9 left and centre) derive from Piero della Francesca's *De prospectiva pingendi* (Mss. Regg. A 41/2, fols. 8v-9r); the figures 39 and ✠ (Fig. 2.9 right and Fig. 2.10) correspond to a broadening of the topic by Barbaro.

at *k*, *i*, *h* and *g*. It is necessary to carry into degraded the divisions of the perfect. So, I say, from the points *g*, *h*, *i*, *k* of the perfect on line *ab*, which is common to both squares, draw the lines to point *a*, where the eye is located, as seen in figure 37 (Fig. 2.9, left). If the line on the perfect divides the diagonals into equal segments, then the divisions [of the diagonal] in the degraded will also be equal; if the divisions are unequal, [then they will be] unequal.

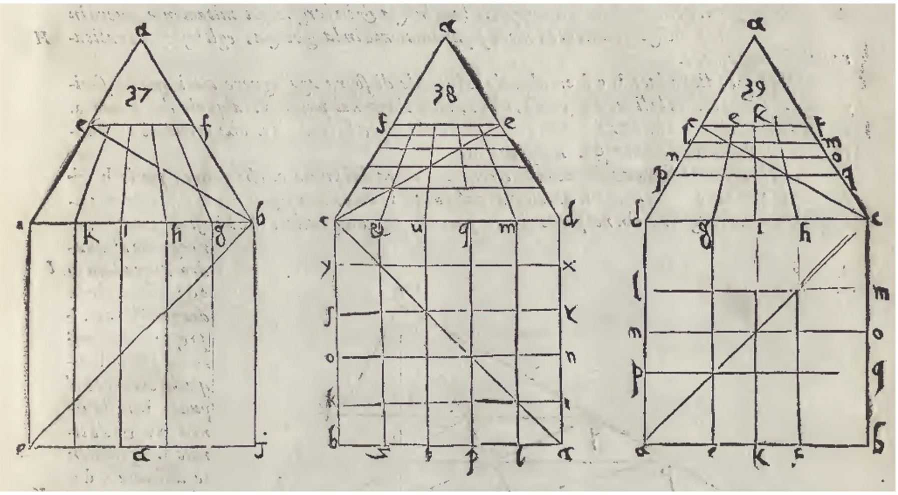

Fig. 2.9 **[p. 33]**

Just as in Chap. VI above [in Part I] we divided the perfect into similar surfaces, the same can also be done in the degraded square, and there will be the same ratio of proportion. So, let the perfect square and the corresponding degraded square be *cedf* [*recte cdab*, as in Fig. 2.9, centre] and *cdfe*. Then let the perfect be divided into many similar surfaces, as seen in figure 38 (Fig. 2.9, centre). Carry all the divisions of line *dc*, which are *c*, &, *u*, *q*, *m*, and *d*, to the eye *a*. These lines, carried to the eye *a*, will cut the line *fe* of the degraded square into parts similar to those on *cd*, which is common to both squares, which are *i*, *p*, *t*, and *z*.[16] If you want to carry the transverse lines *yx*, *sr*, *on* and *ki* from the perfect to the degraded, just as in the perfect you made them pass through the cuts of the diagonals, so too you will do in the degraded, with lines parallel to the line *cd*.

---

[16] In the Errata on an unnumbered page at the end of the book it is remarked that these letters are missing in figure 38 and should appear between *f* and *e*.

If the division of the perfect is divided into unequal parts, as in figure 39 (Fig. 2.9, right), you will do the same things, taking note of the cuts that the transverse lines in the perfect make on the diagonal, and making the same cuts in the degraded.

Not only will you observe these rules when the eye is in the centre and in front of the square, but also when the eye is beside it. This is seen in figure ✠ (Fig. 2.10), where the perfect square is *abdc*.

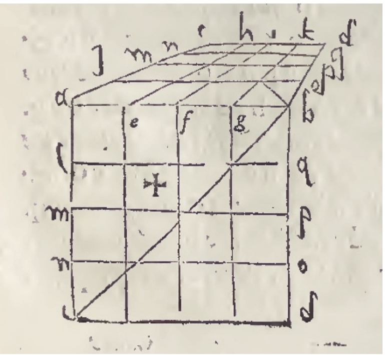

Fig. 2.10 **[p. 33]**

It is divided into 16 squares with the lines *eh*, *fi*, *gk* [points *h*, *i* and *k* on line *cd* of the perfect are not shown in the figure] on one side and lines *no*, *mp* and *lq* on the other. The degraded, likewise beside it, is divided into 16 proportional squares by the lines corresponding to the lines of the perfect, according to the cuts of the diagonal. That is all that needs to be said about the division of degraded squares according to the proportions of the perfects.

Now you will learn how it is possible to add to or take away some parts from the **[p. 34]** degraded square, according to need.

Chap. VI. The way to add to or take away from the degraded square[17]

It is necessary to know how it is possible to add to or take away from the degraded square should the occasion arise to make the square larger or smaller with respect to some edifice that we wish to place beyond the first square or with respect to the little space in which you have to place the plane that is very far away. So, the addition of both length and width together, or length and width alone, must be done in the following way.

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[17] This same construction is explained in Piero della Francesca (Mss. Regg. A 41/2, fol. 14r).

Let the degraded square be *bcde*, to which it is intended to add other similar squares. Draw the diagonal lines, which cross at point *f*. To this from point *a*, where the eye is, draw a line that falls at right angles on *bc*. This terminates at point *g* and also cuts line *de* at point *h*.

Now if we wish to add an upper square [to the degraded square], a line will be drawn from point *b* passing through point *h*, and will cut line *ac* in point *i*. Then from point *i* you will draw a line parallel to line *de*, which will cut *ab* in point *k*, making the degraded square *deki*, which is similar – that is, equal to – the degraded square *bcde*. Since the diagonal *ag* is proportionally in point *h*, which concurs in point *i* in the line *ac*, and [the line through] *i* was drawn parallel to *de*, touching *ab* in point *k*, the ratio of *ak* to *ab* is the same as that of *ki* to *bc*, and the ratio of *ak* to *ad* is the same as *ki* to *de*. Thus to plane *bcde* we have added in length the equal and similar plane *deki*.

If it is desired to add in width, from point *f* will be drawn a line parallel to line *bc*, which will cut *bd* in point *l* and *ce* in point *m* [not shown in Fig. 2.11[18]].

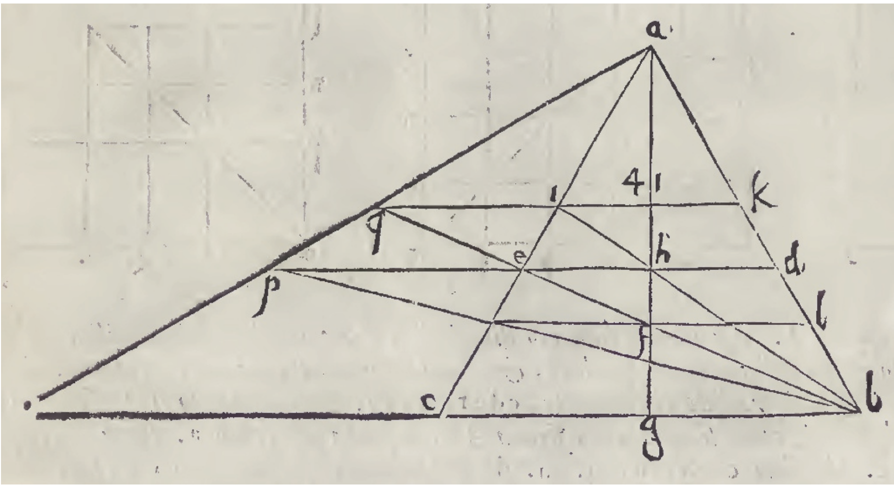

Fig. 2.11 **[p. 34]**

Then draw *de* out to point *p*, which will make *ep* equal to *de*. Then from point *b* draw a line that passes through point *m* out to point *p*, and from point *a* draw a line through point *p* out to point *o*, to which line *bc* is extended.[19] I say that *co* is equal to *bc*, because we set *ep* equal to *de*, and the ratio of *de* to *bc* is the same as that of *ep* to *co*, and that same ratio is of *dp* to *bo*, so they are equal. So this has carried out the intent to add plane *coep* in width to plane *bcde*.

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[18] In contrast to the woodcut of Fig. 2.11, the letters are all found in their correct positions in the pencil drawings in the Italian menabò manuscript It. IV, 40=5447 (fol. 18v) and the extended Italian manuscript It. IV, 39=5446 (fol. 270v). See Monteleone (2020, p. 413).

If it is desired to add in both width and length, making a square of 4 squares equal to *bcde*, take *co* of the same length as *bc*, which will be *co*, and from point *a* draw *ao*. Then from *b* draw the diagonal through *e* out to line *ao*, to point *q*. From that point draw a line parallel to line *bc*, which will cut *ab* in *k*. In this way you will have made square *boqk*, which is composed of 4 squares, which was what we set out to do, as seen in figure 41 (Fig. 2.11).

[p. 35]

Chap. VII. How to cut a square from a quadrangular surface that is wider than it is long

Let the plane be 50 palms long and 10 palms wide; from this I intend to cut a square plane. First, let the degraded plane be *bcde*, of which, before it was degraded, *bc* was 10 palms and *bp* 50 palms. Draw the diagonal *be*. Since the width, which is 10, will go 5 times into the length, which is 50, *bc* will be divided into 5 equal parts, which will be *f*, *g*, *h* and *i*. Draw [a line from] *f* to point *a* [the eye, not marked in Fig. 2.12[20]] which will cut the diagonal at point *k*, and from point *k* draw the line parallel to *bc*, which will cut *bd* at point *l* and *ca* at point *m*. I say you will have cut a square plane, which is *blcm*, from the degraded square. This is proved in the following way.

Make a quadrilateral *nopq* in true form that is 50 palms long and 10 wide, letting *np* be 50 and *no* 10. Draw the diagonal *nq* and divide *no* into 5 equal parts at *r*, *s*, *t* and *u*. From point *r* draw a line parallel to *np*, which will cut the diagonal at point *x* and divide line *pq* at point *y*. From point *x* draw a line parallel to *no*, which will divide line *np* at point *z* [not shown in Fig. 2.12] and *oq* at point &. These points will make a square *noz*& in true form [i.e., the perfect] cut from the quadangular plane *nopq*, conducted by the diagonal that passes through point *x* and divides the line *ry*, which is the fifth part of *no*. As it was said that *bcde* was degraded, and line *bc* made equal to line *no*, which is divided into 5 equal parts, and taking one of the parts, which is *bf*, and drawing it to point *a* that divides the diagonal in point *k*, and drawing the parallel line that passes through point *k* and divides *bd* at point *l* and *ce* at point *m*. Since the fifth part of *no* was taken – that is, *r* – and that drawn parallel to *np*, which divides the diagonal at point *x*, and *x* being drawn parallel to *no*, which divides *np* at point *z* and *oq* at point &, and since the diagonal divides the surface in true form [i.e., the perfect] in the fifth part *noz*&, so too the diagonal divides the degraded surface, as seen from the principles.

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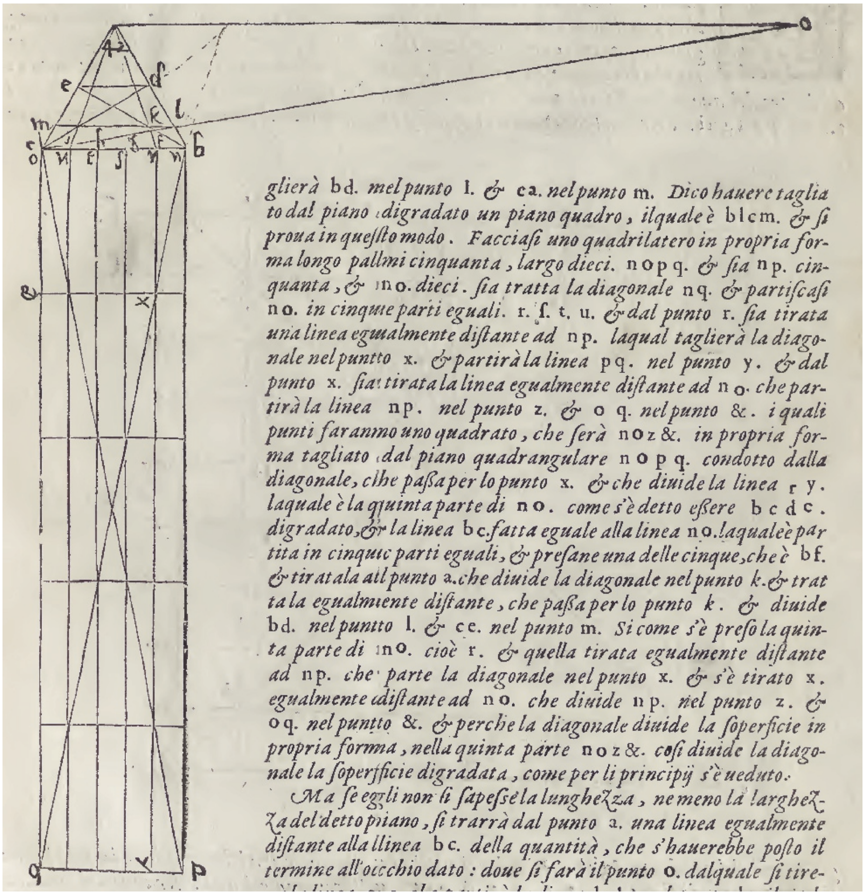
[19] Actually, you draw a line from point *a*, the eye, through point *p* to where it intersects with the extension of line *bc*, which is point *o*.

[20] The letters of Fig. 2.12 are found correctly inserted in the drawing relative to them in the extended Italian manuscript It. IV, 39=5446 (fol. 271v) (see Monteleone 2020, p. 414). The dotted lines that appear in this reproduced figure were added by hand to the printed page and indicate on the horizon *ao* the vanishing point of the lines parallel to the diagonal *op*, which is *cd* in perspective. This leads us to think that these dashed lines were added after the beginning of the seventeenth century since Guidobaldo del Monte dealt with this kind of problem in *Perspectivae libri sex* (1600, pp. 109-110).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glierà bd. *mel punto* l. *&* ca. *nel punto* m. *Dico hauere tagliato dal piano digradato un piano quadro, ilquale è* blcm. *& ſi proua in queſsto modo. Facciaſi uno quadrilatero in propria forma longo pallmi cinquanta, largo dieci.* nopq. *& ſia* np. *cinquanta, &* no. *dieci. ſia tratta la diagonale* nq. *& partiſcaſi* no. *in cinqu&#x200B;e parti eguali.* r. ſ. t. u. *& dal punto* r. *ſia tirata una linea egualmente diſtante ad* np. *laqual taglierà la diagonale nel puntto* x. *& partirà la linea* pq. *nel punto* y. *& dal punto* x. *ſia tirata la linea egualmente diſtante ad* no. *che partirà la linea* np. *nel punto* z. *&* oq. *nel punto* &. *i quali punti faranno uno quadrato, che ſerà* noz&. *in propria forma tagliato dal piano quadrangulare* nopq. *condotto dalla diagonale, clhe paßa per lo punto* x. *& che diuide la linea* ry. *laquale è la quinta parte di* no. *come s'è detto eßere* bcdc. *digradato, & la linea* bc. *fatta eguale alla linea* no. *laquale è partita in cinque parti eguali, & preſane una delle cinque, che è* bf. *& tiratala all punto* a. *che diuide la diagonale nel punto* k. *& trattala egualmente diſtante, che paßa per lo punto* k. *& diuide* bd. *nel puntto* l. *&* ce. *nel punto* m. *Si come s'è preſo la quinta parte di* no. *cioè* r. *& quella tirata egualmente diſtante ad* np. *che parte la diagonale nel punto* x. *& s'è tirato* x. *egualmente diſtante ad* no. *che diuide* np. *nel punto* z. *&* oq. *nel puntto* &. *& perche la diagonale diuide la ſoperficie in propria forma, nella quinta parte* noz&. *coſi diuide la diagonale la ſoperſficie digradata, come per li principij s'è ueduto.*

Ma ſe egli non ſi ſapeſse la lunghezza, ne meno la larghezza del detto piano, ſi trarrà dal punto a. *una linea egualmente diſtante alla llinea* bc. *della quantità, che s'hauerebbe poſto il termine all'occhio dato: doue ſi farà il punto* o. *dalquale ſi tire-*

Fig. 2.12 **[p. 35]**

If neither the length nor the width of said plane is known, draw a line from point *a* parallel to line *bc* out to a length where the eye has been placed, marked as point *o*. From that is drawn line *oc*, which will divide line *bd* at point *l*. That point will take from the degraded plane *bcde* the quantity *bc*, which **[p. 36]** is *bl*. Then from point *l* draw a line parallel to line *bc*, which will cut the diagonal at point *k* and *ce* at point *m*. In this way we will say that *blcm* is the quadrilateral cut from the non-degraded plane *bcde*, since the line that goes from the eye *o* and terminates in *c* and divides *bd* at *l* such that *c* is represented to the eye higher than *b* by the quantity of *bl*, as was demonstrated above.

Chap. VIII. How to respond to those who, in dividing the plane into braccia, come to have a foreshortening that is larger than the perfect

Pietro of Borgo S. Stefano,[21] who has left us some things about perspective from which I have taken some of the descriptions given above, says the following formal words (Fig. 2.13):

> To take away the error of some who are not experts in perspective, and say that many times, in dividing the plane in braccia, they end up with a foreshortening that comes out to be longer than the perfect, I say that the following demonstration can remove the occasion of error. So, let there be made a square that is *bcde*, inside of which is made another whose sides are parallel to the sides of the first; this is *fghi*. You will draw the diagonals, which cross at point *a* and pass through vertices *f*, *g*, h, and *i*. You will then divide the surfaces between the two squares into equal parts and number those on top 1, 2, 3, 4, 5, 6, 7, and 8, opposite which will be the numbers inside 17, 16, 15, 14, 13, 12. Give one vertex the letter *g* and the other the **[p. 37]** letter *f*. Similarly you will divide side *bd* with the numbers 21, 22, 23, 24, 25, 26, 27 and 28, opposite which will be, on side *fh*, the numbers 32, 33 and so on up to 37. All of these divisions are parallel and square, and represented to point *a* (where the eye is located) divided by the diagonals *be* and *dc* into four equal

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[21] This is Piero della Francesca from Borgo San Sepolcro; cor*recte*d in the Errata on an unnumbered page at the end of the book.

parts, each of which, if we want to think of it this way, is an eye. Because the eye is round and outside (as we have said) it does not come to the perfect rotundity of the fourth part of a circle, so we say that point *a* stands for four eyes. One is the part that is opposite line *fh*, the other opposite line *gf*, the third [opposite] the line *gi*; the last is opposite the line *hi*. If there were four men, each of whom was looking at his façade, it would be the same as what we are saying about the eye. We made the eye at point *a* round, and from the intersection of two thin nerves comes the visual virtue at the centre of the crystalline liquid. From that the rays dilate, extending straight out and dividing the fourth of the circle, form a right angle in the centre. Because the lines that come out of the right angle terminate in the points *f* and *g*, we will say that line *fg* is the greatest quantity that the eye can see, since if it were to go beyond the diagonal, it would follow that the other eye would be less than the fourth part of the circle. This cannot be, because the diagonals of the perfect square divide the circle into four equal parts. So, *fg* is the largest termination that the eye can see. For this reason it happens that going beyond that termination the degraded quantity comes out to be greater than that which is not degraded, because it enters into the part that the other eye sees. This is the proof.

Draw lines from *b*, 1, 2, 3, 4, 5, 6, 7, 8 and *c* to point *a*. I say that the diagonal line *ab* will pass through the point *f* of the line *fg*. Add to line *bc* the quantity from *i* to *b*, making *bk*, and add [to line *gf*] the quantity from *f* to 21 to make 21*l*, then draw *kl*, making square *bkl*21. You will see that the foreshortened [*ml*] is greater than the non-degraded *l*21 by the quantity between 21 and *m*, because *kl* is represented equal to *lm*, which is greater than *l*21 (as we have said), the foreshortened being greater than that which is unforeshortened. This cannot be, since the eye cannot see *k* in that termination, which is part of the eye opposite line *fh*. But although the eye sees *fg*, the intellect does not comprehend it or understand its parts, except as a spot seen from far away which it cannot judge to be a man or another animal.[22] Thus are points *fg* seen from point *a*. Because the things that cannot be understood cannot be reasonably degraded, except as spots, it is necessary to take a termination larger than line *fg* so that the eye can more easily receive the things that are opposite it, represented under an angle smaller than a right angle such that three angles compose an equilateral triangle, and each angle has the same force as the others. Since this line involves roots [i.e., irrationals], we will put actual numbers in so that this termination can be more clearly understood. We say that if your work is seven braccia wide, and that you are looking from no less than six braccia away, and thus when it is more, that you are in proportion.[23] But when your work is less than seven braccia you can stay six and seven braccia away to see, but you cannot consider a larger ratio than that of six to seven[24] (as said) because in that termination the eye can see everything without turning. If it had to turn, the terminations would be false, since there would be more views. If, therefore, you observe the rationales that we have stated, you will know that if the thing that is degraded comes out to be larger than the thing not degraded, the defect is of those [who are not experts in perspective], and not of perspective [itself].[25]

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[22] For Barbaro's discussion of indistinct images seen from far away becoming clearer as the distance between observer and object decreases, see Barbaro (1567, p. 124), Williams (2019, p. 220).

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[23] This is based on the argument that the work measuring 7 braccia forms one side of the equilateral triangle, and the distance to the eye forms the altitude of that triangle. The altitude is found by, equal to 6.06. Thus Piero della Francesca bases his argument on a ratio of 7:6.

[24] Recalling that the smaller the denominator, the larger the ratio, the argument cannot be based on a ratio larger than 7:6 (say, 7:5 or 7:4); in other words, you cannot stand at a distance of less than 6 braccia, because the angle of vision would exceed 60°.

[25] This is a direct quotation with minor alterations from Piero's *De perspectiva pingendi* (Mss. Regg A 41/2, fols. 16v-17r).

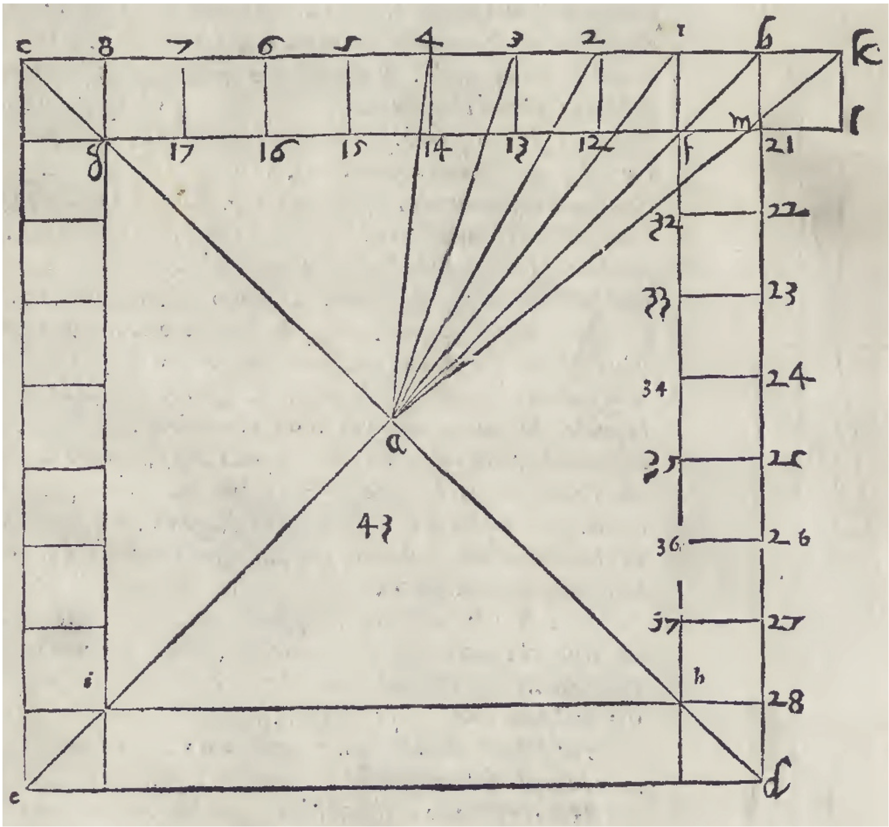

Fig. 2.13 **[p. 36]**

I wanted to quote here the words of the painter Piero in order to show the considerations that those who want to practice the things of perspective must have. Although in the first part of Chap. V [in Part I] where I reasoned about distance, I founded the principle of Piero's proposition, and starting from there I have cor*recte*d what he says in some places, since the circumference of the black [uvea] of the eye is not large enough to embrace the right angle by sight. But it is time to come to the description of the plans according to the order of the regular bodies. These were taken into such great consideration by the ancients that Plato used them to signify the elements of the world and the circle itself, and from the secret intelligence of their forms rose to the summit of speculation about things. By 'regular bodies' he meant bodies of many similar and equal faces and solid angles that are circumscribed by the rotundity of the sphere, as will be seen in what follows.

[p. 38]

Chap. IX. The plan of the triangle, and how the degraded is made from the perfect

The triangle is the first figure that presents itself to us, since every other figure resolves into that, as the mathematicians say. This is the base of the pyramid [i.e., tetrahedron], which is the first body of the regular solids, according to the same rationale by which the triangle is the first figure. The triangular figure thus standing in this way, let the perfect [square] be *abcd* with its diagonals *ac* and *bd*, and let there be formed within it the triangle with equal sides and vertices 1, 2 and 3. Then draw lines from the vertices of the triangle to the sides of the perfect square (as we have said) both to the bottom and to the right side, so that from vertex 1 will fall line 1-1 and from vertex 2 line 2-2. There is no need to draw a line from vertex 3 because it is at the same height in the perfect square as vertex 1, and therefore the same line serves for both. Similarly, draw the lines from the said vertices to the right side, 1-1 and 2-2 as seen in figure E (Fig. 2.14).

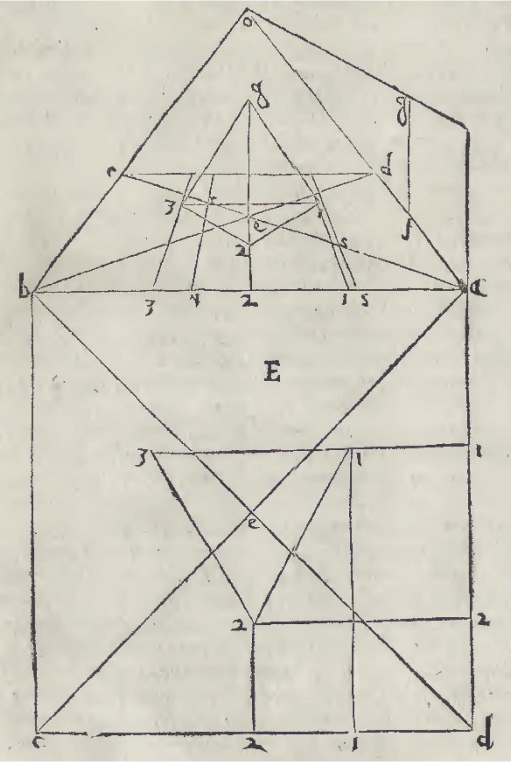

Fig. 2.14 [p. 38]

Next make the degraded square such that the line *ab* of the perfect is in common with the degraded square, which will be *abcd*. Then carry the space 1-*d* of the line *dc* [of the perfect] to the [common] line *ab*, and let that be *ai* [*recte a*1] Carry as well the space that is *d*2 [of the perfect] to [common] line *ab*, and let that be *a*2. Finally, carry the space *di* [*recte d*1] from the line below to the line *ab* and let that be *b*3. From points 1, 2 and 3 marked on line *ab* draw the rays up to the eye *o* until they cut the line *dc* of the degraded plane. I say that on these lines are to be placed the vertices of the triangle 1, 2, 3 of the perfect square, and how this is to be done I will show you now. If you notice, the lines that go from the vertices of the triangle to the bottom of the perfect square can be (and it is better thus) drawn to the top side, which is common to the perfect and the degraded squares. Take from the perfect square the distance *id* [*recte* 1*d*] which is on the line *ad* and carry it to *ab*, the line common to the perfect and the degraded squares, from point *a* towards point *b*; this will be *ar*. Likewise carry the distance 2*d* from the line *ad* to the common line *ab*, and let that be *as*.[26] Now from the points *r* and *s* you will draw occult lines, that is, lines that can be taken away or erased; draw them (I say) to the eye *o*, and where they cross the diagonals [*ac* and *bd* of the degraded] mark the points. Then placing the ruler on those points, draw the line parallel to the common line *ab*, and note where that line, or the ruler, crosses the lines going to the eye **[p. 39]** from the points, or vertices, of the triangle carried to line *ab*. For example, on the line that is born from point 1 marked on the common line *ab* will be placed vertex 1 of the triangle made inside the perfect square. If you want to know in which part of the line is to be placed vertex 1 in the degraded, look to see where the occult line that is born from point *r* cuts the diagonal *ca*, and there mark point *r*. On that you will place the ruler parallel to common line *ab*, and where the ruler cuts the line that comes from vertex 1, make point 1. Because vertex 3 is at the same height, on the line of vertex 3 also mark (I say) 3. In this way you will have marked two vertices of the degraded [triangle], that is, 1 and 3. Now, to place as well[27] the third angle, marked 2, look to see where the occult line that is born from *s* cuts the diagonal *ac* in the degraded square, and mark there the point *s*. On this you will set the ruler parallel to the common side *ab*, and where the ruler cuts the line that is born from point 2, mark 2 in the degraded square.

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[26] The procedure is more clearly explained as marking point *r* where the line between vertices 3 and 1 of the triangle crosses the diagonal *bd*, and point *s* where the line 2-2 crosses on diagonal *bd*, then carrying those up to line *ab*, rather than by operating with distances.

[27] In the Errata on an unnumbered page at the end of the book the original *ponere a., che* is cor*recte*d in *ponere, ancho*.

There is the third vertex of the degraded triangle, which is equal in value to the triangle of the perfect [square].

If you were to ask me the reason why the diagonal *ac* in the degraded square above has been cut by the line that is born from point *r*, and below the same diagonal has been cut at point *s* of the occult line that comes from point *s*, I would respond that all the vertices, all the points and all the lines that in the perfect square correspond to the upper parts of the diagonals have similar relations to the diagonals in the corresponding degraded square, as is known much better through practice than through reasoning. Here then is the rationale by which the plan of the pyramid was positioned in the degraded square, as seen in figure E (Fig. 2.14); this rationale will then serve us in turn in raising and elevating the pyramid.

Chap. X. How to form the plan of a cube in the perfect and degraded squares

Let the perfect square be *abcd*, in which is described the plan of the cube, according to the rationales given above, so that it is a perfect square, and let that be 1234. Draw the diagonals, and from the vertices of the square let fall to the lower side [*dc*] line 1-1, which will also serve for vertex 3, and line 2-2, which will also serve for vertex 4. Do the same to the right side. Then form the degraded square on the common line, to which are carried the points 1, 2, 3 and 4 from line *dc*. By drawing lines from those to the eye until they cut the line *dc* of the degraded square will show, as above, how the vertices of the degraded square are to be made. To position them directly you will do as follows. Take from line *da* of the perfect square the distance, or space, from *d* to 2 and 4, and carry that space to the common line *ab*, from *a* towards *b*. This will come to the middle of the common line, and will cut the diagonals at the point where they cross. On this will be placed the ruler parallel to the common line *ab*. Where the ruler cuts the lines that come from 4 and from 2, you will make the point 2 on the right and 4 on the left. To have the other two vertices, from line *da* in the perfect take the space from *d* to 1 and 1-3 [the single point that counts for both vertices 1 and 3] and carry it to the common line, from *a* to *b* and from *b* to *a*. This will fall on the points 2 and 4 marked previously, the lines of which cut the diagonals below and above. Placing the ruler at those points, you will mark the line that comes to the eye from points 1 and 3, above 3 and below 1, and in this way you will have the two remaining vertices of the degraded plan. When conjoined by lines, these will give you the plan of the degraded cube, as seen in figure I (Fig. 2.15). This will then serve us in raising the cube, so that from this common example is taken, as needed, that which has been said.

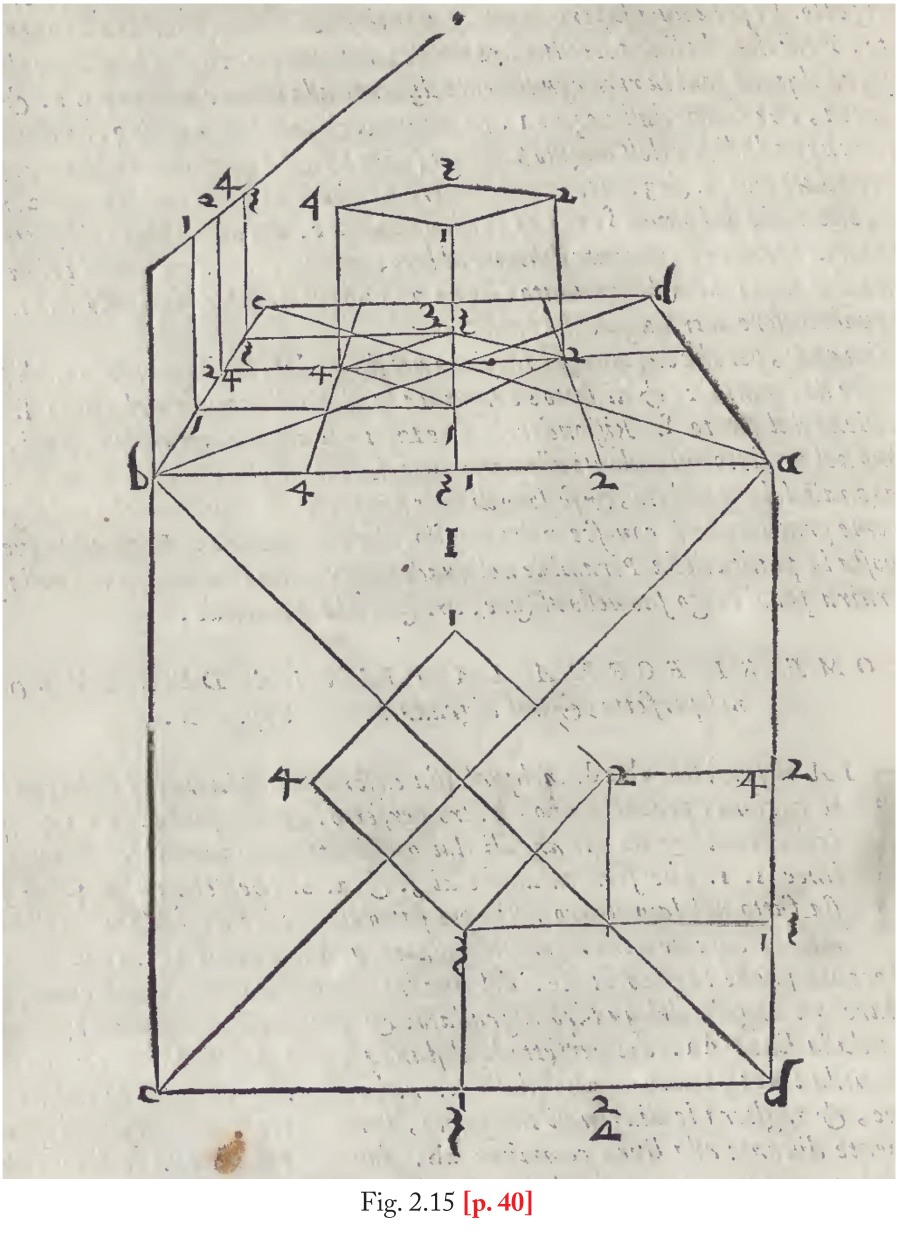
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Fig. 2.15 **[p. 40]**

[p. 40]

Chap. XI. Other ways of making plans

Since from the examples given above can be deduced the way to reduce all surfaces to the degraded square, I will leave for now the degradation of plans of the other regular bodies, also because we will reason about that in the next part, along with the way to raise said plans. Now I will set forth other ways to make plans so that everyone can use one manner or another.

Let there be a given square *abcd* with its diagonals *ad* and *cb*, which cross at &. On & is made a circle in which is inscribed the surface with 5 equal sides **[p. 41]** *efghi*. Let all sides of said surface be extended out to the sides of the perfect *abcd*, as seen in figure 44 (Fig. 2.16): side *ef* will make the line *vefx*; the side *eg* will make the line *kego*; the side *gi* will make the line *ygir* [*recte ygit*]; the side *ih* will make the line *sihz*; the side *hf* will make the line *lfhp*. Then draw line *aq* through point *g* out to side *dc*, and the line *br* through point *h* out to the same side. In this way you will have formed the perfect of said surface with all of its meeting points.

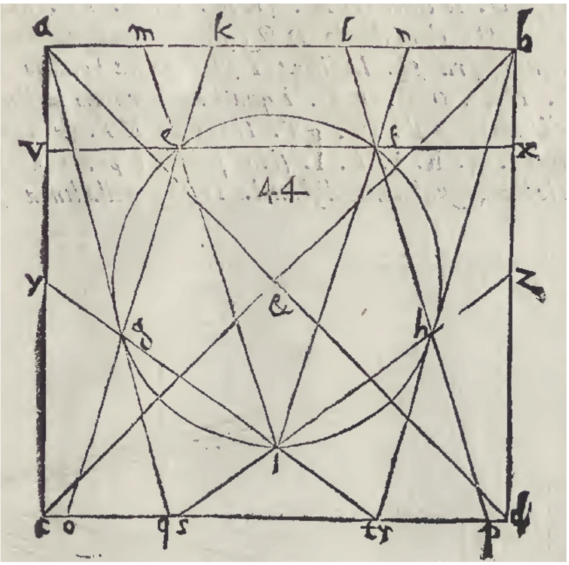

Fig. 2.16[28] **[p. 41]**

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[28] Fig. 2.16 is inaccurately drawn, in that there should be a small space between *t* and *r*, just as there is between *q* and *s*. The Italian menabò manuscript It. IV, 40=5447 (fol. 22v) carries the same imprecision; instead, the space between *t* and *r* is clearly seen in the Latin manuscript Lat. VIII, 41=3069 (fol. 21r) and in the extended Italian manuscript It. IV,

Make the degraded plane *abcd*, with the eye in &, and onto the common line *cd* carry the points *c*, *o*, *q*, *r*, *p* and *d*, and the points *m*, *k*, *l*, and *n*; points *m* and *n* must correspond on the common line *cd* taken from line *ab* of the perfect square. Then draw the lines *m*&, *k*&, *l*& and *n*&; where these lines cut the line *ab* will be carried the points *k*, *m*, *l* and *n*. Draw the line from point *k* of the line *ab* to point *o*, and from point *l* to point *p*, and similarly from point *a* to point *q*, and from *b* to *r*. Where line *aq* cuts line *ko* at point *g*, draw the line *gt*, and where line *br* cuts line *lp* at point *h*, draw the line *hs*. The lines *tg* and *sh* will cut at point *i*. Then draw the lines *im* and *in* to the line *ab*. Where those cut the lines *ko* and *lp* in the points *e* and *f* draw the line *ef*. In this way you will have represented on the degraded plane the pentagonal surface drawn from the perfect (Fig. 2.17).

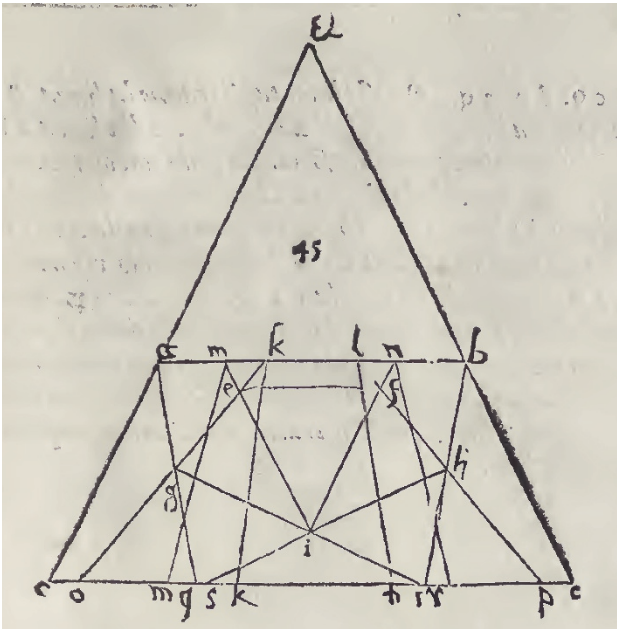

Fig. 2.17 **[p. 41]**

[p. 42] There is another way to carry the surface from the perfect onto the degraded plane, which is as follows. Let the perfect be *ABCD* with the diagonals *AB* and *BC*, which cross at point *E*. On this, after the occult circle is made, is described a surface of 6 equal sides, *fgHIKL*. From these points

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39=5446 (fol. 277r).

fall onto *CD* the lines *HM*, *fKN*, *gLO*, and *IP*. Extend line *CD* by its same length, to the point *Q* by space *DQ*. Let it be divided equally at point *S* by line *RS*. At right angles over *Q* is the line *TQ*, and let the eye be in *T*, to which go the rays *CT*, *MT*, *NT*, *OT* and *PT*, cutting line *BD* at the points *a*, *b*, *c*, *d* and *e*. Then draw lines *AT*, *ft*, *gT*, *HEIT* [not shown in fig. 2.18], *KT* and *LT*, which cut the line *BD* at the points *f*, *g*, *h*, *i*, *k*, [I,] and *l*. Then from points *a*, *b*, *c*, *d*, *e* on line *BD* draw the lines parallel to *DQ* onto the line *QT*; these points will make the lines *am*, *bn*, *co*, *dp*, and *eq*. These lines will cut the line *Rs* in the points *V*, *r*, *s*, *t* and *u*. Then let the space on line *BD* that is from *f* to *I* be carried to line *Vm* from point *V* towards *a* and from the same point [*V*] towards *m*. The space [*f I*] on *am* to the left will be *x* and to the right *y*, and let the space *xy* [not marked on Fig. 2.18] be the top side of the degraded square. Carrying from the line BD the space from *i* to *l*, or from *i* to *g*, which is the same thing, onto the line *dtp*. With the compass, mark *z* to the left and & to the right. In the same way carry the space *ik* from line *BD* onto line *eso*, marking 2 on one side and 𝔭 on the other. The lines *bn* and *cq* will cut the line *RS* in the points *r* and *u*, which are two vertices of said figure; and the others are *z*, &, *2*, and 𝔭, which, when tied together with lines, will represent the degraded figure, as seen in figure 46 (Fig. 2.18).

This is the end of Part II, in which *ichnographia* was treated. This is followed by Part III, which will treat *orthographia*, that is, the elevation of the bodies from their plans.

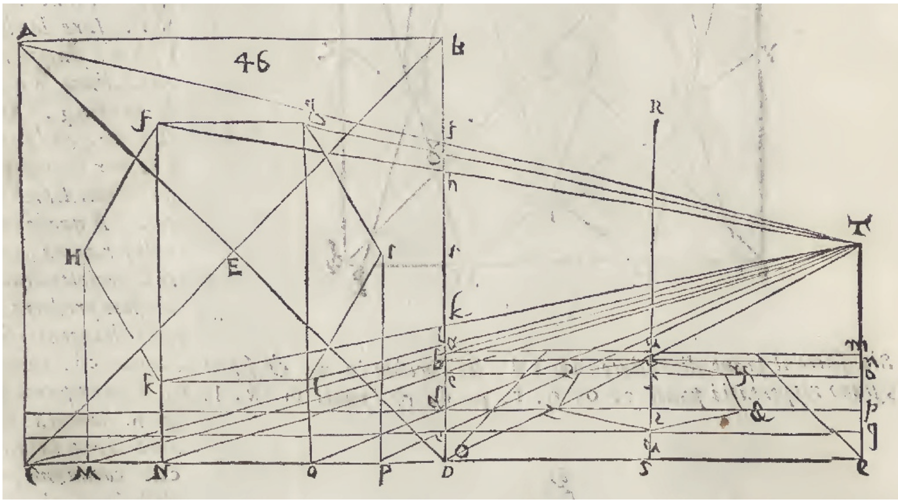
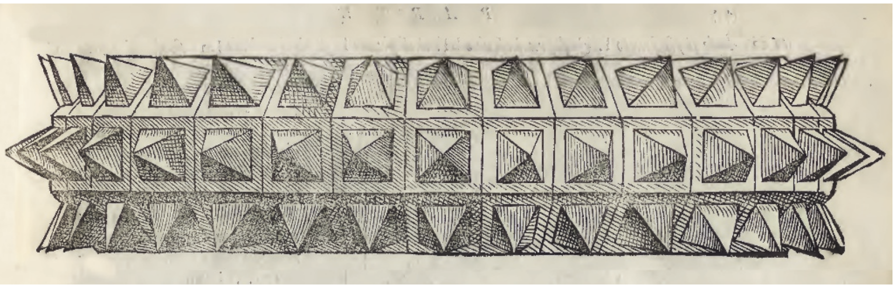


Fig. 2.18 **[p. 42]**



Part III, which treats the ways of raising the body from the plan

Chap. I. Three ways of raising bodies from the plans

[p. 43] There are three ways that are most used to raise bodies from their plans, of which the first is the following. Let there be given the degraded plane *abcd* in which is the degraded triangular plan *egi*; the eye is at *o*, and the centre is *b* [*recte h*].[29] On points *a* and *b* of the degraded square we raise the lines to whatever height we wish the pyramid to be, and those will be *af* and *bk*. Draw line *fk*, which will serve us for the other plane, then draw from the points *k* and *f* the lines – that is, the rays – *ko* and *fo*. Then at points *c* and *d* of the first [lower] degraded plane draw the lines *dm* and *cl* up to the lines *ko* and *fo*. If you draw a line from *l* to *m* you will have degraded the second [upper] plane, which is *fklm*. Let this plane be divided by the diagonals *fm* and *lk* [crossing] at point *n*, from which line *nh* falls to point *h*, which is the crossing of the diagonals of the first plane, and the centre of the base of the pyramid. Then from point *n* draw the lines from the vertices of the degraded base; these will be *ne*, *ng* and *ni*. In this manner you will have raised the pyramid over its degraded plan, as seen in figure A (Fig. 3.1). In this way all bodies can be raised easily.

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[29] The Italian menabò manuscript It. IV, 40=5447, fol. 24r presents, beneath the cutout of the printed figure, a correct figure drawn by hand; in contrast, the extended Italian manuscript It. IV, 39=5446, fol. 27v contains a figure but with different letters, while on fol. 278r the letters are correct; see Monteleone (2020, pp. 216 and 420).

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0_6

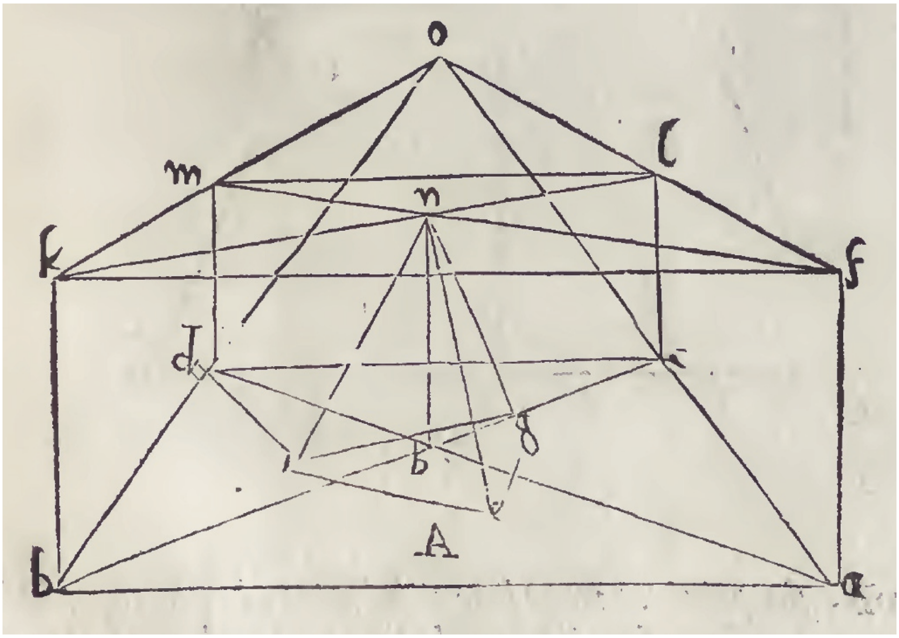
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Fig. 3.1 **[p. 43]**

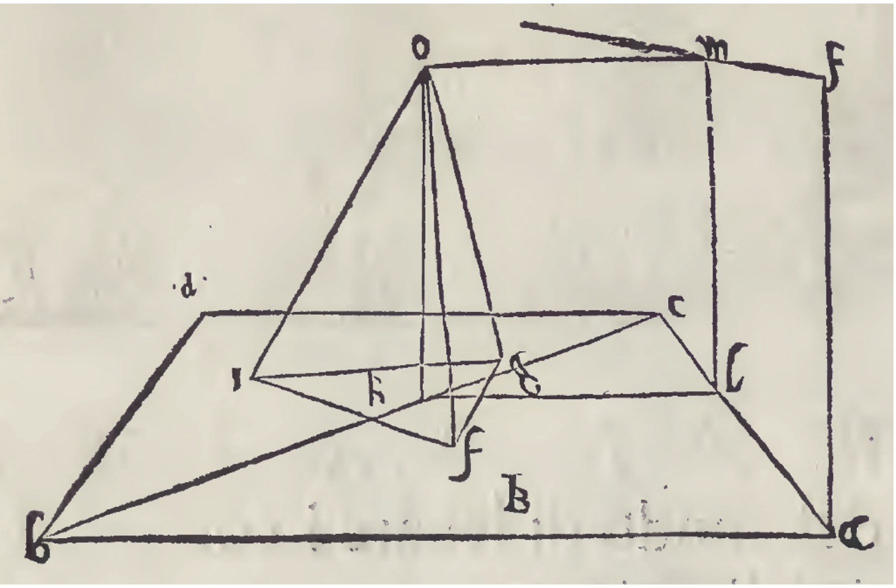
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Fig. 3.2 **[p. 44]**

The second manner is almost the same as the preceding. So, having degraded the square and located within it the base *ifg*, using the rules given above, over point *a* of the common line [*ab*[30]] raise the line *af* to the height that the pyramid is to be, and from point *f* draw a line to the eye *o*.[31] Then from the centre of the base *h* draw a line to the side **[p. 44]** *ca*, making point *l*. On line *hl* from point *l* draw the line *lm* such that the point *m* is on the line *fo*. Similarly, draw a vertical line from point *h* to the height of line *lm*; that is *hn* and point *n* will be the tip of the pyramid.[32] To this, if you draw the lines *fn*, *gn*, and *in* from the vertices of the base, you will have raised the proposed pyramid to the [chosen] height; that pyramid is *ifgn*. This manner is virtually the same as the previous one, and you see that it is born from the choice of the practitioner to do the same thing in a different manner, as seen in figure B (Fig. 3.2).

The third way of raising is this. Let the perfect square be *abcd*, and its degraded plane be made according to the manner described in figure 46 in Part II (Fig. 2.18 **[p. 42]**); let it be *dfgh*. Then, wanting to raise some body in that, take the desired height of that body and place it [on the line] from the point *c* to the point *a* in the perfect square, at point *i*. From point *i* to the eye *o* draw the ray *io*, which will cut the line *db* of the perfect square in *k*. From the point i to the line *db* of the perfect square draw the line *ie* at right angles, and extend it out to point *n*. From the points *e* and *n* draw the rays to point ✠, which will make *e*✠ and *n*✠. Then draw a line from the point *k*, which cuts the line *e*✠ in *l* and *n*✠ in *m*. Thus we will have degraded the upper plane, which is *elmn*. So if in these planes *dfgh* and *elmn* were drawn the surface of 6 sides (as seen in figure 46), and all the vertices of the upper plane were conjoined with lines to **[p. 45]** the corners of the corresponding plan, this would form the body of 8 surfaces [prism with a hexagonal base], of which 6 would be quadrangular and two hexagonal. So, as one plane was raised over the other in figure C (Fig. 3.3), in the same way is raised any figure described within it.

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[30] Recall from Part II that the ‘common line’ is that shared by the perfect square and the degraded square. Here the perfect square is not shown explicitly but is implied.

[31] The eye is not shown in the figure; the tip of the pyramid, *n*, is erroneously marked *o*. The Italian menabò manuscript IV, 40=5447, fol. 24v contains the same imprecisions, while in the Latin manuscript Lat. VIII, 41=3069, fol. 28v, and the extended Italian manuscript It. IV, 39=5446, fol. 278v, the construction shows all the letters, although marked differently; see Montelone (2020, p. 421). The method of the foreshortened height was used by Piero della Francesca in *De prospectiva pingendi*, for example, in the perspective of a cube (Mss. Regg. A41/2, fol. 21v).

[32] Marked O in Fig. 3.1.

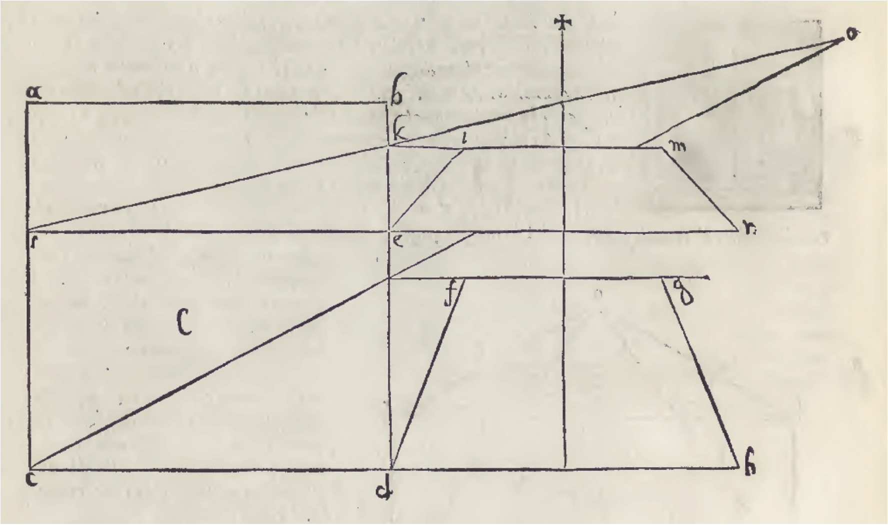

Fig. 3.3 **[p. 44]**

Chap. II. Unfolding, raising and shadowing the pyramid

The following order will serve in describing the bodies: in the first place we will set forth their unfoldings, then their perfect and degraded plans, and finally their raisings (*dritti*[33]) and their shadowings. By 'unfolding' I mean the descriptions of the opened figures from which the solid figures are made by folding them together to demonstrate of the true form, something that is truly convenient for practice and delightful for forming many bodies in lanterns and other uses for pleasure.

So, the pyramid [tetrahedron] is a body with 4 triangular faces, and equal angles and edges, whose unfolding is the figure D (Fig. 3.4).

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[33] The term *dritto* (sometimes also called *impiè* by Barbaro), here translated as 'raising', is a scheme generated directly on the perfect, through which Barbaro obtains the heights of the vertices of the solid. Barbaro uses the raising for substituting the elevations of polyedra.

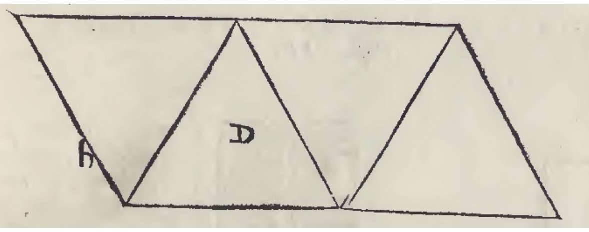

Fig. 3.4 **[p. 45]**

If the paper as drawn is folded and the sides of it carefully glued together, the true pyramid will be formed. The exemplar of this is composed of 4 triangular planes (as said) and has 4 acute solid angles, 6 edges, and 12 acute plane angles. So, having formed the plan of the pyramid in both the perfect square and the degraded square according to the rule given earlier in figure E (Fig. 2.14 **[p. 38]**), it is necessary to raise it appropriately, for which the following description is of benefit. Let there be given a semicircle with centre *a* whose diameter is *de*. On the semidiameter *da* make a triangle of equal sides, *abd*; on semidiameter *ab* form the triangle *abc*; finally, on line *ac* form the triangle *ace*. Then draw one line from *d* to *c* and another from *e* to *b*; these will cut the centre of the triangle *abc* in point *i*. From this point is drawn a line towards side *bd* out to the circumference in point *h*; this line *hi* will be the perfect height of the pyramidal body, as shown in figure F (Fig. 3.5)

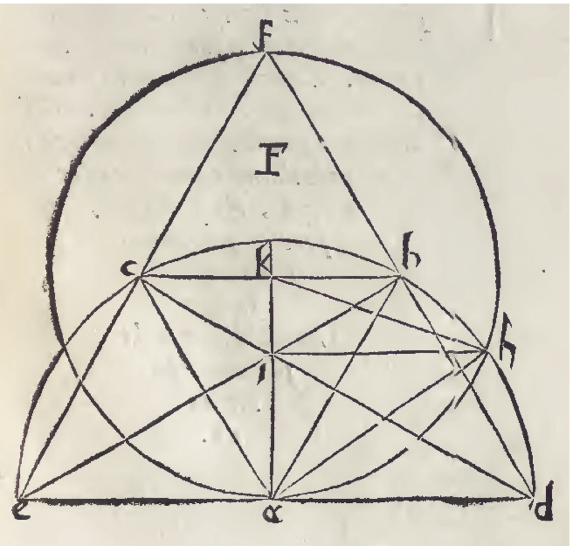

Fig. 3.5 **[p. 45]**

That it is true that the line *hi* is the height of the pyramid, is known in this manner. Let there be drawn from angle *a* through centre *i* a line to the line *bc* to point *k*. With centre *k*, space [radius] *ka*, draw the circle *ah*. From *k* draw the line *kh*, and from *h* the line *ha*. The lines *kh* and *ha* [*recte ka*] being from the centre to the circumference of circle *ah*, they are equal. Similarly, the lines *ab* and *ah* being from the centre to the circumference of the semicircle *de*, they are equal. Line *ak*, the diametrical line [median] of the triangle *abc*, and *ab*, the lateral line, and *kh* being equal to *ka* and *ah* equal to *ab*, it follows that the line *hi* is the true height, since the pyramidal body has the lateral lines opposite its surfaces, or faces. Since it rests as shown in figure F above (Fig. 3.5), drawing from it the unfolding of the pyramid, I will say that making **[p. 46]** a triangle of equal sides on the line *bc*, we have the figure in which are comprised 4 triangles, that is, *abd*, *abc*, *ace*, and *bcf*. These, when folded and glued together, will form the solid of the pyramid. From figure E above (Fig. 2.14 **[p. 38]**) you have the plan and the raising of the pyramid. It is true that the height of that was set without its true measure,[34] but the manner is the same.

The shadowing of the pyramid is easy, as seen in figure G (Fig. 3.6), in which the pyramid is placed in perspective and shadowed as it appears.

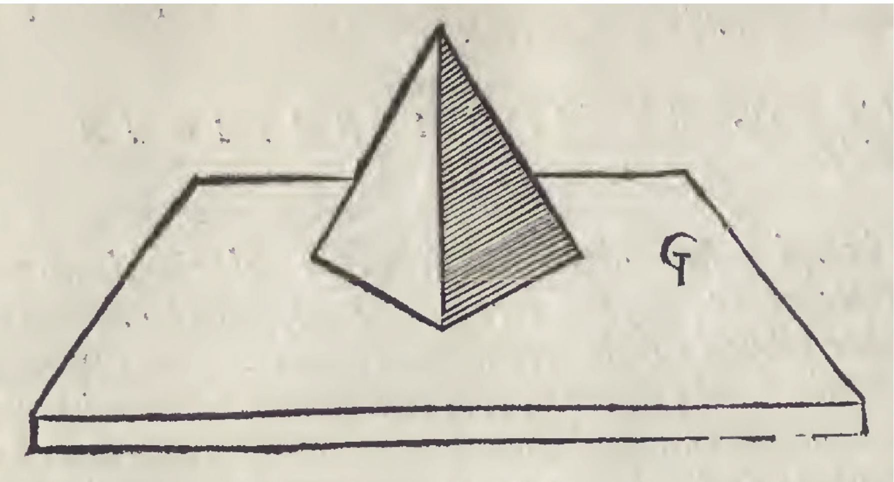

Fig. 3.6 **[p. 46]**

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[34] That is, the drawing does not show a regular tetrahedron because the height is incorrect.

Chap. III. Unfolding, raising and shadowing the cube

The cube is formed of 6 surfaces, all perfect squares, and 8 right solid angles, 24 plane angles, and 12 edges, as its unfolding demonstrates in figure H (Fig. 3.7); this, when closed into a body, will represent the true cube.

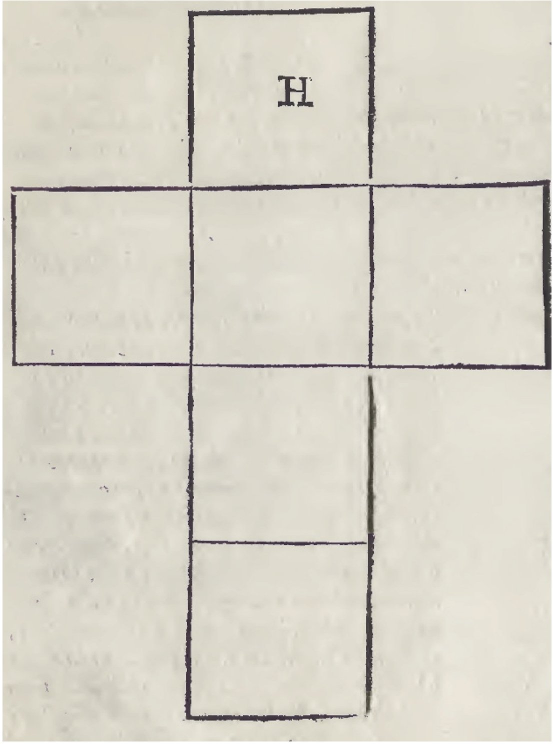

Fig. 3.7 **[p. 46]**

That which appertains to the perfect and the degraded and the raising (*impiè*) can be deduced from figure I (Fig. 2.5 **[p. 40]**) from Chap. X of Part II. The shadowing is shown here in figure K (Fig. 3.9). The height of the cube is equal to one of its edges. Wishing to show its firmness and solidity, the ancients gave the cube to the earth,[35] because when the cube is thrown it stops and becomes immobile of its own accord, like a die does.[36]

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[35] See Plato, *Timaeus* 55d-56c.

[36] Barbaro made a similar remark in his commentary on Vitruvius; see Barbaro (1567, p. 205), Williams (2019, pp. 337-338).

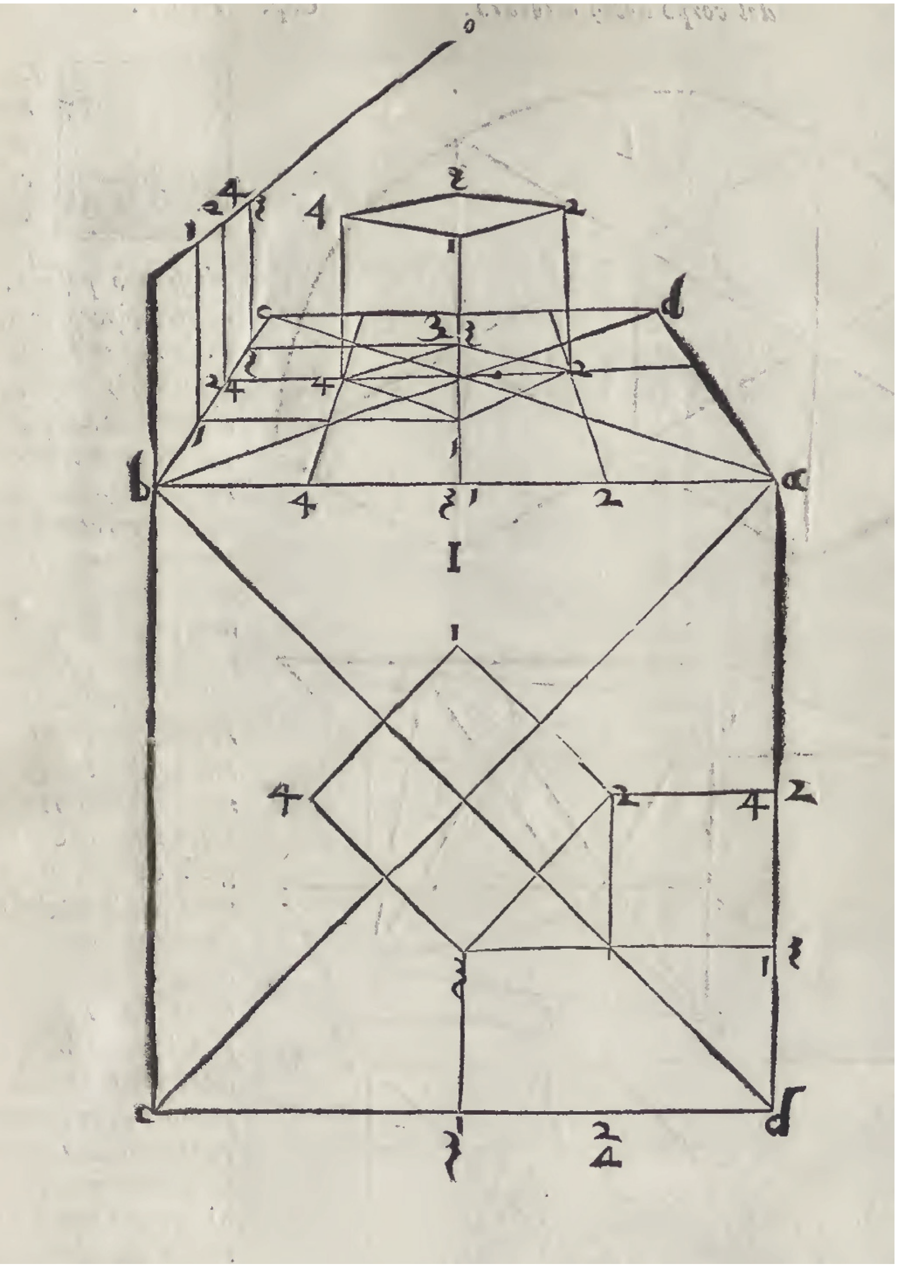
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Fig. 3.8 [p. 47]

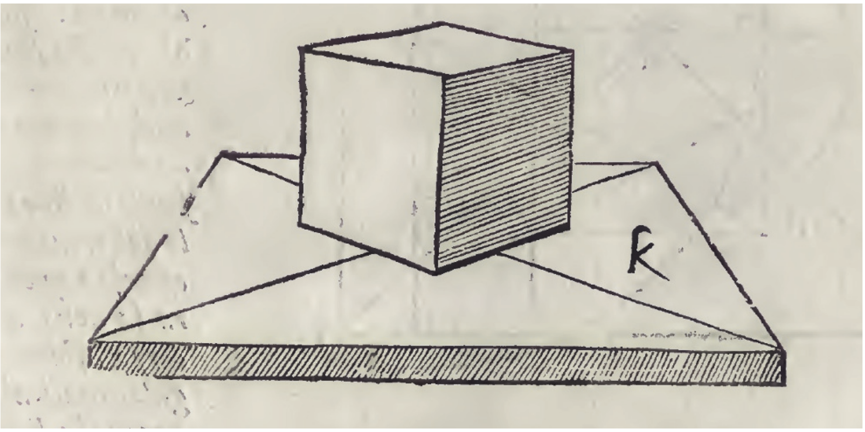

Fig. 3.9 **[p. 47]**

[p. 48]

Chap. IV. Unfolding, raising and shadowing the body called 'octahedron'

The body called 'octahedron' by the mathematicians, because it has 8 faces on which it can rest, is formed of 8 triangular surfaces of equal sides, and contains 12 edges, 6 right solid angles, and 24 acute plane angles. Its unfolding is in figure L (Fig. 3.10).

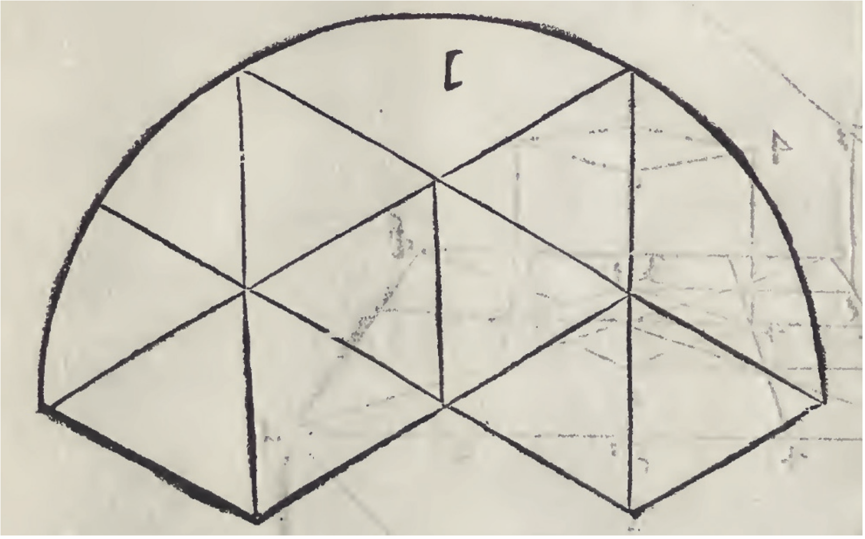

Fig. 3.10 **[p. 48]**

The manner in which its perfect is found is this. Let the surface of 6 equal sides and angles *bcdefg* be set within the perfect square, and then form the triangles *cfe* and *bdg* by means of alternate points, as seen in figure M (Fig. 3.11).

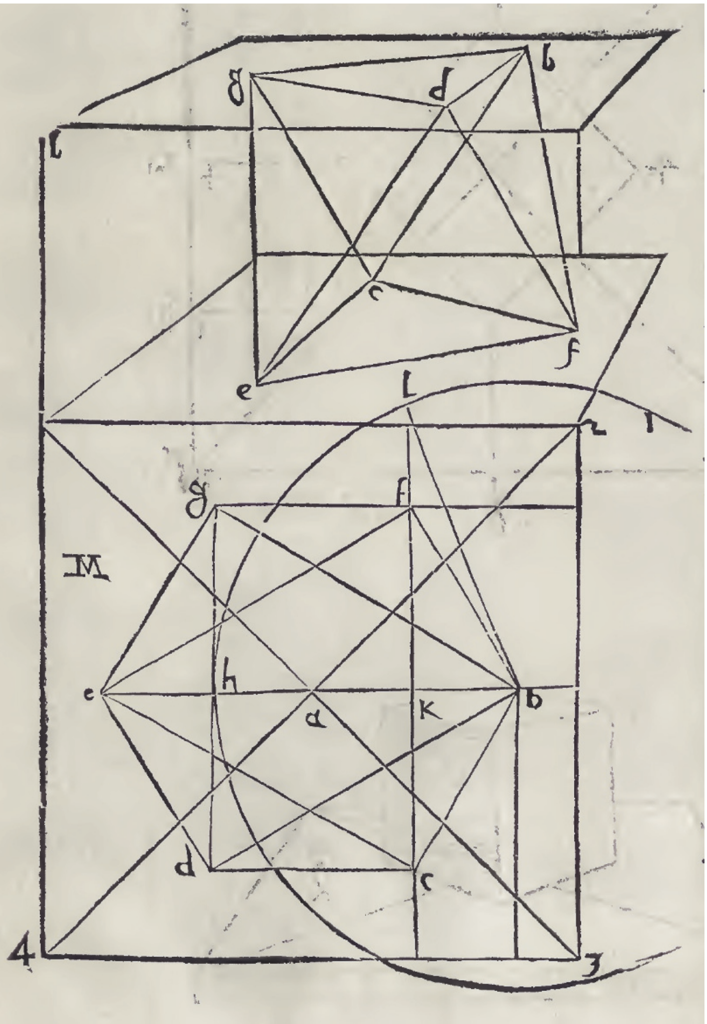

Fig. 3.11 **[p. 48]**

These 2 triangles *cfe* and *bdg* will form 8 triangles, which are *cfe*, *bdg*, *bfg*, *ged*, *edc*, *dcb*, [*efg*,] and *cbf*. Draw a line from point *b* to point *e*, which will cut the the line *cf* in point *k* and the line *dg* in point *h*. On point *b*, with space [radius] *bh*, draw the circle *bi*, and from the point *k* through point *f* draw the line to point *l* out to the circumference of the circle *hi*. This line *kl* will be the height of the octahedral body. That this is true is thus demonstrated. Draw the line *bl*, which will be equal to the line *bh* because both go from the centre of the circle to the circumference of the same circle. Line *bh* being the diametrical line [median] of the triangle *bdg*, and *bl* equal to that, the line *lk* will be the true height of that body formed of 8 triangles equal to triangle *cef*, since the line *bh* is the width but not at right angles **[p. 49]**, since if it were at right angles, the point *k* would be over the point *b*. So, let the perfect square be 1234, within which is the plan of this body *bcdefg*. Let the eye be where the lines that begin in points 1 and 2 concur [not shown in Fig. 3.12]. Let the height of the other plane be equal to the length of the line *kl*, or *bh*, from point *i* to point *f*, from which is drawn the line of the plane 5-6, so as to form the degraded plane 5678,[37] in which is formed the triangle *bdg*. Since in the degraded plane is described the triangle *cef*, you draw lines connecting the vertices of the triangle above with the vertices of the triangle below: *d* with *e* and *f*; *b* with *c* and *f*; *g* with *c* and *e*, as seen in the degraded plane in figure M (Fig. 3.11).

The shadowing of this body is shown in figure N (Fig. 3.12).

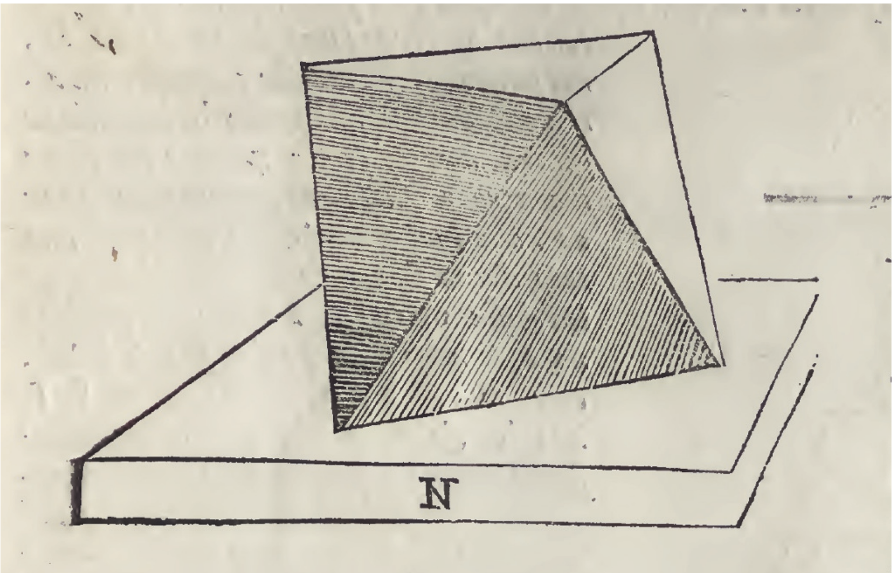

Fig. 3.12 **[p. 49]**

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[37] These numbers are not shown in Fig. 3.11.

Chap. V. Unfolding, raising and shadowing the dodecahedral body

Next is formed a regular body of 12 faces of 5 sides each, which is called a 'dodecahedron' because it has 12 planes of equal sides and angles, 30 edges, 20 obtuse solid angles, and 60 obtuse plane angles. Its unfolding is in figure O (Fig. 3.13).

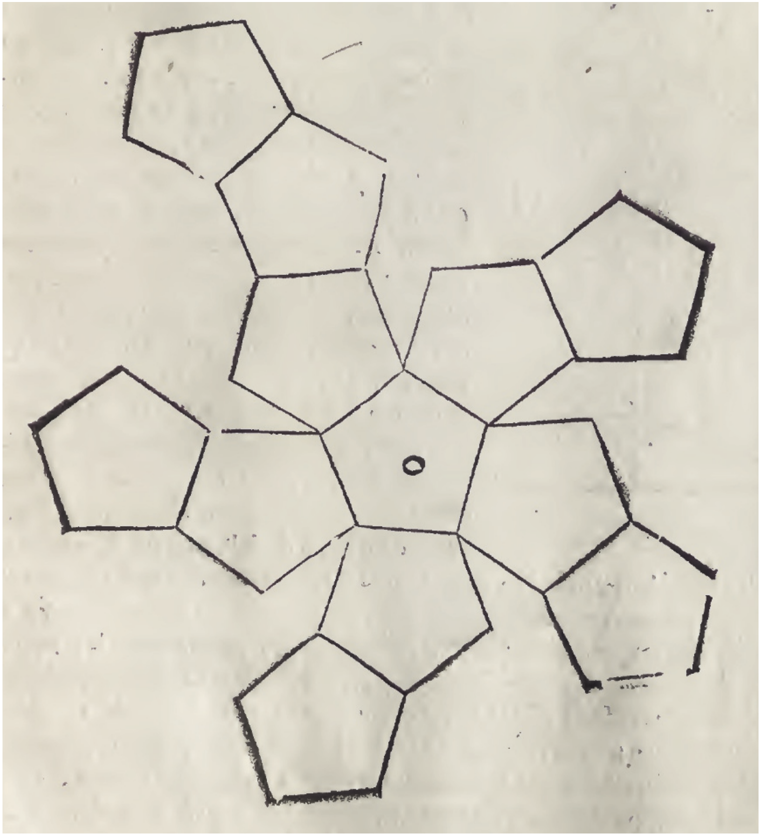

Fig. 3.13 **[p. 49]**

The perfect of this body is made in one manner in practice, but in demonstrating it with the rationales of geometry it is done in another way. Here I will set forth the one that appertains to practice. Since all of these regular bodies are circumscribed by the sphere, that is, with all of their vertices they would touch the concavity of a sphere in which they were enclosed, so in the formation of their perfect plans they are enclosed in a circle.

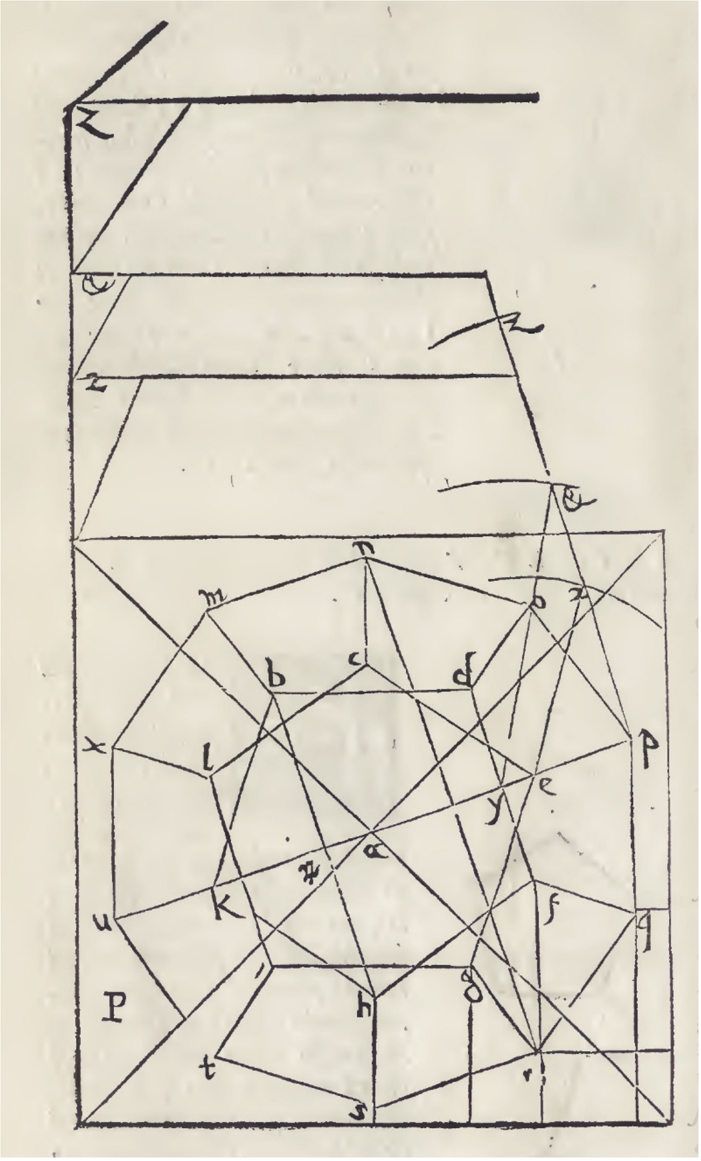

Fig. 3.14 **[p. 50]**

[p. 50] So, make a circle on centre *a*, and divide it into 10 equal parts at *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, and *l* (Fig. 3.14). Alternating over those parts make 2 surfaces of 5 equal sides, one of which is *bdfhk*, the other *cegil*. This is for the plan of that surface that rests on the plane and the *bdfhk* for the plan of the upper surface.

From point *k* through centre *a* draw a line to the side *df* at point *y*, and also draw another one from point *b* to point *h*, which will cut *ky* at point ƶ. Then on centre *a* make a circle as large as the line *bh*. In that let there be one side of the surface of 5 equal sides; this will be *moqsu*, whose vertices correspond to the vertices of the pentagon *bdfhk*; that is, *m* [corresponds] to *b*; *o* to *d*; *q* to *f*; *s* to *h*; and *u* to *k*. These points correspond in the plan to those vertices in the middle of this body, which are raised from the plan. Since there is another level of raised vertices in the perfect plan, these are made in this way. In the larger circle make another pentagon whose vertices are *n*, *p*, *r*, *t*, and *x*, which correspond to the vertices of the smaller pentagon [*cegil*]: *n* to *c*; *p* to *e*; *r* to *g*; *t* to *i*; *x* to *l*. However, it must be noted that the larger pentagons must not be conjoined in the plan with lateral lines; instead, it is sufficient to locate their vertices on the circumference of the large circle, which must also be drawn occultly, that is, in such a way that it can be removed, as they are seen removed in figure P (Fig. 3.14), in which no circles, either large or small, are seen. It is good to draw the apparent lines *bm*, *cn*, *do*, *ep*, *fq*, *gr*, *hs*, *it*, *ku* and *lx*, and similarly *mn*, *no*, *op*, *pq*, *qr*, *rs*, *st*, *tu*, *ux*, *xm*. In this manner is described in the perfect plan of this body, which will have 12 surfaces, that is, *cegil*, *bdfhk*, *mnobd*, *nopce*, *opqfd*, *pqrge*, *fstgi* [*recte rstgi*], *stukh*, *tuxil*, *uxmbk*, *xmncl*, and *qrsfh*, as seen in figure P.

Having finished the plan, it is necessary to find the suitable heights, so let the arc of a circle be drawn to the upper right over centre *e* with space *ec*. Then over *y* with space *yk* let there be drawn another on the same side as the first. Let there be raised over point *p* a line at right angles, which is *pz*. This will cut the arcs drawn previously over *e* and *y* in the points & and 2. Its length must be equal to the space *pk*, since that is the height of the whole body. These cuts *z*, & and *2* will be the heights, such that *p* is the lower plane, 2 the first raised, & the second level, and *z* the third. Thus in the plane of *p* will be degraded the surface *cegil*; in the plane 2 are set the points *n*, *p*, *r*, *t*, and *x*; in the plane & the points *m*, *o*, *q*, *s*, *u*; and in the plane *z* the surface *bafhk* [*recte bdfhk*]. Then by drawing the lines, as said, the form of this body will be raised in perspective **[p. 51]**, as seen in figure T (Fig. 3.15).

That points *z*, & and 2 are the heights of this body will be found by drawing the line *e*2 equal to *eo* [*recte ec*], as lines drawn from the centre to the circumference of the same circle. The point 2 will be the first height, since the first height is equal to the side of the pentagon but not at right angles. Since the line *e*2 is not set at right angles on the point *e*, its height is equal to *p*2, since 2 is at right angles over *p*.

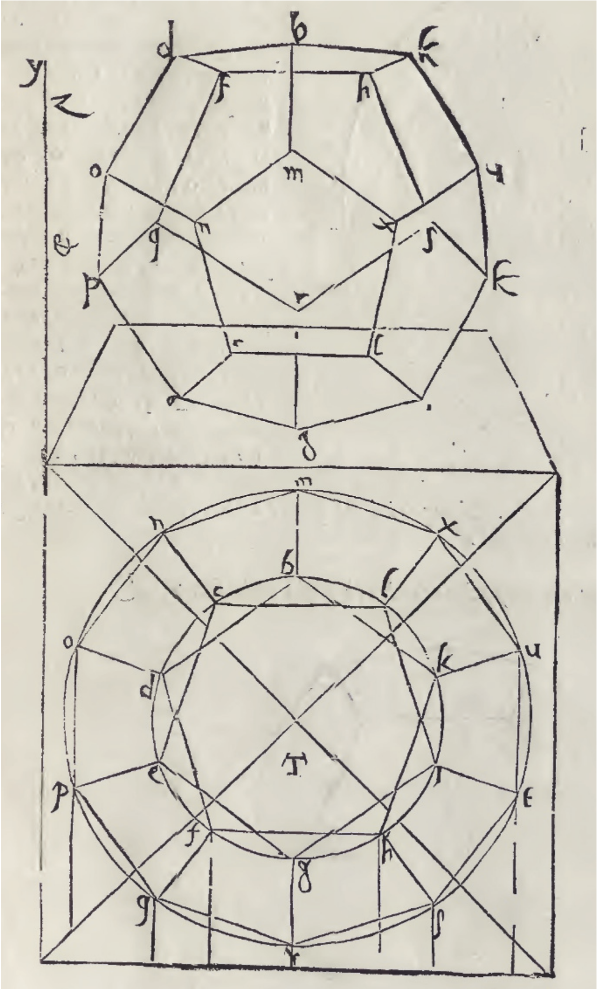

Fig. 3.15 **[p. 51]**

That *p* is in the circle born from centre *a* and cannot be extended beyond that line 2*p*, because if it were extended the body would not be perfect. This can be wonderfully known when the perfect is before our eyes. It occurs similarly to the line *ky*, raised to the sign & with the line *y*&, by which it is proven that the point & is the second height, since the distance from the middle of side *fd* to point & is the same as that line *ky*. Line *ky* is found to be equal to *y*&, since both are lines from the same centre to the circumference; nor can it be extended beyond &2. Since *z*& is equal to *p*2, the point 2 is the other height, because &2 is the side of the pentagon, as is *p*2, which is seen in figure P (Fig. 3.14).

In order to better state what has been said regarding the heights, I will say the following. In the perfect of figure P, let there be drawn the line *nr*, from which and from side *df* is made, in a separate drawing, a quadrangular figure, whose sides are *abcd*; let a circle be made about its centre *e* with space *ea*. This circle will come out much larger than the one made first, due to the fact that if the said dodecahedron were placed in a spherical body so that with all of its vertices it touched the spherical surfaces, its larger circle would be the above-mentioned circumference. This proceeds from the lines *ab* and *ac*, which, reduced into the quadrangle, form that circle, as seen in the material body. So, on the points *a*, *b*, *c* and *d* of the quadrangle with the space of *ky*, taken from figure P, let there be drawn 4 occult arcs, which cut each other in the points *f* and *g*. Then draw the lines *bf*, *fd*, *cg* and *ga*. Then from point *a* towards *g*, carry the line *k*♃ from the figure P, onto the point *h* of the figure R (Fig. 3.16). Carry the same from point *b* **[p. 52]** towards *f* onto *i*, and from *c* towards *g* onto *k*, and from *d* towards *f* onto *l*. Then draw the lines *gi*, *gl*, *kf*, and *hf*. Here the lines *gi* and *hf* will cut each other in the point *m*, and *gl* and *kf* will cut each other in the point *n*. Then draw the line *mn*, and 4 pentagons will be formed, that is, *abimh*, *mnhgk*, *cdkln*, and *mnifl*, and 4 others opposite these. The other 4 occupy the lines *bf*, *fd*, *cg* and *ga*. Then draw a line from point *f* to the line *kc* at right angles in point *o*; this line *fo* is the height of the whole body, and will be equal to *p*2 [*recte pz*] in the other figure; *nd* in this one is equal to *e*2 in the other; and *df* in this one is equal to *y*& in the other. Then draw the lines *al* and *hd*, which will cut the line *fo* in the points *p* and *q* [not shown in Fig. 3.16]. Line *oq* in this figure is equal to *p*2 in the other; *qp* in this one is equal to 2& in the other; *pf* in this one is equal to *z*& in the other. So said points *z*, &, and 2 are the true heights of this body, as seen in figure K [*recte* R[38]] (Fig. 3.16).
The shadowing of the dodecahedral body is set out in figure Q (Fig. 3.17).

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[38] This correction is noted in the Errata on an unnumbered page at the end of the book.

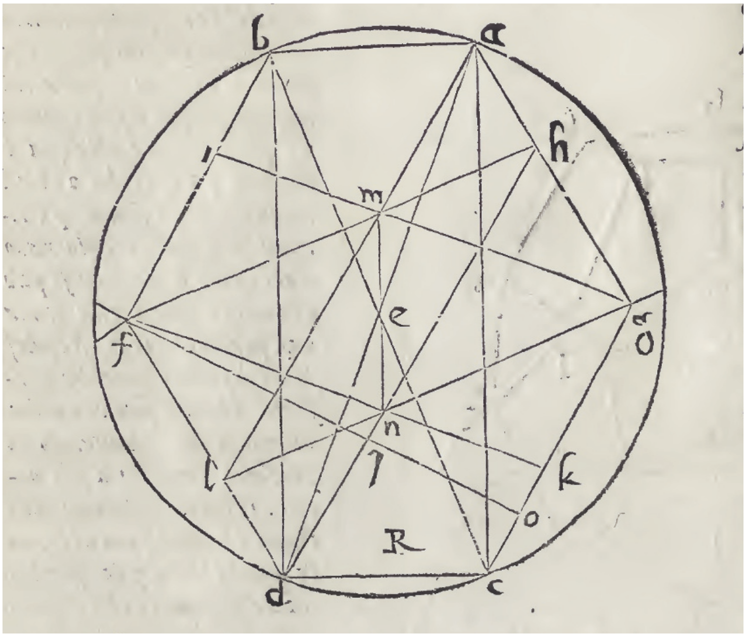
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Fig. 3.16 [p. 52]

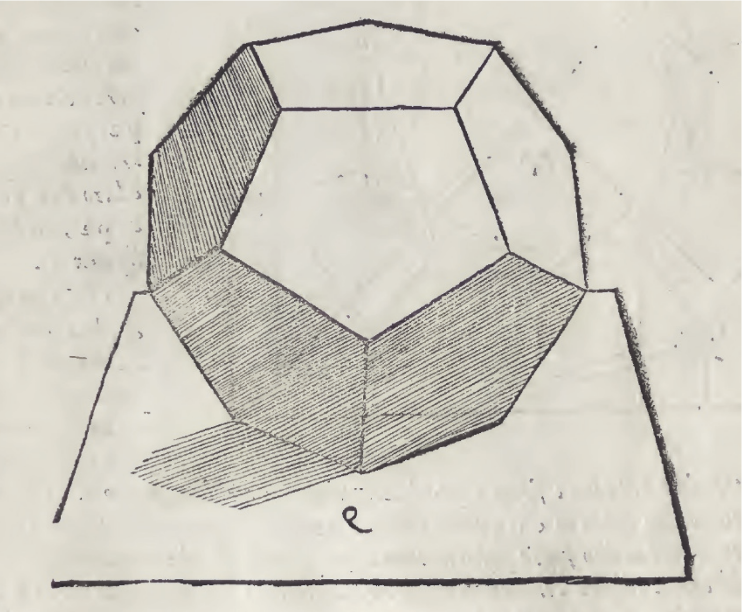

Fig. 3.17 [p. 52]

[p. 53]

Chap. VI. Unfolding, raising and shadowing the body called 'icosahedron'

The icosahedral body, so called for having 20 triangular faces, is subject to obtuse and acute angles. In its edges it is equal to the dodecahedral body, since both have 30 edges, but in their bases and solid angles they are not alike, because that [dodecahedron] has 12 bases and 20 solid angles, and this [icosahedron] has 12 angles and 20 bases; that [dodecahedron] has 60 obtuse angles, and this [icosahedron] has 60 acute angles. The unfolding of this body is seen in figure V (Fig. 3.18).

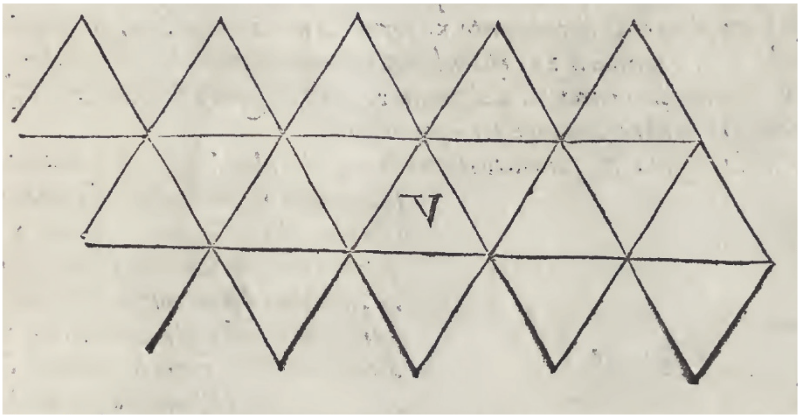

Fig. 3.18 **[p. 53]**

The perfect of this body is formed in this way. Let a circle be made over centre *a* in which is described a pentagon of equal sides *bcdef*. On *ce* be make a triangle with equal sides *ceg*; around that is made an occult circle whose centre is *h*, and in it is described another triangle *ikl*, whose vertices are equidistant from the vertices *c*, *e* and *g* of the first triangle. Then draw the line *bf* and the line *bm* [not shown in Fig. 3.19] which fall on *b* at right angles. Also draw the line *hc*, extended to point *m*, and with *h* as the centre and space *hm*, draw the circle in which is described a surface of 6 equal sides *mnopqr*. Then draw the lines *rk*, *rc*, [*re*,] *rm*, *rq*, *nl*, *nc*, *nm*, *no*, *ng*, *pi*, *po*, *pg*, *pq*, *pe*, *mk*, *mc*, [*ml*, *og*,] *oi*, *ol*, [*qi*,] *qe* [and *qk*], and you will have described the perfect with its 20 triangles, as seen in figure X (Fig. 3.19).

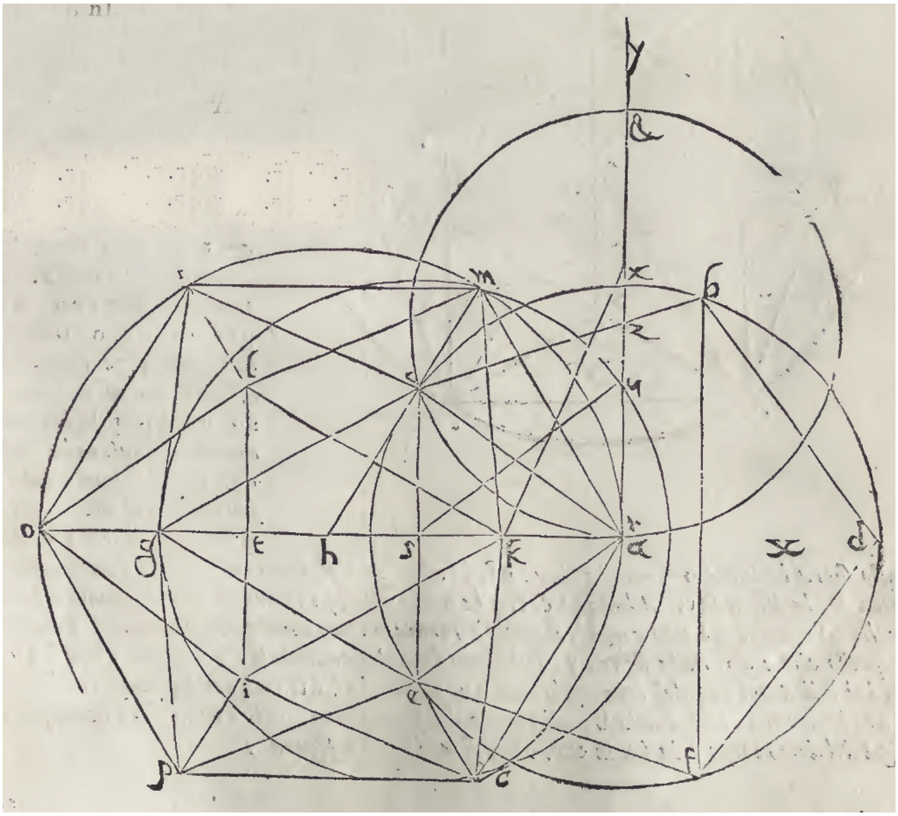

Fig. 3.19 **[p. 53]**

The planes and the heights of this body are actually investigated in the following way.

[p. 54] Draw the line *rg*, which will cut *ce* in point [*s*], and *ih* [*recte il*] in point *t*. Place line *yr* on point *r* at right angles, and with *s* as the centre and space [radius] *sg*, make a circle, which will cut *yr* in point *u*. With *k* as the centre and space *ki*, make another circle which will cut *yu* in point *x*. Then divide the line *xu* equally [in two] at point *z*. With that [*z*] as the centre and space *zr* draw the circle that will cut *yx* in point &. These points *r*, *u*, *x* and & are the planes of this body, such that if you carry into plane *r* the triangle *kil*, into plane *u* the points *m*, *o*, and *q*, into plane *x* the points *n*, *p*, and *r*, and into plane & the triangle *cge*, and then you draw the lines as demonstrated in the perfect drawn above, these will form the icosahedral body.

That the points *x*, *y* [*recte u*], and & are the heights of this body is proved in this way. In the afore-mentioned figure X (Fig. 3.19) draw the lines *us* and *xk*, and since *us* is equal to *sg* and *yh* [*recte kx*] is equal to *ki*, and the first height in the real body being *sg*, which is equal to *kr* [*recte su*], whose point *u* terminates on the line *xr*, it follows that said point *u* is the termination of the first height. Since in the real body the second height is *ki*, which is equal to *kx*, whose point *x* terminates in the line &*u*, that point *x* is the second height. Since *z*& is equal to *zr*, because in the material body the third height [&] is as far from the second [height] *x* as the first [height] *u* is from the point *r*, so the point & will be the third height. With the following example this will be more clearly practiced.

With the lines *bf* and *ce* from figure X (Fig. 3.19) form the quadrangle *abcd*, and find its centre *e*, on which is drawn the circle that passes through point *a*. This circle would be the largest if the said body were circumscribed by the sphere. Then from figure X take the space *gs*, and making centres of the points *a*, *b*, *c* and *d* of the quadrangle draw the arcs that cross each other on all sides: *fg* and *hi* on one side, and *kl* and *lm* on the other. These will cut each other in the points *o* and *p*. Then draw the lines *ao*, *oc*, *bp*, *pd*, *ap*, *cp*, [*do*] and *ho* [*recte bo*]. Here the lines *ap* and *cp* cross with the lines *bo* and *do* in the points *q* and *r*, and thus in another way is formed the [icosahedral] body. The 20 triangles are *abq*, *bqp*, *pqr*, *prd*, *drc*, *cro*, *orq*, *aoq* and the other 8 that lie underneath these, and 4 are hidden by the lines *ao*, *oc bp* and *pd*, which are diametrical lines [medians] of the triangle, while *ab* and *cd* are the sides which are bases of this body. Then draw the line *ps*[39] at right angles to the line *oc*; this line *ps* is the height of the body and it is equal to the line &*r* in figure X (Fig. 3.19). Thus &*r* is the true height, being equal to line *ps*, since line *ps* is born from a body that is formed, with that proof [which comes] from the figure X, and that which is *ab* in figure Y (Fig. 3.20), here [in figure X (Fig. 3.19)] is one side of a triangle. That which is *ao* in this [figure Y] is there [in figure X] the diameter [height] of the triangle; *bo* in this one and *ds* in that one is a diameter [median] of the pentagon; *ac* in this one and *bf* in that one is a diameter [height] of the pentagon. It is therefore manifest that the height of this body is the line &*r*, as seen in figure Y (Fig. 3.20).

[p. 55] Now practicing further I will prove the things said above. On centre *a* let there be made a circle in which is described the figure with 5 equal sides *bcdef*. Draw the line *bf*, as see in figure Z (Fig. 3.21).

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[39] Point s is not shown in Fig. 3.20.

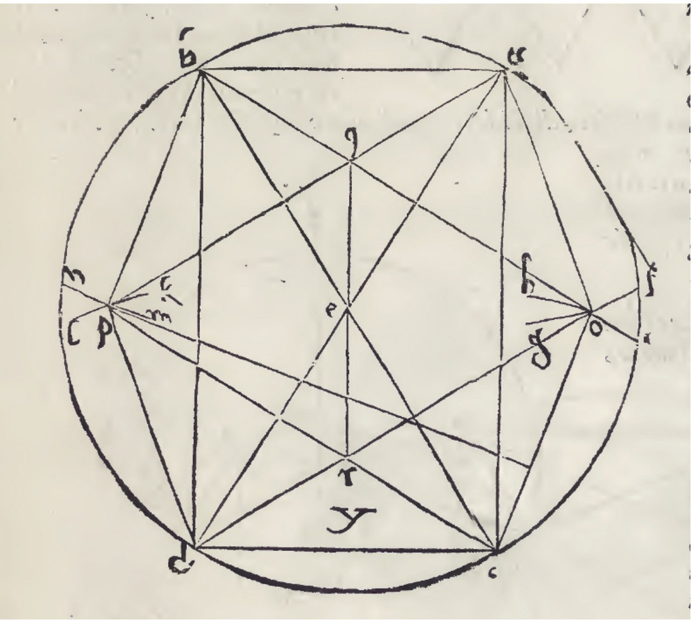
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Fig. 3.20 **[p. 54]**

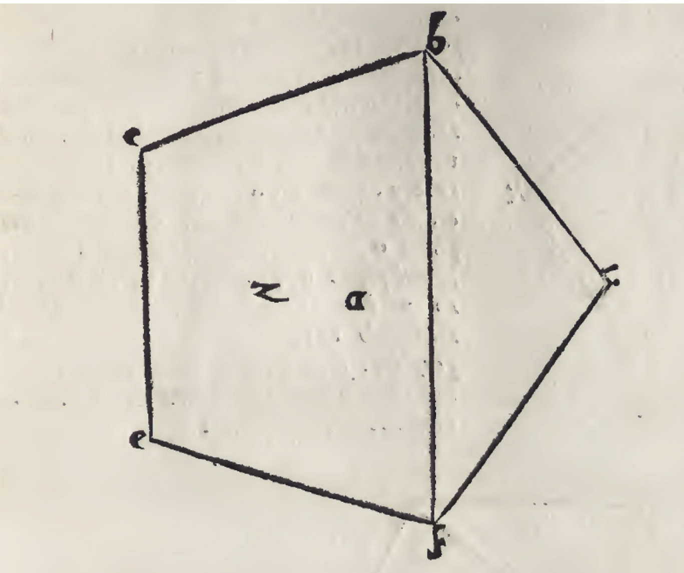

Fig. 3.21 **[p. 55]**

In another place make a circle about point *h*, and let one side of the pentagon of figure Z (Fig. 3.21) be the side of a triangle of equal sides drawn in that circle. Let 2 triangles of equal sides be drawn in that circle, the vertices of which are equidistant from each other; let one be *cge* and the other *kil*. Then with that centre *h* draw a circle of circumference such that the line *bf* of figure Z is one side of a triangle inscribed in it. In that circle are found only the vertices of 2 triangles of equal sides drawn in the circumference, equidistant from each other; these correspond to the triangles first described, as demonstrated in figure X. So, let the vertices of one triangle be *moq*, and those of the other be *npr*. Then draw the lines, as in figure X, thus forming the plan in the perfect as in figure & (Fig. 3.22).

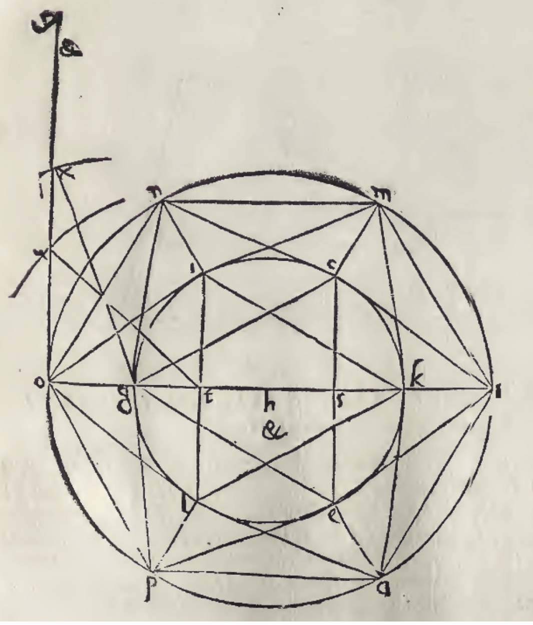

Fig. 3.22 **[p. 55]**

In drawing this, the pentagonal form is necessary because adding together 5 of its triangles to make the body results in the pentagonal form.

Now to understand its heights, in figure & (Fig. 3.22) draw the line *kg*, which will cut *ce* in *s* and *il* in *t*. Then draw line *yo* at right angles over point *o*, and with centre *t* and space *tk*, draw an arc of a circle, which will cut the line *yo* in point *u*. With *g* as the centre and space *ge*, draw an arc of a circle, which will cut *yu* in *x*. **[p. 56]** Then carry the distance of *ou* from *x* to &, and you will have the planes of this body: *o*, *u*, *x* and &. Distance *ou* is equal to &*x*.

Now, to see that these planes are the true heights of this body, draw lines *tu* and *gy* [*recte gx*]. Since *tu* is equal to *tk*, and *tk* is the first height, so *u* is the first height. Since *tx* [*recte gx*] is equal to *gc*, and *gc* is the second height, so *x* is the second height. Since &, which is the third height, is as far from *x*, the second height, as *u*, the first height, is from *o*, then &*x* being equal to *uo*, it follows that the point & is the third height. Therefore, if in plane *o* we form the triangle *cge* [*recte ikl*], in plane *u* points *m*, *o*, and *q*, in plane *x* the points *n*, *p* and *r*, and in plane & the triangle *ikl* [*recte cge*], and we draw the lines as in the perfect, we will have formed this icosahedral body, as seen in figure & [*recte* Q] (Fig. 3.23). The shadowing is shown in figure 2 (Fig. 3.24).

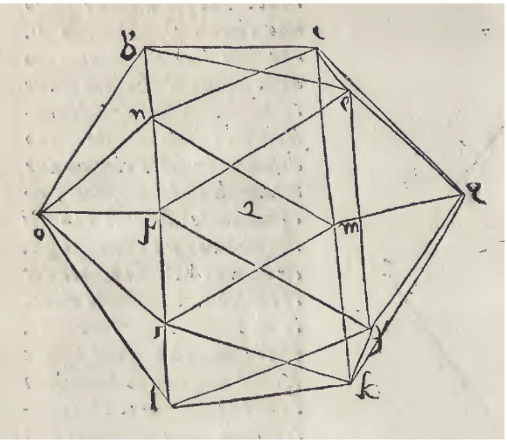

Fig. 3.23 **[p. 56]**

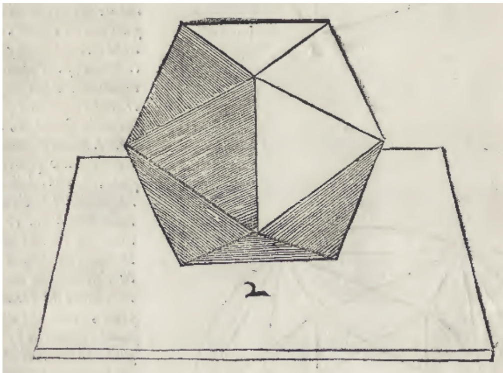

Fig. 3.24 **[p. 56]**

Chap. VII. Description of the irregular bodies that are born from the regular bodies [truncated tetrahedron]

From the regular bodies set out above, of which there are no more than 5 (not counting the sphere), are formed many other irregular bodies thanks to the mutation of the plane and solid angles. This is seen in their unfoldings, which show how these bodies are composed. First, from the pyramidal body is born the body made of 4 surfaces of 6 edges and 4 triangles of equal sides [truncated tetrahedron]. It has 24 obtuse [plane] angles and 12 acute plane angles, 12 solid angles, and 18 edges, as seen in the figure of its unfolding in figure 4 (Fig. 3.25).

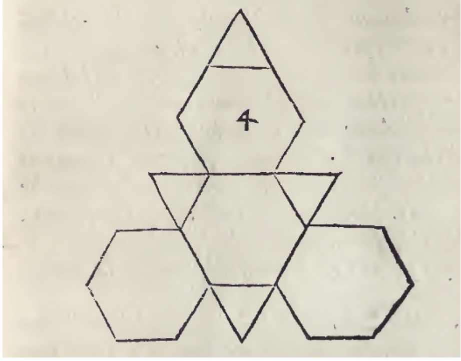

Fig. 3.25 **[p. 57]**

The perfect of this is formed in practice in this manner. Let there be formed a square, in the middle of which, at centre *a*, is made a **[p. 57]** circle, in which is described a surface of 6 equal sides *bcdefg*. Then draw the line *bg*, and over centre *a* make another circle big enough so that line *bg* makes in it the points [i.e., *bg* is the side] of an equilateral triangle. Let these points [of the triangle] be *h*, *i* and *k*. Draw the lines *cd*, *cg*, *dg*, *bf*, *be* and *cf* [*recte ef*]. Make the lines of another surface of 6 equal sides, whose points are *l*, *m*, *n*, *o*, *p*, and *q*. Then draw the lines *mn*, *mq*, *nq*, forming a triangle of equal sides whose vertices are *m*, *n*, and *q*. Then draw the lines *hm*, *hb*, *hd*, *ic*, *in*, *ie*, *kg*, *kq* and *kf*. Thus will be described the perfect in which the 4 hexagons are *bcdefg*, *bchimn*, *hdmfqk*, *ineqgk* and the 4 triangles are *mnq*, *hbd*, *ice*, *kfg*.

The heights and planes of this body are made in this manner. Take the line *qk* and place it at point *r*, where it [*qk*] is divided by *fg*, and then carry that line [subdivision] *qrk* onto the lower line of the square onto the points *s*, *t* and *u*. Then let the line *bc* be divided equally [into two] at point *x*, and draw the line *xq*, which will cut *mn* in *y*. Then with space [radius] *qy* and centre *t*, draw the circular line, which will cut the right line of the square in point *z*. Then with space [radius] *xr* and centre *u*, draw the circular line that will cut said line to the right in &, so that points &, *z* and *s* are the points of the heights. So drawing the line &*u* and *zt*, that line &*u* is equal to *xr*, and *zt* is equal to *yq*. Since in the solid body *yq* is the first height, but not at right angles, so *zs*, which is at right angles with *st*, that point *z* is the first height and the second plane. Since in the solid *xr* is the second height but not at right angles, so &*s*, which is at right angles with *su*, that point & is the second height and the third plane. Therefore, if in plane *s* is located the surface of 6 sides *bcdefg*, in plane *z* the points *h*, *i*, and *k*, and in plane & the triangle *mnq*, and the lines lines *hb*, *hm*, *hd*, *ic*, *in*, *ie*, *kf*, *kq*, and *kg* are drawn, these will form the irregular body cut from the pyramid, as seen in figure 5 (Fig. 3.26).

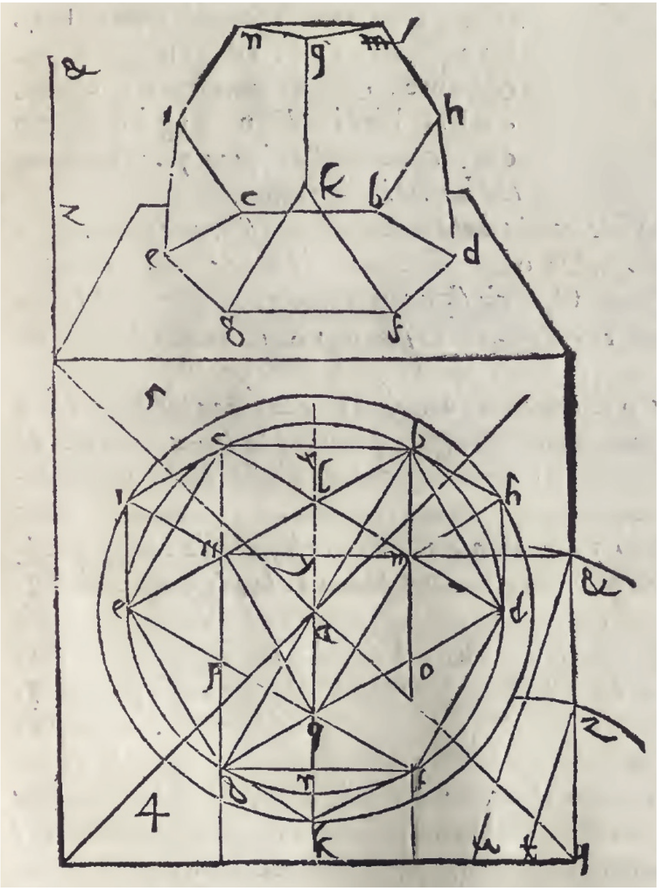

Fig. 3.26 [p. 57]

Since it is formed of surfaces of 6 and 3 sides, it can be rested on both the triangular [face] and the hexagon. So, if in plane *s* is described the triangle *mnq*, and in plane *z* the points *h*, *i*, and *k*, and in plane & the surface *bcdefg*, and the lines drawn (as said) in the perfect, it will rest on the triangular base. The shadowing is understood clearly from figure 5[40] (Fig. 3.26), which shows the perfect in plan, and the raised figure degraded.

[p. 58]

Chap. VIII. Description of a body that is born from the cube and from the octahedron, and its unfolding [cuboctahedron]

There is a body born from the cube and from the octahedron which contains within itself the surfaces of those bodies, that is, 8 triangles and 6 squares [cuboctahedron]. It is made by cutting the edges of both into 2 equal parts and then drawing lines from the midpoint of those parts to another, taking away the solid angles of said bodies to the endpoints of the lines, such that cutting the solid angles of the octahedron, of which there are 6, forms 6 square surfaces and leaves 8 triangles of equal sides, and the [solid] angles of the cube, of which there are 8, forms 8 triangles and leaves 6 squares. This body (as said) has 6 squares and 8 triangles of equal sides, 24 acute [plane] angles and as many again right plane angles, and 12 solid angles formed of right and obtuse angles, and 24 edges.
The unfolding of this body is shown in figure 6 (Fig. 3.27).

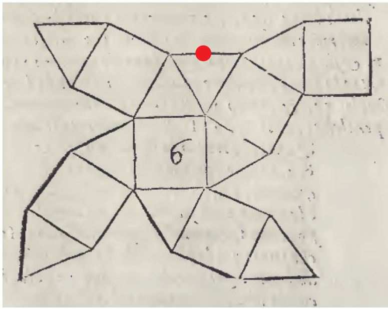

Fig. 3.27[41] **[p. 58]**

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[40] The shadowing has been left out of the figure but the Latin manuscript Lat. VIII, 41=3069, fol. 32v, shows the two degraded forms and shadowings of the truncated tetrahedron in both positions, i.e., resting on one of its hexagonal faces or resting on a triangle.

[41] This unfolding of the cuboctahedron has an extra line that we have indicated with a red point. The correct unfolding is shown in the Latin manuscript Lat. VIII, 41=3069, fol. 33r, and in the extended Italian manuscript It. IV, 39=5446, fol. 45v; see Monteleone (2020, p. 228).

In practice the description of the perfect is this. Over centre *a* let there be made a circle in which is described a surface of 6 equal sides *bcdefg*. Draw the lines *be*, *bf*, *ef*, *cd*, *eg*, and *dg*, which will form another surface of 6 equal sides *hlmikn*, by whose points *h*, *i*, *k* and *l*, *m* and *n* are formed 2 triangles of equal sides, *hilk* [*recte hik*] and *lmn*. Thus you will have the perfect, whose 8 triangles are *kik* [*recte hik*[42]], *lmn*, *hcb*, *lbd*, *idf*, *nfg*, *kge* and *cme*, and whose 6 squares are *chek*, *cmbl*, *bhdi*, *dlfn*, *figk*, and *gnem*, as in figure 7 (Fig. 3.28), which also shows the degraded (Fig. 3.29).

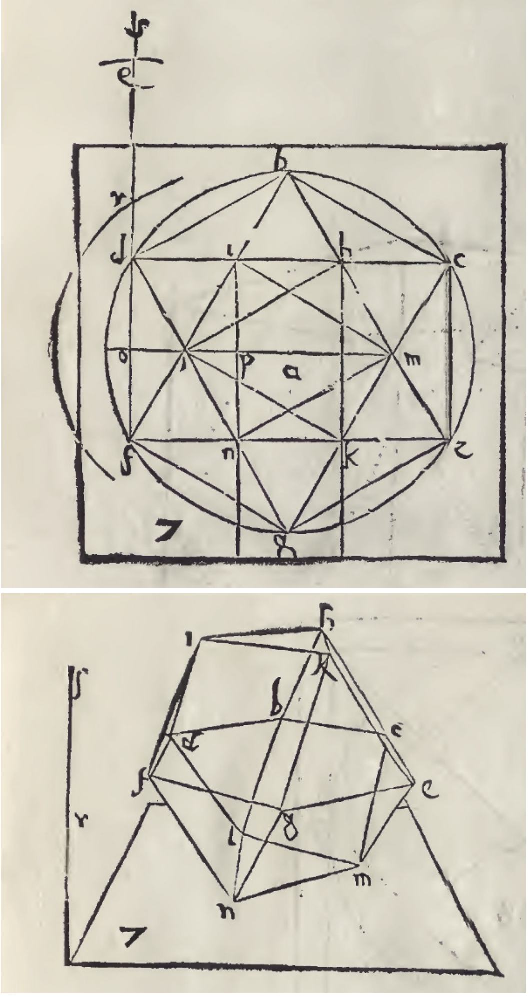

Fig. 3.28-3.29 **[p. 59]**

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[42] This correction is noted in the Errata on an unnumbered page at the end of the book.

The height of this body when it is set on a triangular base is found in this manner. Let line *df* be divided equally [into two] at point *o*; draw line *mo*, which will cut *ln* in *p*. Then through *o* draw line *sdo*, and with *p* as the centre and space *df*, draw the circle which will cut *sd* in point *r*. With centre *r* and space *ro*, draw the arc which will cut *sr* in *q*. These points *o*, *r* and *q* are the planes and the heights of the proposed body. So, drawing the line *pr*, the distance between *p* and *r* will be equal to the distance between *d* and *f*; *df* is the lateral line [side] of a square, and the first height of said body is equal to the lateral line of one of its squares, but not at right angles, so *pr* is not at right angles over *p*. Since in said body the third height [*q*] is as far from the second height [*r*] as the second height is from plane *o*, the point *q*, which is as far from point *r* as *r* is from *o*, will be the third height. Therefore, if in plane *o* is located the triangle *lmn*, in plane *r* the hexagon *bcdefg*, and in plane *q* the triangle *hik*, and then lines are drawn from the vertices of said triangles to the vertices of the hexagon, that is, *ld*, *lb*, *nf*, *ng*, *mc*, *me*, *hb*, *hc*, *id*, *if*, *kg*, and *ke*, this will form the said body degraded resting on the triangular base, as seen in figure 7 (Fig. 3.29).

When it rests on a square base, it can be degraded in two manners. The first is from the body of the cube demonstrated above, from which it is born by dividing each of its edges equally [into two] and drawing lines to the ends of those parts – that is, from one division to the other – which will make the description of that body in the cube. The second is actually made with the perfect drawn another way, which is very easy in both consideration and practice. So, let there be given a circle on centre *a* in which are described the squares *bcde* and *fghi*, one set diagonally within the other. This is the perfect plan of this body (Fig. 3.30).

[p. 59] The first plane is the point *i*; the second *d*; the third *g*. So if the square *bcde* is formed in the planes *i* and *g*, and the point of the square *fghi* are found in the plane *d*, and then lines *hi*, *de*, *ce*, *id*, *ch*, eh, he, hc, *bf*, *cf*, *fb*, *fe*, *bg dg*, *gb* and *gd* [*recte di*, *ei*, *id*, *ie*, *eh*, *ch*, *he*, *hc*, *bf*, *cf*, *fb*, *fc*, *bg*, *dg*, *gb*, *gd*] are drawn, said body will have been formed. This is seen in figure 8 degraded (Fig. 3.31), where the planes *i*, *d* and *g* are the same as the line *idg* in the perfect.

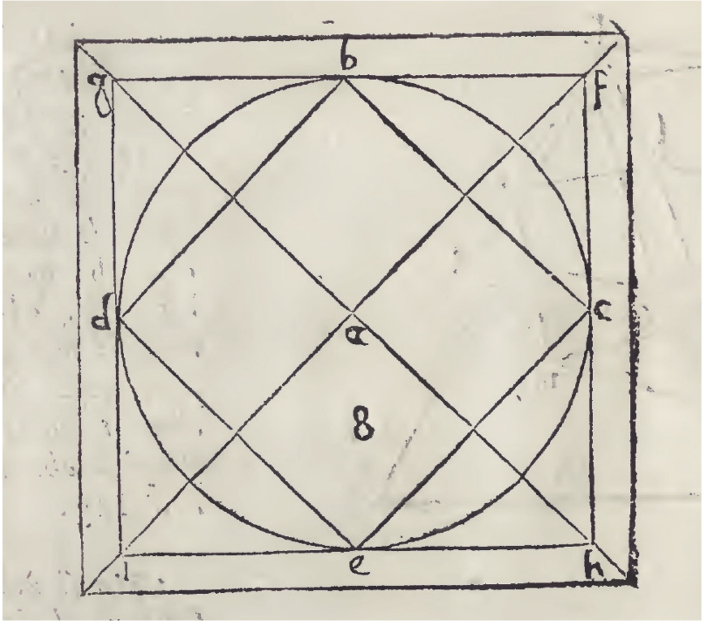

Fig. 3.30 **[p. 60]**

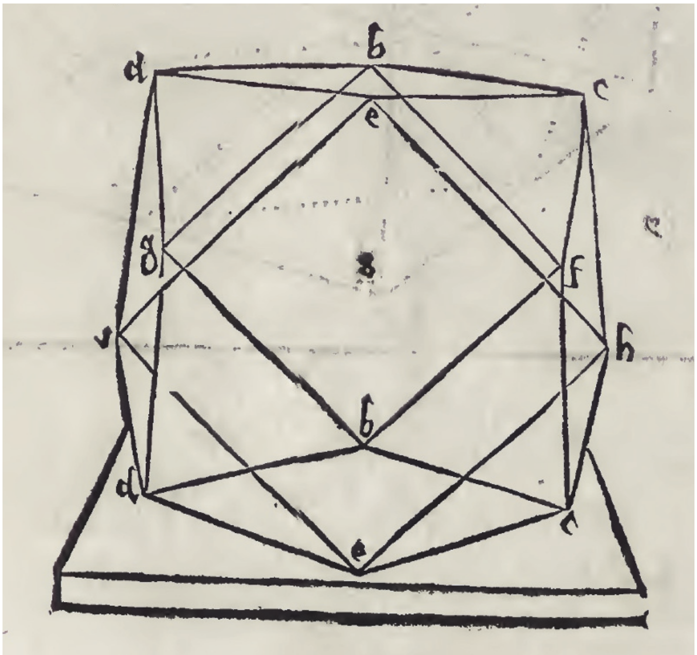
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Fig. 3.31 **[p. 59]**

If that body is to be described in the cube, this is quite easily done by working according to the rules given above. So that this body is well distinguished from the cube, it is a good idea to make the cube a different colour than the body, or darker or lighter, according to which turns out better, as seen in figure 9 (Fig. 3.32).[43] The shadowing of this body can be deduced from its degraded by concealing those parts that must be concealed and allowing those that are in front of the eyes to be shown, and shadowing where [the body is] seen.[44]

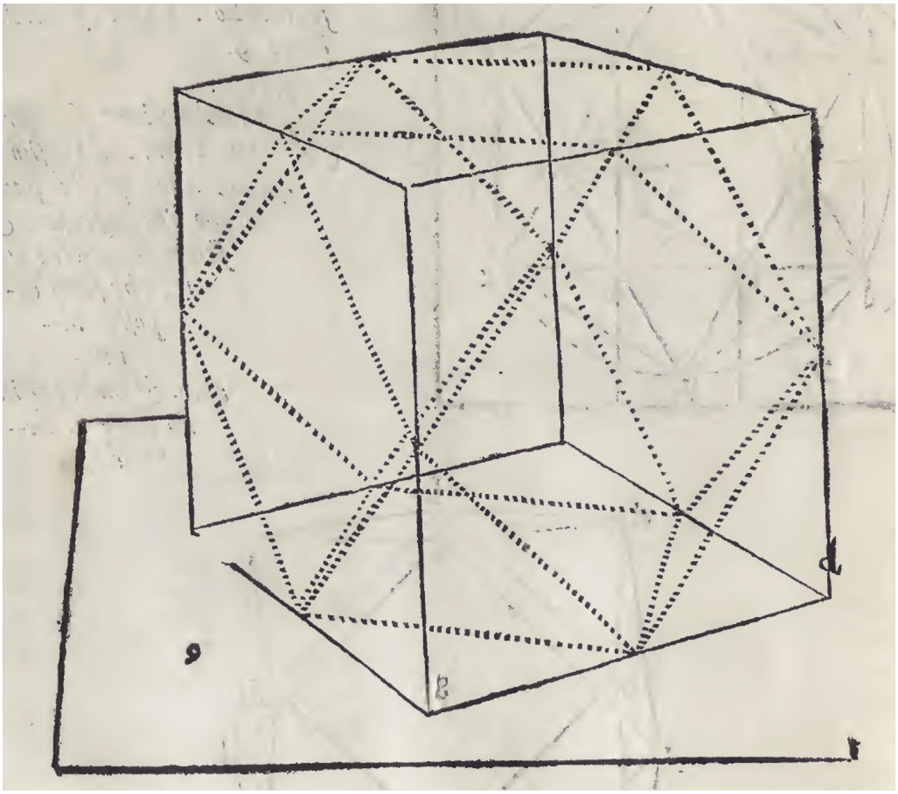

Fig. 3.32 **[p. 60]**

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[43] In Barbaro's figure 9 (Fig. 3.32) to make a clearer distinction between the two solids the cuboctahedron is drawn with dotted black lines while the cube is drawn with continuous black lines. In the extended Italian manuscript (It. IV, 39=5446, fol. 47r) the distinction is made between blue (cuboctahedron) and sepia (cube), while in the Latin manuscript (Lat. VIII, 41=3069, fol. 34r) the distinction is made between sepia (cuboctahedron) and red (cube).

[44] The shadowings of the cubotahedron resting both on a triangle and on a square are in the Latin manuscript (Lat. VIII, 41=3069, 33v).

[p. 61]

Chap. IX. Description of another irregular body that is born from the cube [truncated cube]

There is another body born from the cube, which has 8 triangular and 6 octagonal surfaces [truncated cube]. It is formed in this way. Find the centre of a surface of the cube, and from that centre draw a line to one of the vertices of that surface. You will carry this line from each of its vertices onto each of the lines of its edges, and where those terminate, take away each solid angle of the cube. So the solid angles taken away will form 8 triangles and 6 surfaces of 8 equal angles and sides. It has 96 edges, 24 solid angles, 24 acute plane angles and 48 obtuse plane angles. The unfolding of this body is in figure 10 (Fig. 3.33).

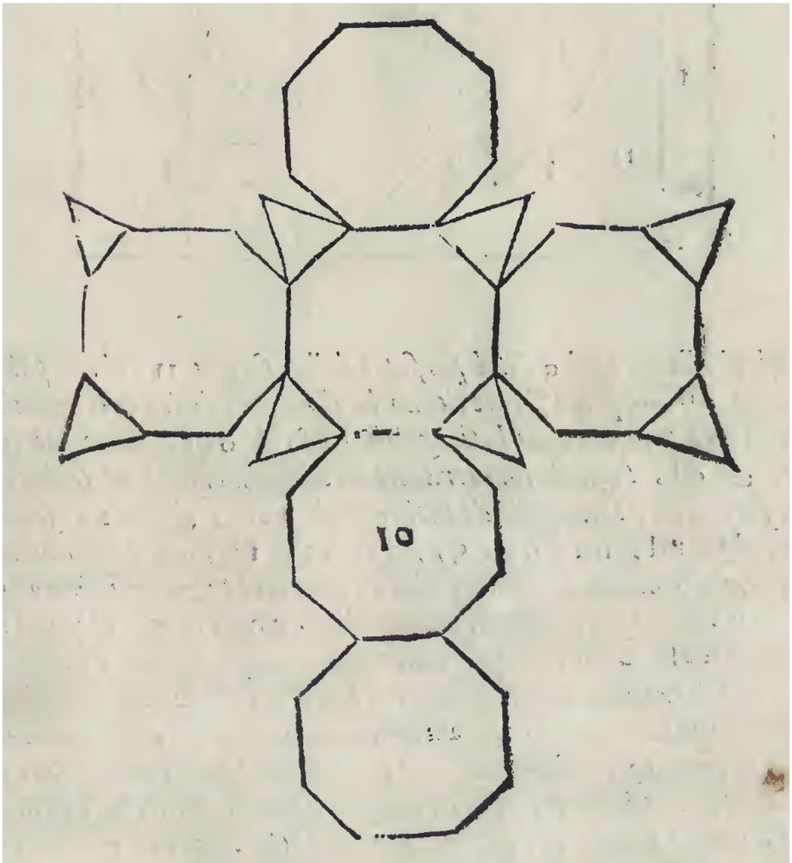

Fig. 3.33 [p. 61]

The perfect is formed in two manners, since it can rest on both the surface of 3 vertices and that of 8. If it rests on the surface of 8, then on centre *a* make a circle in which are inscribed 2 squares, *bcde* and *fghi*, with their vertices at equal distances from each other. These will form the surface of 8 edges *klmnopqr*, which with the points *f*, *g*, *h*, and *i* will be the perfect of that body. The 8 triangles are *fkm*, *lgn*, *ohq*, *pri* and 4 others that are opposite these, since when it is projected onto a plane each triangle is on top of another. The lines *fg*, *fh*, *cg* [*recte ig*] and *ch* [*recte ih*] each of which is equal to *kq*, are for 4 surfaces of 8 sides, and the points *k*, *l*, *m*, *n*, *o*, *p*, *q* and *r* are for the other two, because each is opposite another.

The heights and the planes are the points *h*, *o*, *m*, and *f* such that if said surface of 8 sides is described on the points [*recte* planes] *h* and *f*, and the points *f*, *g*, *h*, and *i* are located on the planes *o* and *m*, and the lines *kf*, *mf*, *lg*, *ng*, *pi*, *ri*, *qh*, *oh*, *fk*, *fm*, *gi* [*recte gl*], *gn*, *ip*, *ir*, *hq*, and *ho* are drawn, said body will be formed, as seen in figure 11 (Fig. 3.34).

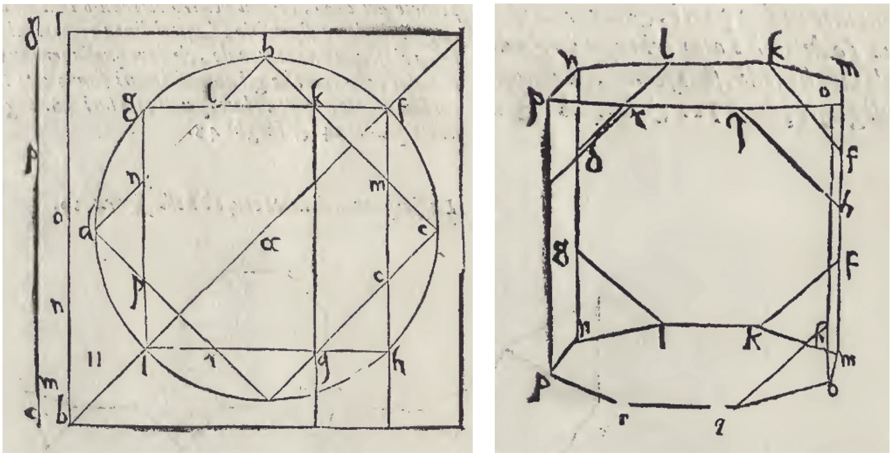

Fig. 3.34. left, perfect **[p. 62]**; right, degraded **[p. 63]**

[p. 62] If this body is resting on the triangular figure, then let there be made on centre *a* a circle whose circumference is large enough so that within it can be described 2 surfaces of 6 unequal sides *bcdlkp* and *ghimef*, equidistant from each other.[45] Their smaller sides are equal to the line *kl*, and the larger ones equal to *kq* of the perfect in figure 11 (Fig. 3.35). Then draw the lines *ki*, *eb*, *fl*, *ph*, *dg* and *no* [*recte cm*], which will form the surface of 6 equal sides *noqrst*, in which are inscribed 2 triangles of equal sides, *nrs* and *oqt*. Then draw the lines *pm*, *bi*, *gk*, *hl*, *fc* and *cd* [*recte ed*], which will form another surface of 6 equal sides *uxyz*&*2*, in which are inscribed 2 triangles of equal sides *uz*& and *yx*2. Then draw the lines *ke*, *fp*, *hc*, *di*, *ml*, *nu*, *ox*, *qy*, *rz*, *t*2, and *s*&. In this manner is described the perfect of said body with the triangular base, of which the 8 triangles are *kln*, *miq*, *dcs*, *ght*, *phr* [*recte pbr*] *eof*, and *u*2& [*recte uz*&, *xy2*], and the 6 octagons are *zunkefp*ꝝ [*recte zunkefpr*], *xyqmlkeo*, *n*&*s-dimln* [*recte u*&*sdimln*], *y*ꝝ*thcdiq* [*recte y*2*thcdiq*], &*zrbghcs*, and 2*xofpbqt* [*recte* 2*xofpbgt*], as seen in figure 12 (Fig. 3.35).

---

[45] The 6 surfaces of unequal sides are extractions from a dodecagon inscribed in the circle (or 3 squares *lfgd*, *mebc*, and *ikpc*). Barbaro certainly knew this construction from the design of the Roman theatre based on 4 squares in the circle. See Williams (2019, p. 422).

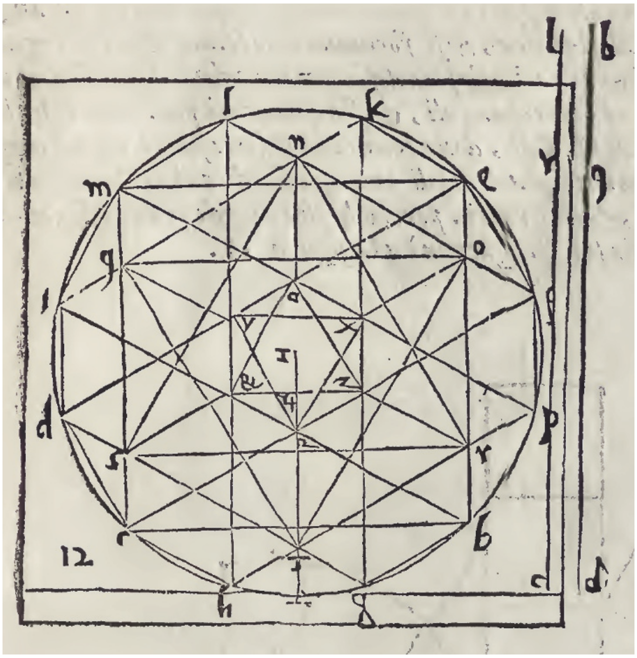

Fig. 3.35 **[p. 63]**

The heights and planes are found in this manner. Draw the line *a*2, which will cut *z*& in point ꝝ. Divide the line *gh* in *a*, and draw the line *ta*. Carry the line ꝝ2*ta* down into figure 13 (Fig. 3.36b)[46] onto the line *BCDE*. On these points draw vertically the lines *FB*, *GC*, *ND* [*recte HD*] and *LE*. At right angles and on the point *D* of figure 13 with space [radius] *yx* of the figure 12 draw an arc *M* which will cut *GC* in *M*. With centre *M* and space *u*𝔷 draw arc *N*, which will cut *FB* in *N*. On the point *E* in figure 13 with space *ny* draw the arc *O*, which will cut *FB* in *O*. On *O* with space *u*𝔷 draw the arc *P*, which will cut *GC* in *P*. With this [*P*] as the centre and space *xy*, draw the arc *Q*, which will cut the line *ND* [*recte HD*] in *Q*. With the point *N* as centre and space *nr* make the arc *R*, which will cut *LE* in *R*. These points *D*, *M*, *N*, *O*, *P* and *Q* are the planes of this body. So, if in plane *D* is described the triangle *xy*9 [*recte xy*2] in plane M the points of the triangle *oqt*, in plane N the points of the hexagon *ghimef*, in plane O the points of the hexagon *pbcdlk*, in plane P the points of the triangles *n*[, *s*,] and *r*, and in plane Q the triangle *nz*& [*recte* uz&] and the lines *un*, *zr*, &s, *rp*, *rb*, *pb*, *sc*, *sd*, *dc*[, *di*], *qm*, *qi*, *mi*, *tg*, *th*, *gh*, *ch*, *ox*[, *t*2, *of*, *oe*, *ef*, *pf*, *nl*, *kn*, *kl*, *ek*, *bg*, *lm*] and *qy* are drawn, the body will be degraded, as seen in figure 12 (Fig. 3.36a). It is degraded since when the lines *DM*, *MN*, *EO*, *NR*, *OP* and *PQ* are drawn, the lines *DM* and *PQ* are equal, and are the edges of said body. *D* is the first plane and *Q* the last. *MN* and *OP* are equal, and are the diametral lines [medians] of the triangles of this body. *M* is the second plane, and *P* the next to last. *EO* and *NR* are the diametral lines **[p. 63]** of the octagons, and *N* is the third plane. *NR* being equal to *EO*, the point *O* is the fourth plane, as you will see with delight when you have the solid in front of you. The error of the engraver in figure 12 will be cor*recte*d with the rules that have been given.

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[46] There is no figure 13 corresponding to this description in the text but it is found in the extended Italian manuscript (It. IV, 39=5446, fol. 50r; see Monteleone 2020, p. 231) and the Latin manuscript Lat VIII, 41=3069, fol. 35r.

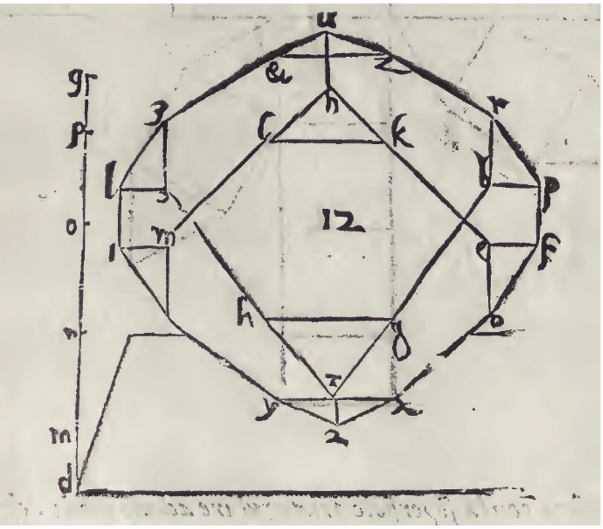

Fig. 3.36a **[p. 63]**

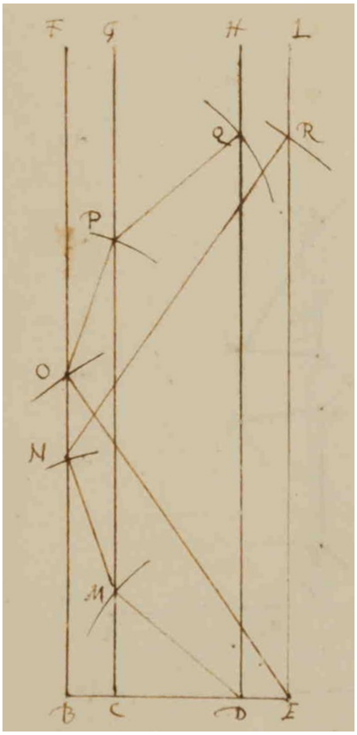

Fig. 3.36b. From Barbaro's extended Italian manuscript It. IV, 39=5446, fol. 50r

[p. 64]

Chap. X. Description and unfolding of another body that is born from the cube [small rhombicuboctahedron]

Another body [small rhombicuboctahedron] is born from the cube when its edges are divided into 3 [unequal] parts, on each of its surfaces at the ends of said parts are formed 5 squares and 4 quadrangles. Of the 5 squares, the one in the middle is the largest, as the example shows (Fig. 3.38).[47] From its vertices of that larger square to the vertices of the other squares are drawn lines, and thus are formed 8 triangles of equal sides and 12 other similar squares. This body will be inscribed in the cube, and is also born from the body formed of 8 triangles and 6 squares [cuboctahedron, Chap. VIII] when each of its side is divided into 2 parts and the solid vertices where those parts terminate are taken away. This body (as said) has 8 triangles and 18 squares of equal sides, of which the 24 obtuse solid angles are formed of right angles and acute angles, so that it has 24 acute plane angles, 72 right plane angles, and 48 edges. Its unfolding is in figure 13 (Fig. 3.37).

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[47] The square in the middle is the largest because the construction is a perfect octagon; therefore the edges are not divided into three equal parts. Barbaro is referring to the subdivision of the side of a square he used for the previous description of the truncated cube.

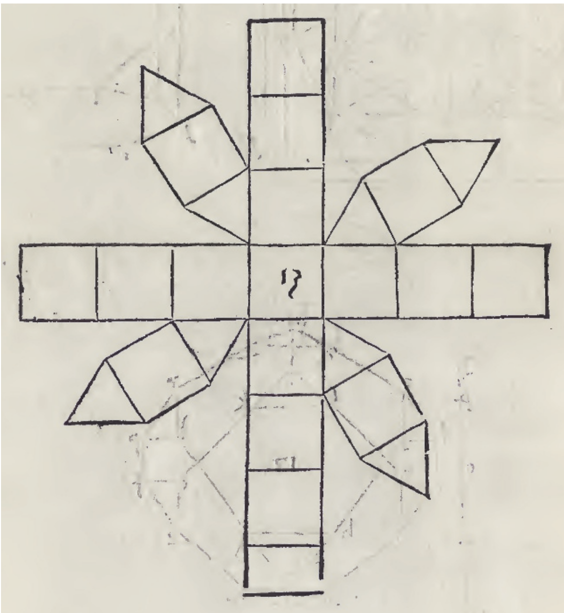

Fig. 3.37 **[p. 64]**

This body can rest in one manner on a triangular surface, and in two manners on a square surface. If it is in the first manner on a square, make in a square *abcd* a surface with 8 equal sides *efghiklm*, as seen in figure 14 (Fig. 3.38, left).

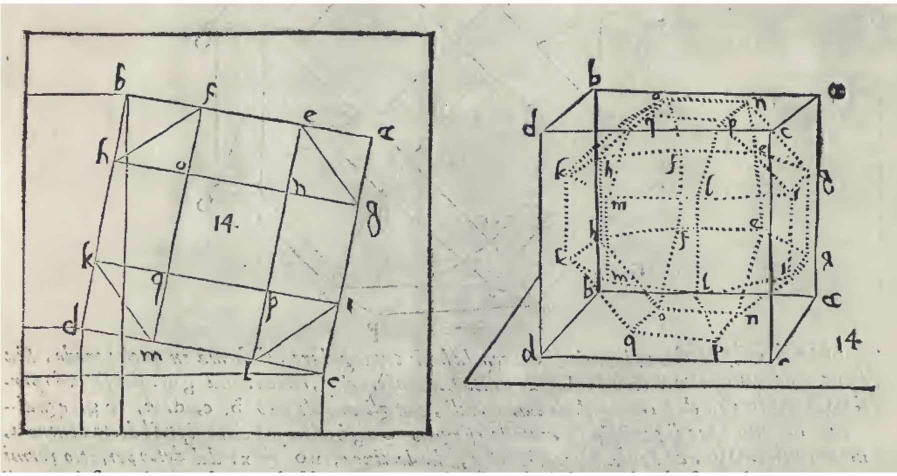

Fig. 3.38 **[p. 65]**

Then draw the lines *el*, *fm*, *gh* and *ih* [*recte ik*[48]] which will form 5 squares and 4 quadrangles. The squares are *aegn*, *fboh*, *nopq* [centre, the largest], *ipcl*, and *qkmd*; the quadrangles are *efno*, *gnip*, *ohqk* and *pqlm*. This octagon divided in such a way is one of the perfects, which has a square base whose 8 triangles are *egn*, *foh*, *ipl*, *kqm* doubled, because they fall on top of each other. The squares are on the 4 **[p. 65]** said quadrangles, and the square *nopq* doubled so that there are 10, because they fall on top of each other, and the other 8 are the sides of that octagon, so there are 18 in all.

The planes are the points *c*, *i*, *g* and *a*, such that if on the planes *c* and *a* is degraded the square *nopq*, and on the plane *i* and *g* the octagon *efghiklm*, and then lines *ne*, *of*, *oh*, *qk*, *qm*, *pl*, *pi*, *ng*, *ee*, *ff*, *hh*, *kk*, *mm*, *ll*, *ii*, *gg*, *en*, *gn*, *fo*, *ho*, *ip*, *lp*, *mq*, and *kq* are drawn, said body will have been raised and degraded, as seen in the degraded in figure 14 (Fig. 3.38, right).

If about square *abcd* (as said) is described the cube on plane *C*, there is no doubt that this cube will circumscribe said body. Therefore it is clearly seen that this body depends on the cube. This is proved because if the diagonals are drawn through every surface of the cube, those same diagonals will be the diagonals of 6 square surfaces of that body circumscribed by the cube.

The second perfect with a square base is made in the following manner, although it can be done differently. In the given square *abcd* let there be inscribed an octagon *efghiklm*. Then draw the lines *el*, *fm*, *gh* and *ik*, forming the square *nopq* [*p* not shown in Fig. 3.41, left]. Then the lines *ek*, *gm*, *fi*, and *hl*, forming the square rstu [*u* is not shown in Fig. 3.41, left]. Then through the points *s* and *t* draw the lines *xy* [*recte xz*] and *y*& parallel to the lines *el* and *fm*, forming quadrangle *xyz*&. Divide equally [into two] the lines *ef* in point 2 and *lm* in point ꝶ. Then draw lines 2*n*, 2*o*, ꝶ*p*, ꝶ*q*, *gx*, *yh*, *iz* and *k*&. Thus will be described the perfect of said body, whose points ꝶ, *u*, *p*, *s*, *n*, *r* and 2 are its planes. The 8 triangles are 2*no* and *pq*ꝶ doubled, which are 4 because they are opposite each other; the other 4 are *gx*, *yh*, *iz* and *k*&, that is, the diametral lines [medians] of said triangles.

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[48] This correction is noted in the Errata on an unnumbered page at the end of the book.

This is proved, since having formed the triangle of equal sides on the lateral line of the square *nopq*, its diametral line [median] will be equal to *gx*. The 18 squares are thus *x*2*gn*, 2*yoh*, *gnip*, *nopq*, *ohqk*, *ipz*ꝶ, *qk*& [*recte qk*ꝶ&], which are 7, and the other 7 are opposite those; the other 4 are the lines of said squares *gi*, *hk*, *xy* and *z*&, that is, *gi* and *hk* are the laterals [sides] and *xy* and *z*& are the diagonals of those squares. This is proved because, the square *nopq* being equal to the square *rstu*, the diagonal of one is equal to that of the other. Line *xy* is the diagonal line of the square *rstu*, as can be clearly seen. Therefore *xy* is the diagonal line of the square *noqp*. It is also clear that *gi* is the lateral line [side]. Therefore, if in planes ℞ [*recte* ꝶ] and 2 is located the square *nopq*, in the planes *u* and *r* the points 2 and ℞ [*recte* ꝶ], in the planes *n* and *p* the points *g*, *h*, *i* and *k*, and in the plane *s* the points *x*, *y*, *z* and &, and the lines *oq* [*recte o*2], *nq* [*recte n*2], *p*℞ [*recte p*ꝶ], *q*℞ [*recte* qꝶ], *ng*, *pi*, *qn* [*recte qk*], *oh*, *gg*, *ii*, *kk*, *hh*, *gi*, *hk*, *gi*, *hk*, *ip*, *gn*, *kq*, *ho*, *o*2, *n*2, *q*℞ [*recte q*ꝶ], *p*℞ [*recte p*ꝶ], *gx*, *xg*, *iz*, *zi*, *x*2 **[p. 66]** 2*x*, ℞*z* [*recte* ꝶ*z*], *z*℞ [*recte z*ꝶ], 2*y*, *y*2, *hy*, *yh*, ℞& [*recte* ꝶ&], &℞ [*recte* &ꝶ], *k*& and &*k* are drawn, you will have the degraded body as seen in the degraded figure 15 (Fig. 3.39), which has been raised from a plan that is larger than its perfect.[49]

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[49] Figure 15 (Fig. 3.39) contains several errors: point *q* should be 2; line *iz* is not drawn and lines *hy* and *h*& are interrupted on line ꝶ&. The text related to this figure has problems too: very often point ꝶ is substituted by ℞. Since this figure and the related text are correct in the Latin manuscript Lat VIII, 41=3069, fol. 36r and the extended Italian manuscript (It. IV, 39=5446, fol. 53v; Monteleone 2020, p. 233), we must blame the printer for the errors.

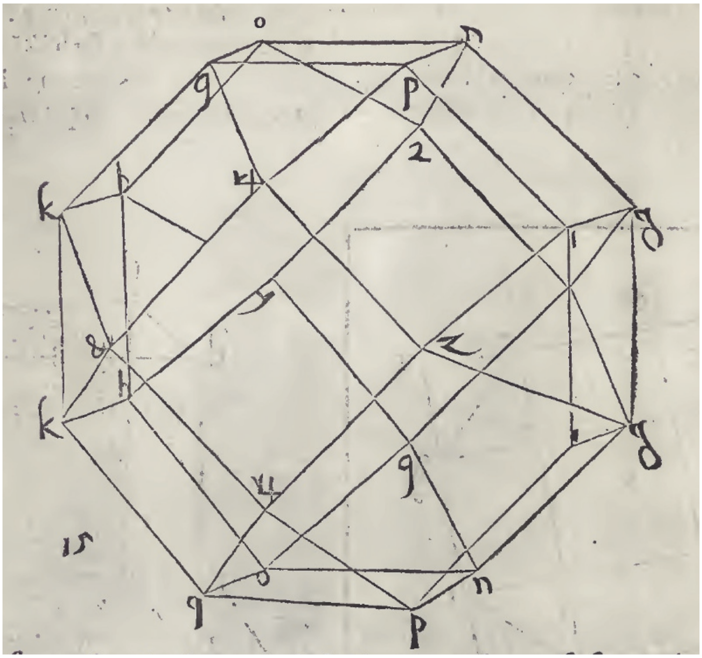

Fig. 3.39 **[p. 66]**

The third perfect, which rests on a triangular base, is made in this manner. On centre *a* let there be made the circle *b* of a circumference such that the line *gm* of the second perfect forms in it 2 triangles of equal sides, with their vertices *b*, *c*, *d*, *e*, *f*, and *g* equally distant, by which is formed the hexagon *ecgdefb* [*recte cgdefb*[50]]. Let there be given another circle *h* on that centre *a* of a circumference such that the lines *no* and *xy* of said perfect form in it 2 hexagons of equal sides *himnlk* and *otsrqp*,[51] whose vertices are equally distant, such that the point *b* [*recte h*] is as far from point *e* as point *o* is from point *b*. Then draw the lines *on*, *pm*, *kt* and *ls*, which will form another hexagon of equal sides *uy*&2*zx*, in which let there be drawn the triangle *uz*& and *xy*2. Thus will be described the third perfect, whose 8 triangles are *uz*&, *xy*2, *hei*, *tcs*, *ngm*, *rdq*, *lfk*, and *pbo*. The 18 squares are *uts*&, *ynm*2, &*rqz*, 2*lkx*, *zpou*, *xhiy*, *etuo*, *cnyi*, *gr*&*s*, *dl*2*m*, *fpqz*, and *bhxk*, which are 12; the others are actually the lateral lines of the hexagon *ecgdfb*, and are thus *bkfp*, *fqdl*, *dmgr*, *gsnc*, *etic*, and *eobh*, as seen in the perfect of figure 16 (Fig. 3.41, right).

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[50] This correction is noted in the Errata on an unnumbered page at the end of the book.

[51] In the Errata on an unnumbered page at the end of the book, it is noted that in figure 16 (Fig. 3.41, right) points *q* and 2 are exchanged, and a line must be drawn from 2 to *e*.

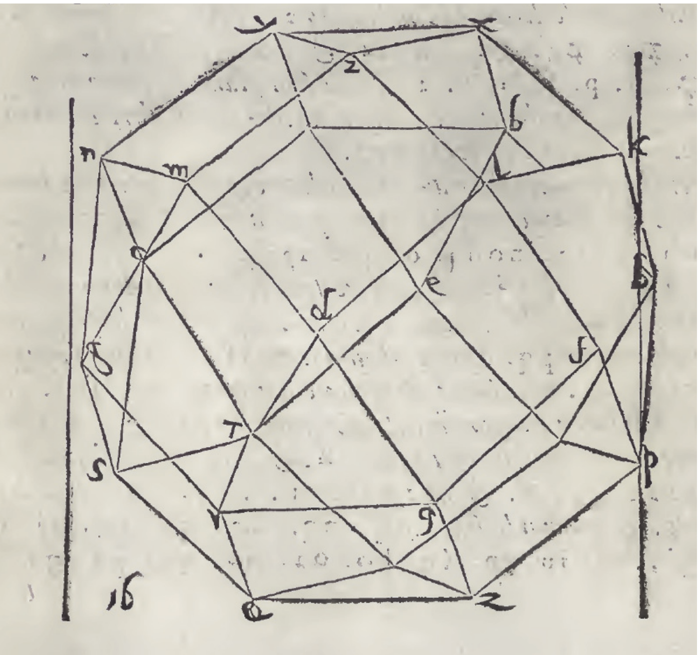

Fig. 3.40 **[p. 66]**

[p. 67] To find the planes draw the line *e*2, which will cut *hi* in *a* and *xy* in ƶ. Let there be given, in a separate drawing, the line *CD*, onto which are carried the points *e*, *a*, *u*, and ƶ. On points *e*, *a*, and *u* draw at right angles the lines *ef*, *ga*, and *mu*. Then with point ƶ as the centre and space [radius] *hi* taken from the plan of 16 (Fig. 3.41, right), draw the arc *n*, which will cut *ga* in *n*. Then with *u* as centre and space *ot*, draw the arc *o*, which will cut *fe* in *o*. With *n* as centre and space 2ƶ, let arc *p* cut *fo* [*recte fe*] in *p*. With *o* as centre and space *q*℞ [*recte* 2ƶ], let arc *q* cut *gn* [*recte ga*] in *q*. With *p* as centre and space *ot*, let arc *r* cut *mu* in *r*. These points *m* [*recte* ƶ], *n*, *o*, *p*, *q* and *r*[52] are the desired planes (Fig. 3.41, centre), since drawing the lines ℞*n* [*recte* ƶ*n*], *no*, *np*, *oq*, and *pr* the line ƶ*n* is equal to *eh*, the side of one square of said body, and *no* is equal to *et*, the diagonal line of that square, and *np* and *oq* are equal to *q*℞ [*recte* 2ƶ], the diametral line [median] of a triangular base of that body. *pr* is equal to *ot*, the afore-mentioned diagonal. All of these lines circumscribe this body, such that if in the plane *a* [or ƶ, which is the same] you locate *u*, & and *z*, in point [*recte* plane] *n* the lines *qr*, *st*, *op*, in plane *o* the points *e*, *f* and *g*, and in plane *p* the points *b*, *c* and *d*, in plane *q* the lines *mn*, *ih*, *kl*, and in plane *r* the triangle *xy*2, and then draw lines *zq*, *zp*, ℞*r* [*recte* &*s*], &*s*, *uo*, *ut*, *dq*, *dr*, *cs*, *ct*, *bp*, *bo*, *fp*, *fq*, *gr*, *gs*, *eo*, *et*, *dl*, *dm*, *ci*, *cn*, *bh*, *kx*, *l*2, *m*2, *ny*, *iy*, *fk*, *fl*, *gm*, *gn*, *eh*, and *ei*, you will form that body resting on the triangular base, as shown in figure 16 (Fig. 3.40), which has been made larger than its perfect plan, and must be cor*recte*d in some places by the diligence of the reader.

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[52] The letters *m*, *n*, *o*, *p*, *q* and *r* are indicated as upper case in the text, but appear as lower case in the figure; here they have been changed to lower case to facilitate comprehension.

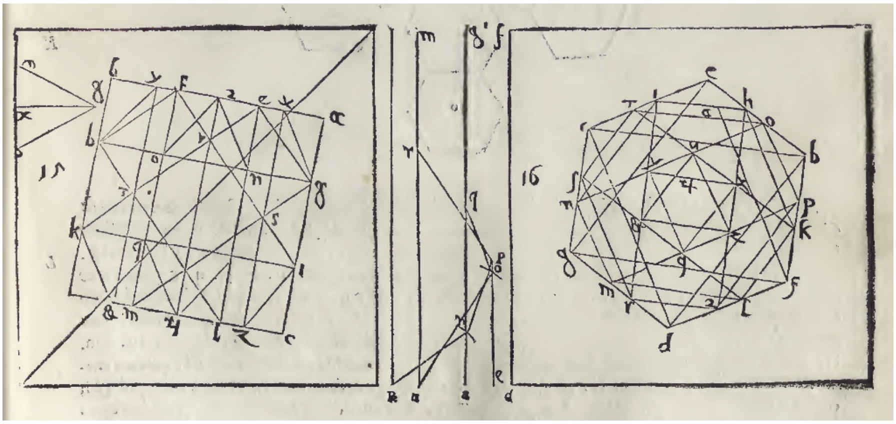

Fig. 3.41 **[p. 67]**

[p. 68]

Chap. XI. Unfolding and description of a body that is born from the octahedron [truncated octahedron]

From the octahedron, dividing its edges into 3 equal parts and taking away the solid angles where those parts terminate is born another body [truncated octahedron] that is very similar to the second one that is born from the cube [small rhombicuboctahedron], because the numbers of the surfaces, edges, solid angles and obtuse plane angles agree with that, but that one has 8 triangular and 6 octangular surfaces, while this one has 6 squares and 8 surfaces of 6 sides. That one has 24 acute plane angles, and this one has the same number of right angles; that one has 28 plane angles, and this one has 48; this one has 36 edges, and 24 edges formed of obtuse and right angles, as will be seen in its surface. Its unfolding is in figure 17 (Fig. 3.42), in which the one marked with the letter o is meant to be a square.

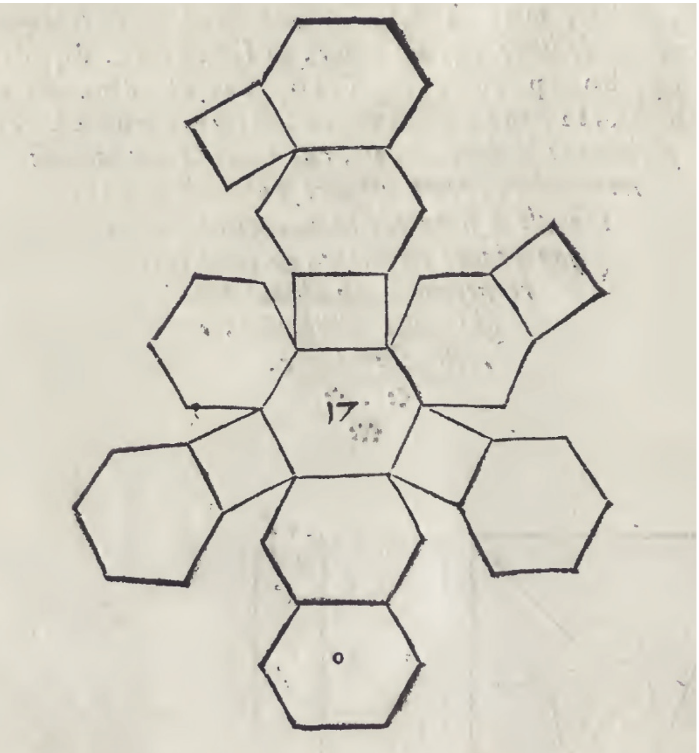

Fig. 3.42 **[p. 68]**

This body can rest on surfaces of both 4 and 6 sides. If it rests on the square, let there be given 5 equal squares *abdc*, *aefb*, *bhkd*, *dmlc*, and *ciga*, and draw the lines *fh*, *km*, *li* and *ge*. This will form an octagon of unequal sides *efhkmlig*. Then divide the lines *fh*, *km*, *li* and *ge* equally at points *n*, *o*, *p*, and *q* and draw the lines *no*, *oq*, *qp*, *pn*, *nq* and *op*. Of these *nq* and *op* will cut each other at point *t*, and in this manner is formed the perfect of this body. Of the 6 squares, 2 are the square *abdc* since one of these squares falls on top of another. The other 4 are the lines *fh*, *km*, *li* and *ge*, which are the diagonals of said squares, by which the diagonal *ad* is equal to each of said lines *fh*, *km*, *li* and *ge*. The 8 hexagons are *acpign*, *cd2mlp* [*recte cdqmlp*], *dqkhob*, and *bofena* doubled, because they fall on top of each other. That the said hexagons would be of equal sides if they were perfectly described is proved with the above-mentioned squares, by the line *ab*, which is doubled with *ac*. Being the double, there is no doubt that said hexagon, being drawn of equal sides, all the squares would also be of equal sides. This is also proved because by making circle *u* with centre *t* and space *ab*, and in that describing the hexagon of equal sides *nxyz*&2 [*recte uxyz*2&[53]]: drawing the line *n*2 [*recte u*2[54]], it will be equal to *np*,[55] as seen in figure 18 (Fig. 3.43).

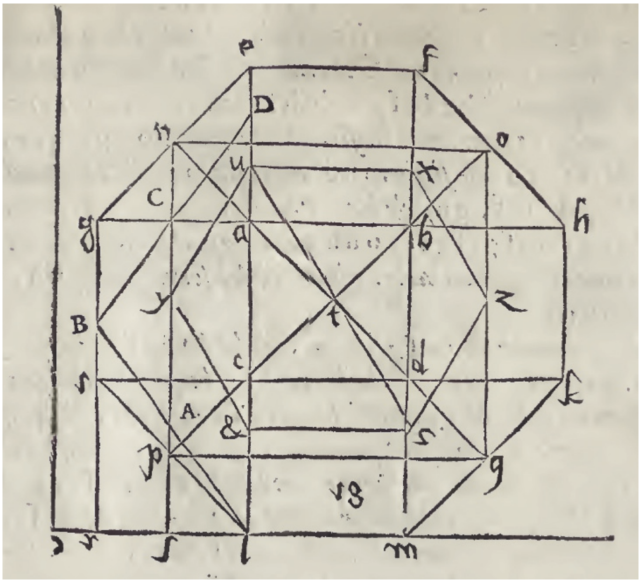

Fig. 3.43 **[p. 69]**

[p. 69] The planes and the heights of this body are found in this manner. Draw the lines *ir* and *lr*, such that *r* is a right angle. Also draw the line *ps* at a right angle to *lr*. With centre *l* and space *u*&, draw the arc *B*, which will cut *gi* [*recte gr*[56]] in *B*. Draw the line *Bl*, which will cut *py* in point *A*. Then with centre *B* and space *Bl*, draw the arc *D*, which will cut *el* in *D*, and draw line *BD*, which will cut *ny* in *C*. These points are its planes, that is, *l*, *A*, *B*, *C* and *D*, such that if in the planes *l* and *D* is described the square *abcd*, in planes *A* and *C* the points *n*, *o*, *p* and *q*, in the plane *B* the lines *gi*, *lm*, *kh* and *fe*, and the lines *an*, *bo*, *cp*, *dq*, *ng*, *ne*, *of*, *oh*, *pi*, *pl*, *qm*, *qk*, *lp*, *mq*, *kq*, *ho*, *fo*, *en*, *gn*, *ip*, *na*, *ob*, *pc*, and *qd* are drawn, the result will be the degraded body taken from a larger plan, as seen in figure 18 degraded (Fig. 3.44).

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[53] This correction is noted in the Errata on an unnumbered page at the end of the book.

[54] This correction is noted in the Errata on an unnumbered page at the end of the book.

[55] Line *u*2 is the diagonal of hexagon *uxz*2&*y* inscribed circle *u* on centre *t*; *np* is the side of square *noqp* circumscribing that same circle *u*; both are equal to the diameter of the circle, and they are therefore equal to each other.

[56] This correction is noted in the Errata on an unnumbered page at the end of the book.

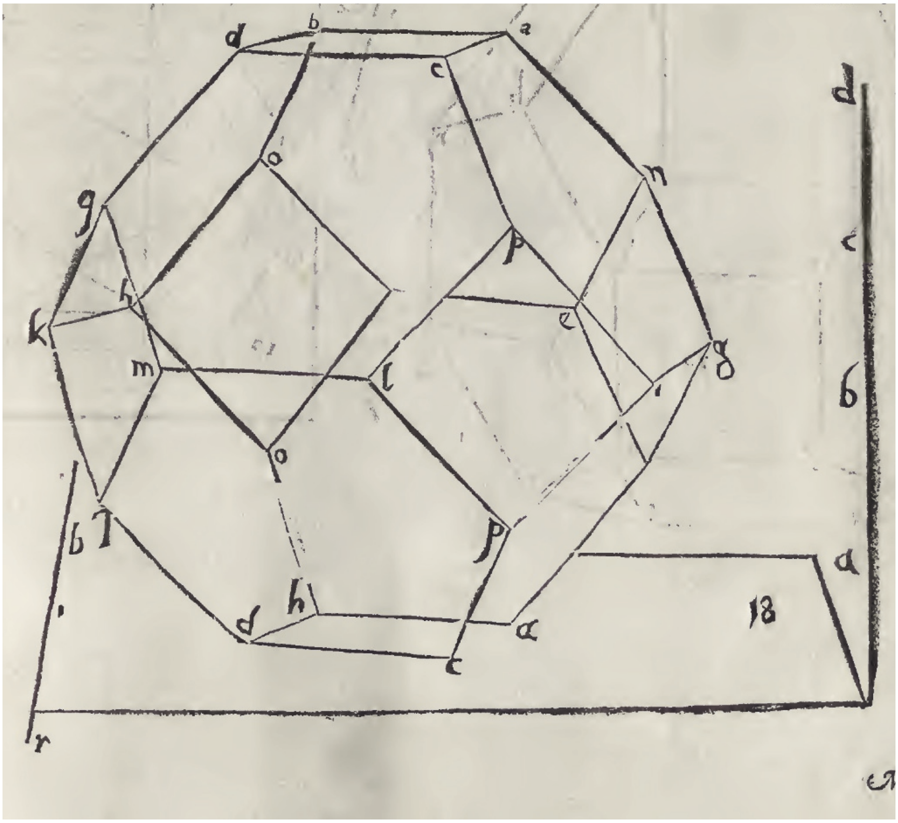

Fig. 3.44 **[p. 69]**

[p. 70] If said body rests on a surface of 6 sides, in the perfect given above let there be drawn the line *u*2. In a separate drawing make a circle with centre *a* of a circumference such that the lines *nx* [*recte ux*] and *u*2 of the perfect form in that circle 2 hexagons with sides that are unequal sides but equally distant from each other, that is, *bcdefg* and *hiklmn*. Then draw lines *bm*, *cl*, *ig*, *kf*, *dh* and *en*, which will cut each other in the points *o*, *p*, *q*, *r*, *s*, and *t*. Then draw lines *op*, *pq*, *qr*, *rs*, *st*, and *to*, which will form a hexagon of equal sides *opqrst*. Then draw the lines *hb*, *ci*, *kd*, *el*, *mf*, and *gn*, and you will have the perfect of that body. Its 6 squares are *bcop*, *ikqp*, *derq*, *lmsr*, *fgts*, and *onht*. Of the 8 hexagons 2 are *opqrst*, since one falls on top of each other in this resting position. The other 6 are *bcipoh*, *ikdqpc*, *delrqk*, *lmfsre*, *fgntsm*, and *gnhbot*.

To know its planes, extend *fg* in *u* [*recte y*] and the line *st* in *x*.[57] With centre *s* and space *so*, cut *gu* [*recte gy*] in *y*. Draw the line *ys*, which will cut *nm* in *z*. With centre *y* and space *st*, cut *tx* in & [shown as 'et' in the figure]. These points *s*, *z*, *y*, and & are the planes of this body, so that drawing the line &*y* makes side of the square and *ys* makes the diameter of the hexagon. This is seen in figure 19 (Fig. 3.45), which due to error is missing line *st* extended to point *x*.

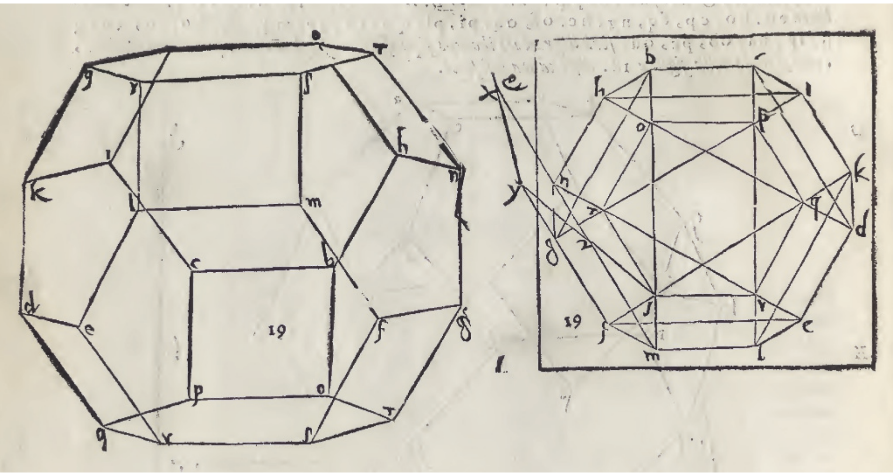

Fig. 3.45 **[p. 70]**

Therefore, if on planes *s* and *x* [*recte* &] is degraded the hexagon *opqrst*, on the plane [*z*[58]] are located the lines *gf*, *ed*, and *cb*, on the plane *y* the lines *ml*, *ki*, and *hn*, and then lines *tn*, *sm*, *rl*, *qk*, *pi*, *oh*, *ng*, *hb*, *mf*, *le*, *kd*, *ic*, *gt*, *fs*, *er*, *dq*, *cp*, and *bo* are drawn, the result is the degradation of this body resting on a hexagonal base on a plane, as figure 19 shows clearly, although it is larger than the plan that we have made.

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[57] The figure erroneously shows *mn* extended instead of *st*; the error is acknowledged at the end of the paragraph. This correction is also noted in the Errata on an unnumbered page at the end of the book, which says, '*fg* in *u*. It is necessary to continue the line *fg* in *u* in the figure, and make *st* in *x*'.

[58] A blank space was left instead of the letter *z*.

[p. 71]

Chap. XII. Unfolding and description of a body that is born from the dodecahedron and from the icosahedron [icosidodecahedron]

From the body of 12 5-sided surfaces [dodecahedron, Chap. V], and from the body of 20 triangular surfaces [icosahedron, Chap. VI] is born another body that contains the surfaces, edges and plane angles of those bodies [icosidodecahedron], since it is formed of 12 pentagons and 30 edges of the dodecahedron, and of 20 triangles and 30 edges of the icosahedron. Thus it has 32 surfaces, 60 edges, 60 obtuse [plane] angles, 60 acute plane angles, and 30 solid angles formed by those plane angles. It is formed by dividing the edges of the two bodies into two equal parts and then taking away the solid angles where those parts terminate, so that the solid angles of the dodecahedron will form 20 triangles and 12 pentagons, and those of the icosahedron will form 12 pentagons and 20 triangles. The unfolding of this body is in figure 20 (Fig. 3.46).

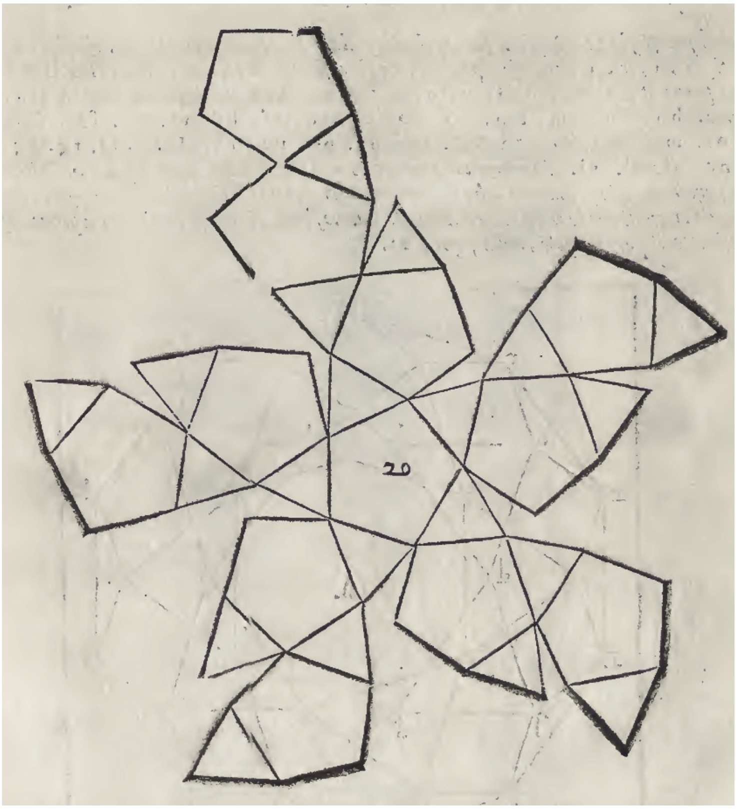

Fig. 3.46 [p. 71]

This body has two resting positions, one on a triangular face and the other on one with 5 sides, so it will be described in both, and first in that of 5 sides, as seen in figure 21 (Fig. 3.47), which shows both the perfect and the degraded, with the description of the planes and heights of this body.

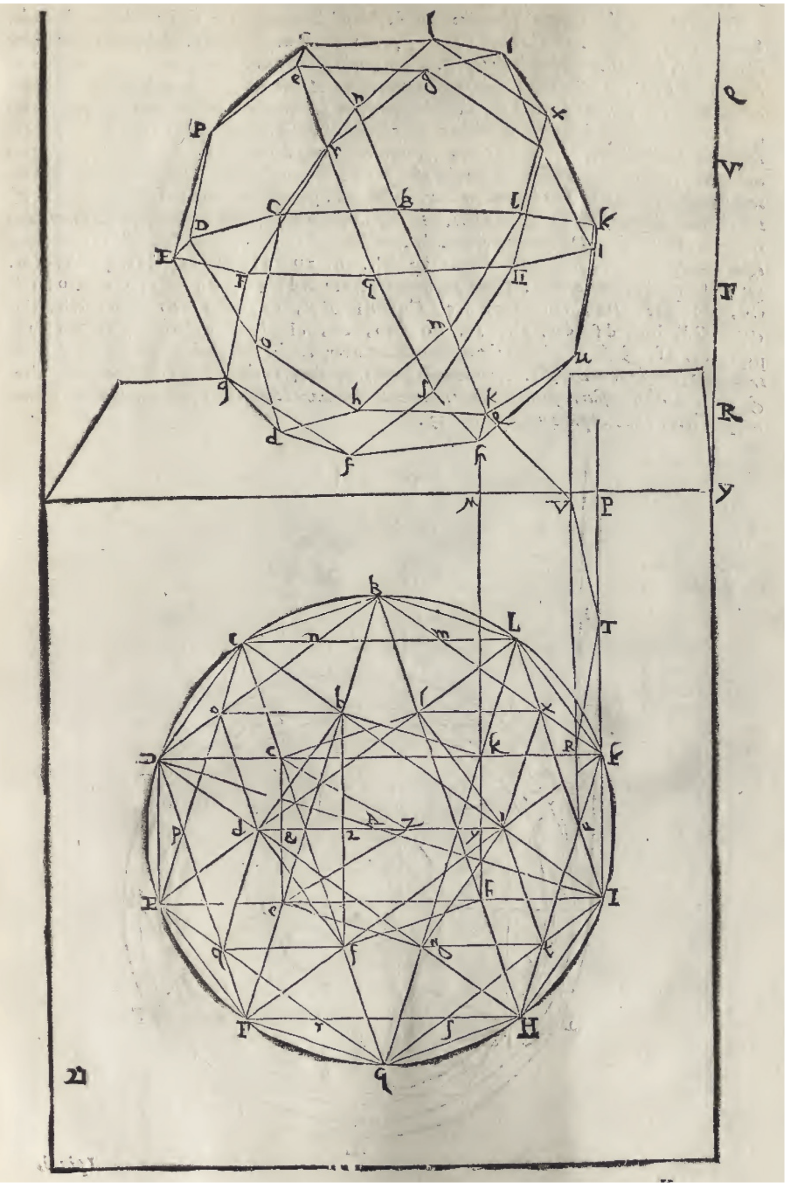

Fig. 3.47 **[p. 73]**

So, let there be made on centre *a* the circle *B* in which is described a surface of 10 equal sides *BCDEFGHIKL*, and draw the lines *BF*, *CG*, *DH*, *EL* [*recte EI*], *EK* [*recte FK*], **[p. 72]** *GL*, *HB*, *IC*, *KD* and *LE*. These lines will cut in the points *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k* and *l*. When connected by lines alternately they will make 2 pentagons of equal sides *lcegi* and *bafhl* [*recte bdfhk*]. Then draw the lines *DB*, *CE*, *DF*, *EG*, *FH*, *GI*, *HK*, *IL*, *KB* and *IC* [*recte LC*], which will cut each other in the points *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, and *x*. Then draw the lines *mB*, *mL*, *mK* [*recte mk*], *mb*, *nB*, *nc* [*recte nC*], *nL* [*recte nl*], [*nc*], *oC*, *oD*, *od*, *ob*, *pq* [*recte pD*], *pe*, *pc*, [*pE*,] *qF*, *qf*, *qe* [*recte qE*], *qd*, *rF*, *rG*, *rg*, *re*, *sG*, *sH*, *sh*, *sF* [*recte sf*], *tH*, *tI*, *ti*, *tg*, *ut* [*recte uI*], *uK*, *uk*, *uh*, *xK*, *xL*, *xl* and *xi*,[59] and you will have described the perfect of this body. Its 12 pentagons and 20 triangles are clearly seen and correctly drawn, such that when the line *xt* is drawn in the pentagon *kxitl* [*recte KxitI*], and the line *bf* is drawn in the pentagon *kbdfh*, line *xt* is equal to *bf*, and *ki* [*recte KI*], side of the triangle *KuI*, is equal to *kh*, side of the triangle *ukl* [*recte ukh*], as seen very clearly in figure 21 (Fig. 3.47).[60]

The planes and heights of this body are found in this way. Draw the line *id*, which will cut *kh* in point *y*. On *k* draw the line *MK* [*recte Mk*] at right angles; on *u* draw the line *uV*; and on *K* the line *PK*. Then on *ce* make a triangle of equal sides, *cez*, which is divided equally by *Zd* [*recte zd*] at point &. With centre *y* and space *z*&, draw arc *R* which will cut *uV* in *R*. With centre *R* with that same space [*z*&] an arc will cut *pk* [*recte PK*] in *T*. With centre *T* with the same space, an arc will cut *uV* in *V*. With centre *V* with the same space, an arc will cut *Mk* in point *Q*. These points *Y* [*recte y*], *R*, *T*, *V* and *Q* are the planes of this body.

Therefore if on *Q* is made and degraded the pentagon *ilceg*; on plane *V* the points *x*, *t*, *r*, *p*, and *u* [*recte n*]; on *T* the surface of 10 sides *BCDEFGHIKL*; on plane *R* the points *u*, *s*, *q*, *o* and *m*; and on plane *y* the pentagon *kbdfh*, and the lines *hu*, *ku*, *km*, *bm*, *bo*, *do*, *dq*, *fq*, *fs*, *hs*, *uI*, *uk* [*recte uK*], *sH*, *sG*, *qf* [*recte qF*], *qe* [*recte qE*], *od* [*recte oD*], *oe* [*recte oC*], *mB*, *Ml* [*recte mL*], *tL* [*recte tI*], *tn* [*recte tH*], *rG*, *rF*, *pe* [*recte pE*], *po* [*recte pD*], *ne* [*recte nC*], *nB*, *xl* [*recte xL*], *xk* [*recte xK*], *ti*, *tg*, *rg*, *re*, *pe*, *pc*, *nc*, *nl*, *xl*, and *xi*, the result will be the degraded figure as shown in figure 21 (Fig. 3.47).

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[59] Lines *mk*, *mb*, *nl*, *nc*, *pe*, *pc*, *re*, *rg*, *sh*, *sf*, *uh*, *uk*, are not shown in Fig. 3.47.

[60] This is seen much more clearly in figure 22 (Fig. 3.48), in which the construction lines are canceled and all the edges of the solid are drawn.

So that it is better known that the perfect described has in it 12 pentagons and 20 triangles, which the many lines that must be drawn do not allow to be fully discerned, the perfect has been drawn by itself in figure 22 (Fig. 3.48).[61]

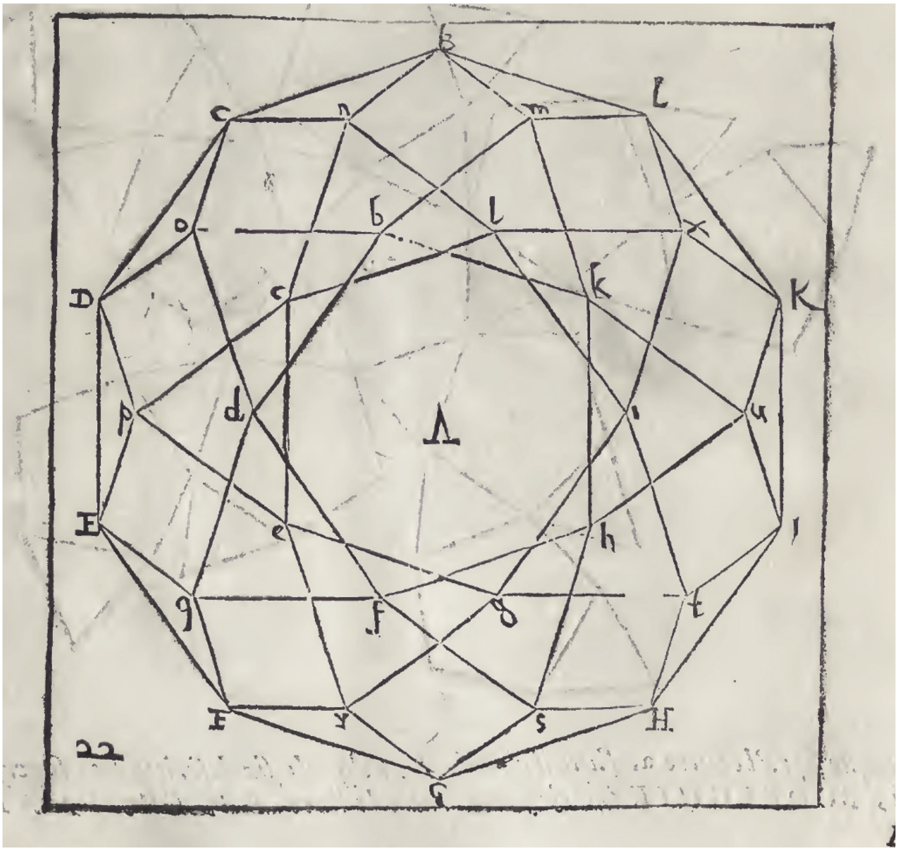

Fig. 3.48 **[p. 72]**

[p. 74] The perfect of this body resting on the triangular surface is formed in this manner. In the other perfect draw the line *IH*[62] and with centre *a* [*recte A*] and space *bf* of the pentagon of before [*kbdfh*] make a circle, which will be *B*. In that are found the point of a hexagon of equal sides *BCDEFG*.

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[61] The 12 pentagons are *cegil*, *IKxit*, *gtHGr*, *DCpcn*, *pEFre*, *BnlxL*, *bdfhk*, *BCobm*, *DEqdo*, *KLmku*, *FGsfq*, and *HIuhs*. The 20 triangles are *gti*, *ixl*, *lnc*, *cpe*, *erg*, *BCn*, *DEp*, *FGr*, *HIt*, *KLx*, *fsh*, *huk*, *kmb*, *bod*, *dqf*, *GHs*, *IKu*, *LBm*, *CDo*, and *EFq*.

[62] *IH* is an edge of the pentagon as well as of a triangle; see Fig. 3.48.

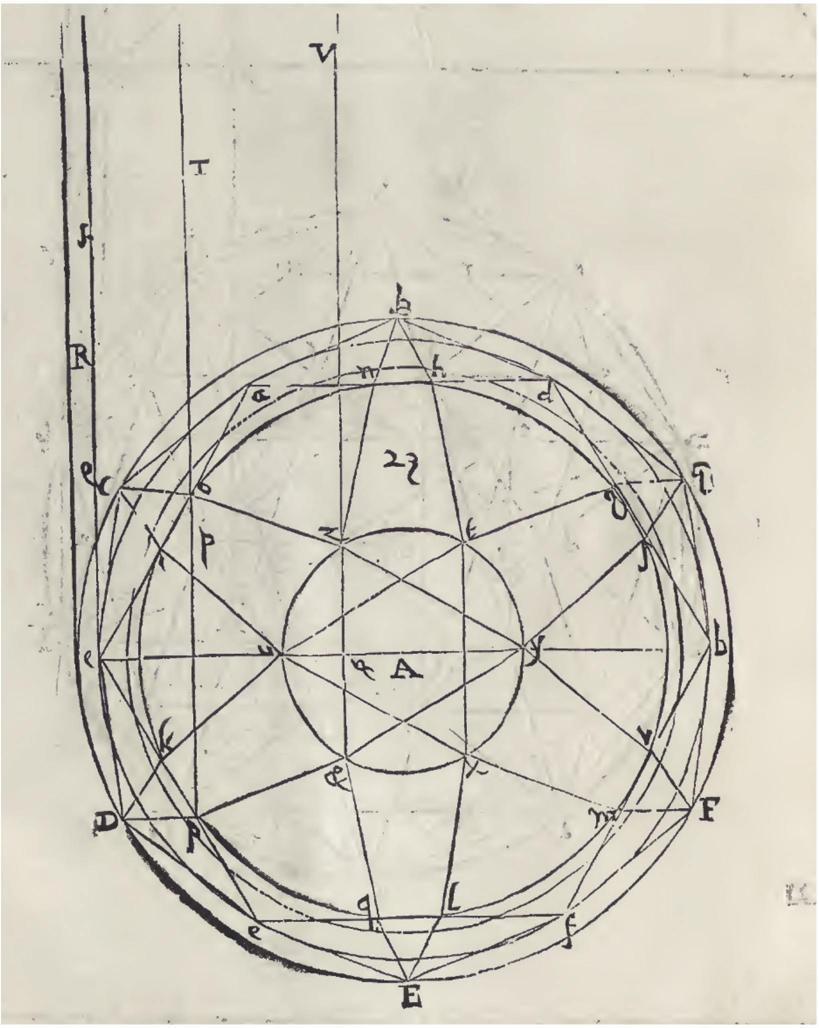

Fig. 3.49a [p. 74]

On that centre *A* make another circle *a* of a circumference such that the line *IH* of the other perfect makes in it 2 triangles, *abc* and *dfe*, of equal sides whose vertices are equally distant from each other and from the vertices of the hexagon [BCDEFG]. Then with the same center, draw a circle *h*, whose circumference is such that the lines *bf* and *bd* of the pentagon of the other perfect make on that circle the points of 2 hexagons of equal sides, *ghiklm* and *nopqrs*, whose points are equally distant from the points of those triangles [*abc*, *dfe*]. Let there then be given another circle on that centre *A* such that the line *bd* mentioned above forms in that 2 triangles, *tux* and *yz*&, of equal sides whose vertices are equally distant from each other, and in that way equally distant so that drawing lines *tg*, *th*, *zn*, *zo*, *ui*, *uk*, &*p*, &*q*, *xl*, *xm*, *yu* [*recte yr*] and *ys*, they are all equal. Then draw lines *Bd*, *Bh*, *Bn*, *Ba*, *Ca*, *Co*, *Ci*, *Ce*, *De*[63], *Dk*, *Dp*, *Dc*, *Ec*, *Eq*, *El*, *Ef*, *Ff*, *Fm*, *Fr*, *Eb* [*recte Fb*], *Gb*, *Gs*, *Gg*, *Gd*, *bg*, *ds*, *du* [*recte dn*], *ah*, *ai*, *ko* [*recte eo*], *ep*, *ck*, *cl*, *fq*, *fr*, *bm*, *gh*, *no*, *ik*, *pq*, *lm*, and *rs*. These will describe the perfect of this body. Its 12 pentagons and 20 triangles are clear in themselves. It must be noted with regard to the triangles *abc* and *def* that it is not necessary to make them, but only to find their points, so they are not shown in the perfect, as seen in figure 23 (Fig. 3.49).

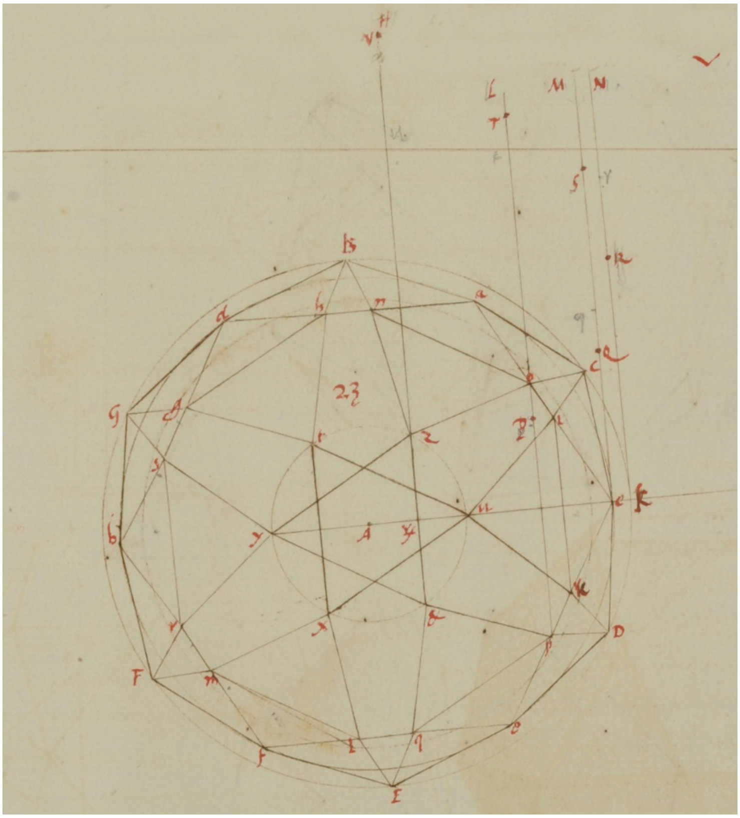

Fig. 3.49b. From Barbaro's Latin manuscript Lat. VIII, 41=3069, fol. 39r[64]

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[63] *De* has been repeated two times in the original text.

[64] Fig. 3.49, printed as figure 23, does not show many of the letters useful for the construction of planes and heights. For this reason we add Fig. 3.49b taken from the Latin manuscript Lat. VIII, 41=3069, fol. 39r.

[p. 75] The planes and the heights are found in this way. In the pentagon of the other perfect (Fig. 3.47), where the line *bf* cuts *zd*, let there be what I call point 2, and then extend the line *uy* in *ue*, such that it cuts *z*& in ℞ [*recte* 𝔷]. Extend *u* out to point *k* of circle *D*, then draw the line *op*. On *z*, set *Hz*, and on *o*, the line *lo* [*recte Lo*], and on *e*, the line *ME* [*recte Me*], and on *k* the line *Nk*. All these 4 lines are at right angles on *k*𝔷. With centre ℞ [*recte* 𝔷] and space *y*2 of the first perfect draw the arc *P*, which cuts *op* in *P*. With centre *P* and space *d*2 of the first perfect cut *Me* in *Q*. With centre *Q* and the same space cut *Nk* in *R*. With centre *R* and the same space cut *MQ* [*recte Me*] in *s* [*recte S*]. With centre *s* [*recte S*] and the same space cut *lo* [*recte Lo*] in *T*. With centre *T* and space *y*2 cut *Nz* [*recte Hz*] in *V*. These points [𝔷], *P*, *Q*, *R*, *S*, *T*, and V are the planes of this body. So if in the first plane ℞ [*recte* 𝔷] is degraded the triangle *zy*&, on plane *P* the lines *no*, *pq*, *rs*, on plane *Q* the points *def*, on plane *R* the points *bcdeg* [*recte BCDEFG*], on plane *S* the points *a*, *b* and *c*, on plane *T* the lines *ik*, *lm*, and *hg*, and on plane *V* the triangle *tux*, and the lines *tg*, *th*, *ui*, *uk*, *xm*, *xl*, *gb*, *gG*, *hb* [*recte hB*], *ha*, *mb*, *mF*, *lE*, *lc*, *ia*, *ic* [*recte iC*], *ke*, *kd* [*recte kD*], *BF* [*recte bF*], *bg* [*recte bG*], *ab* [*recte aB*], *ac* [*recte aC*], *cD*, *bd* [*recte Bd*], *hn* [*recte Bn*], *Co*, *cE*, *De*, *ds*, *Ef*, *E*2 [*recte Eq*], *Ff*, *fr* [*recte Fr*], *Gg* [*recte Gs*], *Gd*, *ds* [*recte pD*], *dn*, *eo*, *ep*, *fr*, *fq*, *nz*, *oz*, *p*&, *q*&, *ry*, and *sy* are drawn, the result is the degraded body, as seen in figure 23 (Fig. 3.50).

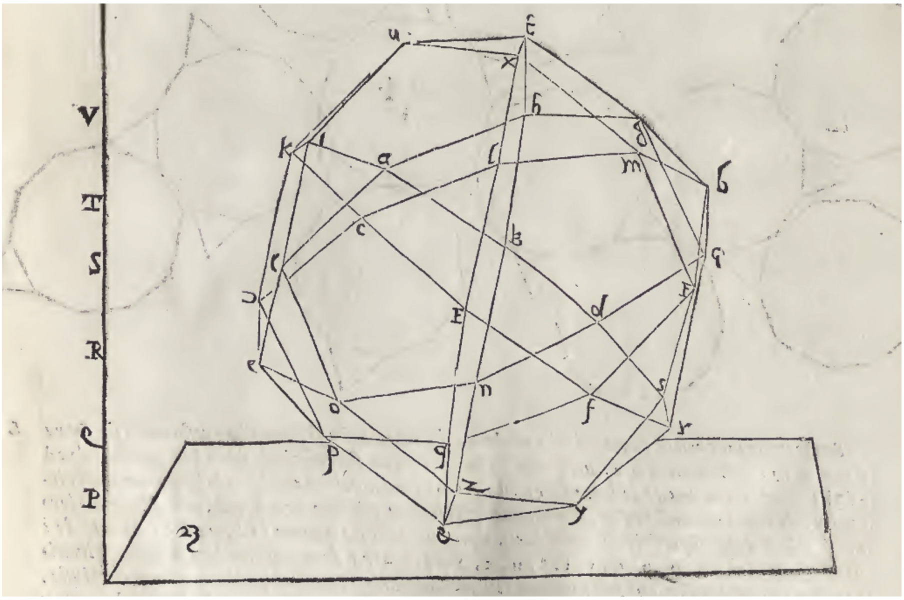

Fig. 3.50 **[p. 75]**

[p. 76]

Chap. XIII. Unfolding and description of another body that is born from the dodecahedron [truncated dodecahedron]

By dividing the edges of the dodecahedron into 3 equal parts and taking away the solid angles where the middle parts terminate is formed a body of 32 faces, which is of 12 decagons and 20 triangles [truncated dodecahedron]. It has 60 solid angles formed of acute and obtuse angles. It has 60 acute plane angles, 120 obtuse plane angles, and 90 edges. Its unfolding is shown in figure 24 (Fig. 3.51).

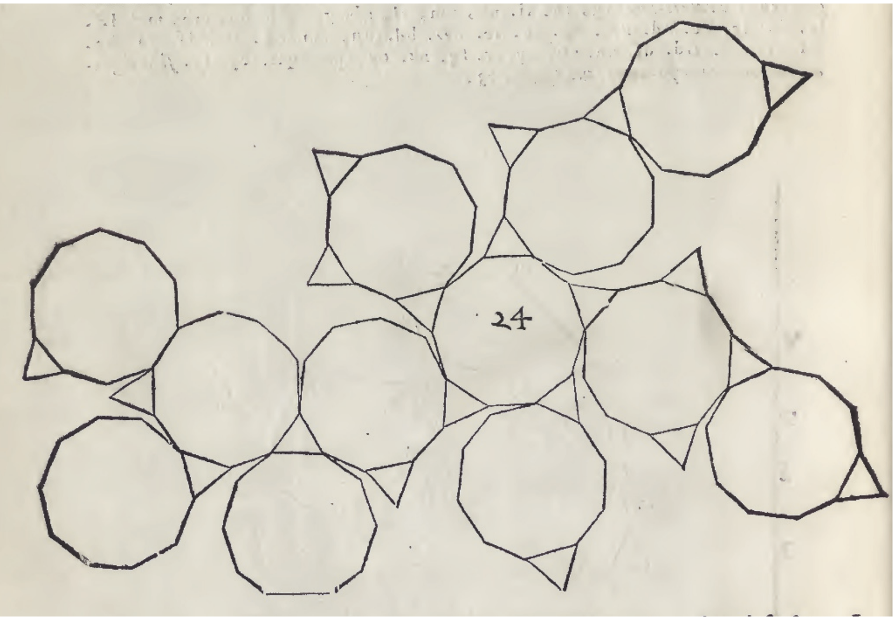

Fig. 3.51 [p. 76]

If this body rests on a face with 10 vertices, the perfect is formed in this way. On centre ✠ is made the circle *a*, in which is inscribed a surface of 10 equal sides *abcdefghik*. Draw the lines *kc*, *id* and *bf*. Also draw on that same centre the circle *r* of a circumference such that, there being inscribed within it 2 pentagons with their vertices towards the sides of the surface of 10 sides, each of their sides is equal to the line *kc*. The points of these [2 pentagons] are *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u* and *x*. Then on that same centre make another circle of a circumference such that, there being 2 pentagons described in it such that their vertices are at right angles of the previous pentagons, each of their sides is equal to line *id*. The points of these are *A*, *B*, *C*, *D*, *E* and *F*, *G*, *H*, *I* and *K*. Then on that same centre make another circle such that the lines *ab* and *kc* each go into it 10 times, that is, *ab* **[p. 77]** forms 10 sides and *kc* forms another 10 sides, alternatingly. The sides drawn from *ab* are 1-2, 3-4, 5-6, 7-8, 9-10, 11-12, 13-14, 15-16, 17-18, and 19-20. The lines drawn from *kc* fall between the points 11 and 8, 2 and 5, 14 and 17. Then are drawn the lines *ar*, *rb*, *bn*, *nc*, *cs*, *sd*, *do*, *oe*, *te*, *tf*, *fp*, *pg*, *gu*, *uh*, *hq*, *qi*, *ix*, *lx* [*xk*], *Km* [*recte km*], *ma*, *rA*, *nG*, *sB*, *oH*, *tC*, *pI*, *uD*, *qK*, *xE*, *mF*, *A*1, *A*2, *G*3, *G*4, *B*5, *B*6, *H*7, *H*8, *C*9, *C*10, *L*11 [*recte I*11], *L*12 [*recte I*12], *D*13, *D*14, *K*15, *K*16, *E*17, *E*18, *F*19, *F*20, 2-3, 4-5, 6-7, 8-9, 10-11, 12-13, 14-15, 16-17, 18-19, and 20-1. The result will be the perfect, as seen in figure 25 (Fig. 3.52).

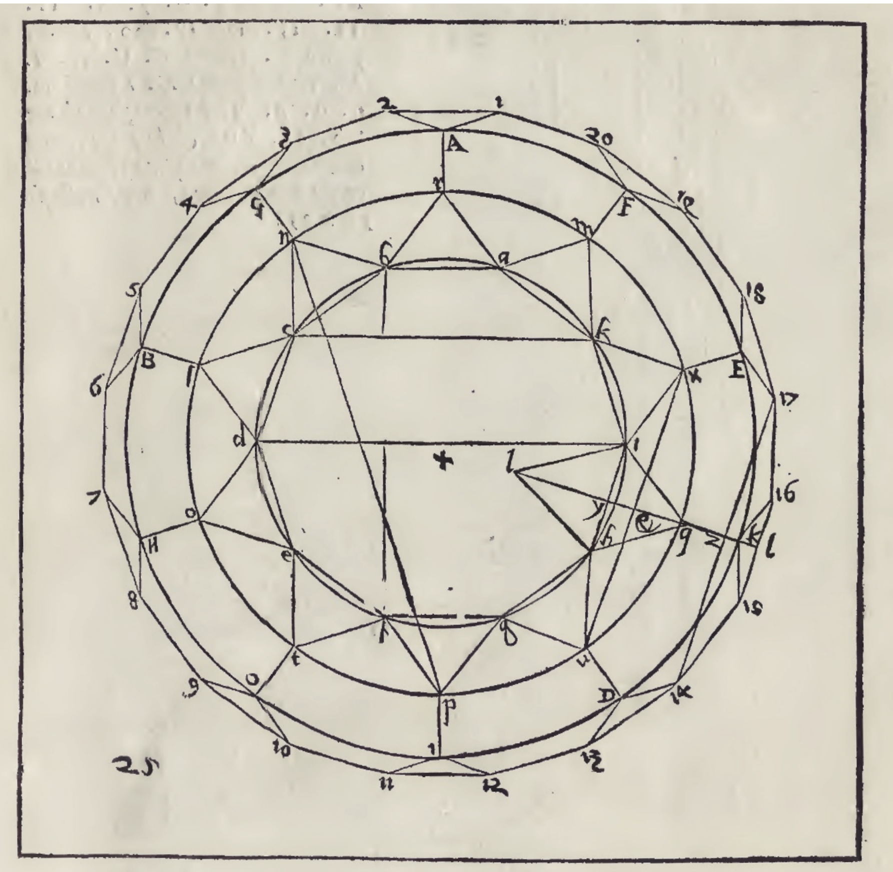
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Fig. 3.52 **[p. 77]**

To know the planes and vertices of this body, on line *ih* in the perfect draw a triangle of equal sides *ikl*. Draw the line *ql*, which will cut *ih* in plane *y*. Then extend *qk* to line 15-16 in point L. Then draw lines 17-14 and *xu*, which will cut *lq* [*recte Lq*] in *z* and *qy* in &. Then carry these points *l* [*recte L*], *k*, *z*, *q*, &, and *y* to another place where there is a line with its letters and on these are drawn the lines at right angles *ML*, *NK* [*recte Nk*], *OZ* [*recte Oz*], *Pq*, Q& and *RY* [*recte* ℞*y*] (Fig. 3.53).

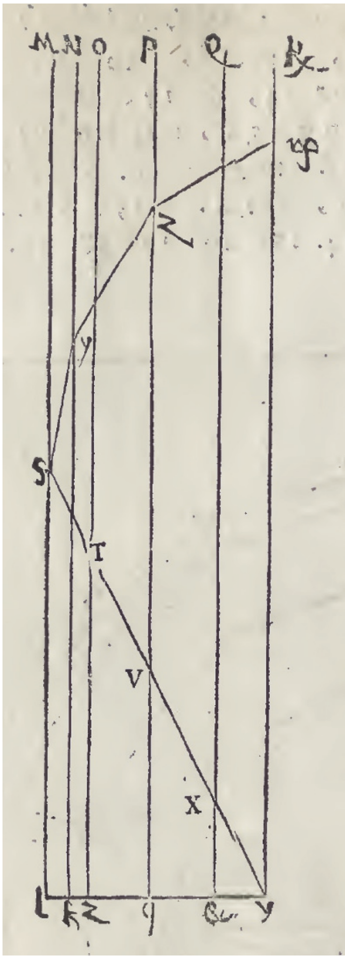

Fig. 3.53 **[p. 78]**

Then with centre *y* and space *bf* cut the line *ML* in *S*. Draw line *Sy*, which will cut the line *OZ* [*recte Oz*] in *T*, line *Pq* in *V*, and *Q&* in *X*. With centre

S and space *yl* [from the perfect] cut *Nk* in *Y*. With centre *Y* and space *Ll*, cut *Pq* in *Z*. With centre *z* and space *yl* cut ℞*y* in 𝔷. These 8 points *y*, *X*, *V*, *T*, *S*, *Y*, *Z* and 𝔷 are the planes of this body. Therefore, if **[p. 78]** on the planes *y* and 𝔷 are degraded the surfaces of 10 sides *abcdefghik*, in plane *X* are set the points *r*, *s*, *t*, *u*, and *x*, in plane *V* the points *A*, *B*, *C*, *D*, and *E*, in plane *F* [*recte T*] the lines 1-2, 5-6, 9-10, 13-14, and 17-18, in plane *S* the lines 3-4, 7-8, 11-12, 15-16, 19-20, in plane *Y* the points *F*, *G*, *H*, *I*, and *K*, in plane *Z* the points *m*, *n*, *o*, *p*, and *q*, and then you draw lines *bn*, *cn*, *do*, and the others as in the perfect, these will form this body, as seen in figure 25 (Fig. 3.54).

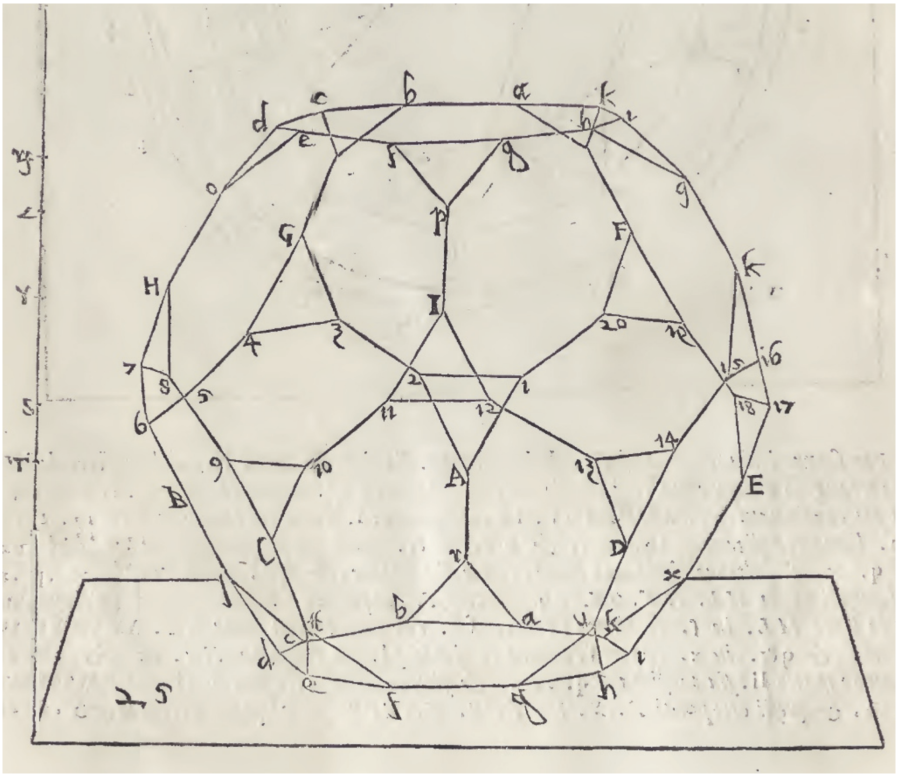

Fig. 3.54 **[p. 78]**

[p. 79] More effort is needed if this body rests on the triangular face, so it is necessary to think about this carefully. So, in the perfect draw the line *np*, which will be the side of a surface of 6 equal sides. Onto its sides carry the lines mentioned above, *ab*, *kc* and *id*, and draw the lines on each side *nn*, *ii*, *kk*, *aa*, *bb*, *cc*, *dd* and *pp*, which will cross each other at many points, but especially in the points 1, 2, 3, 4, 5, up to 54. Then draw lines 1-3, 3-5, 5-1, 6-2, 2-4, 1-7, 4-6, 2-8, 3-9, 4-10, 5-11, 6-12, 7-24, 7-13, 8-14, 8-15, 9-16, 9-17, 10-18, 10-19, 11-20, 11-21, 12-22, 12-23, 24-13, 14-15, 16-17, 18-19, 20-21, 22-23, 25-13, 25-14, 26-15, 26-16, 27-17, 27-18, 28-19, 28-20, 29-21, 29-22, 30-23, 30-24, 31-32, 32-33, 33-34, 34-35, 35-36, 36-37, 37-38, 38-39, 39-40, 40-41, 41-42, 42-43, 43-44, 44-45, 45-46, 46-47, 47-48, 48-49, 49-50, 50-51, 51-52, 52-53, 53-54, [54-31,] 54-25, 25-33, 34-26, 34-26 [repeated], 26-37, 38-27, 27-41, 42-28, 28-45, 46-29, 49-29, 50-30 and 30-53, as seen in figure 26 (Fig. 3.55); the result will be the perfect.

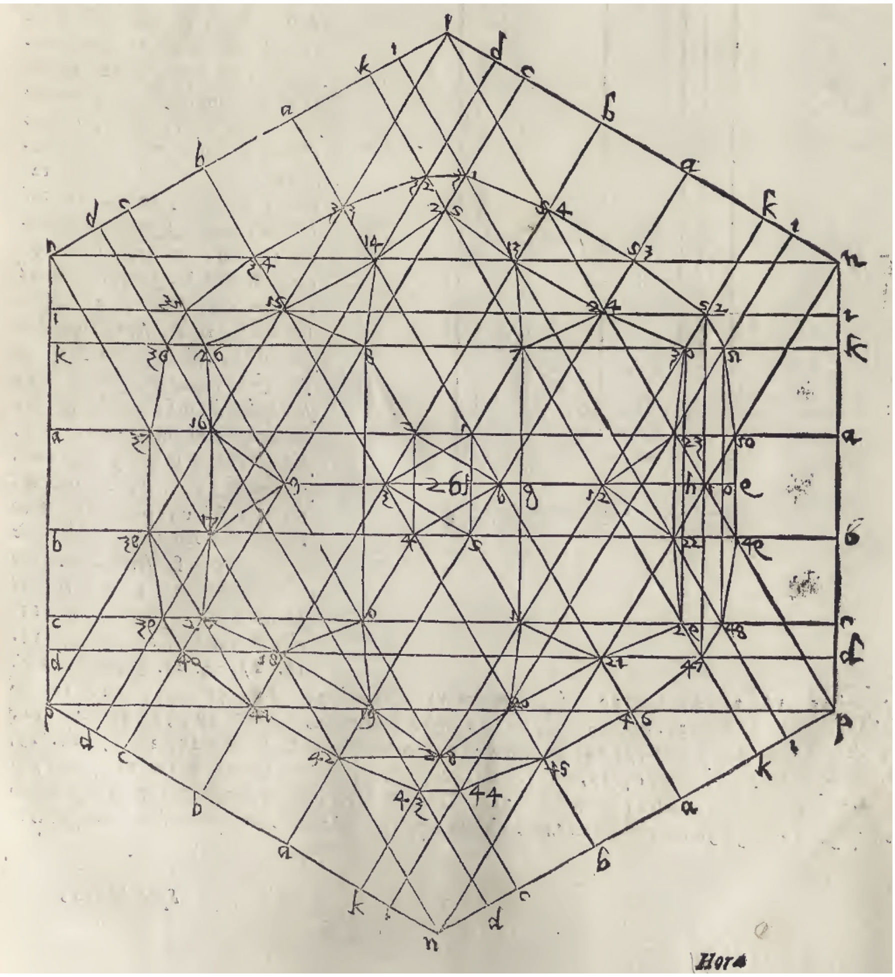
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Fig. 3.55 **[p. 79]**

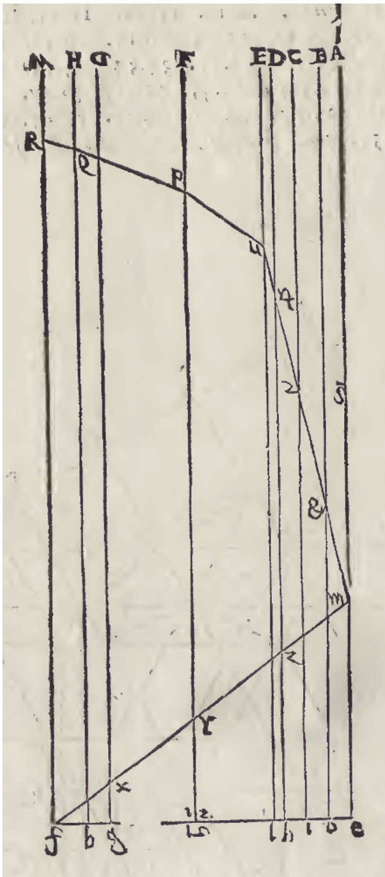

Fig. 3.56 **[p. 80]**

[p. 80] Now the planes are found in this way. Divide line 49-50 in point *e* and draw the line *e*-12, which will cut line 22-23 in point *h*. Then draw the line 6-3, which will cut line 1-5 in *f*. Then draw line 7-11, which will cut the line 12-6 in *g*. Then draw the lines 29-30, 47-52, 48-51, which will cut the line *e*-12 in points *h*, 1 [not labelled on Fig. 3.55], *o*, and *i*. Carry these points *e*, *o*, *i*, *h*, *l*, 12, *g*, *b* [*recte* 6] and *f* onto a line [in a separate place]. On those draw lines *Ae*, *Bo*, *Ci*, *Dh*, *El*, *F*12, *Gg*, *H*6 and *Mf* at right angles (Fig. 3.56). With centre *f* and space *bf* [*recte* 6*f*] from the preceding perfect cut line *Ae* in *m*. With centre *m* and the same space cut *El* in *V* [*recte u*]. Then draw *mf*, which will cut *Gg* in *x*, *F*12 in *y*, and *Dh* in *z*. Then draw the line *mu* which will cut *Bo* in &, *Ci* in 9 [*recte* 2] and *Dh* in ℞ [*recte* 𝖃]. Then with centre *u* and space *f* 3, cut *F*12 in *p*. With *p* as centre and space 1-5, cut *H*6 in *q*. With centre *f* and space *q*6, cut *mf* [*recte Mf*] in *R*. With *R* as centre and space *mf* or *mu*, cut *Am* in 5. These points *f*, *x*, *y*, *z*, *m*, &, 9 [*recte* 2], 5, ℞ [*recte* 𝖃], *u*, *p*, and *q* are the planes of this body. Therefore, if in plane *f* is degraded the triangle 1-3-5, in plane *x* are located points 7, 9, and 11, in plane *y* the points 24, 13, 16, 17, 20, and 21, in plane *z* the points 25, 26, 27, 28, 29, and 30, in plane *m* the points 33, 34, 41, 42, 49, and 50, in plane & the points 35, 40, 43, 48, 51, and 32, in plane 2 the points 36, 39, 44, 47, 52 and 31, in plane 5 the points 37, 38, 45, 46, 53 and 54, in plane 𝖃 the points 25, 26, 27, 28, 29, and 30, in plane *u* the points 14, 15, 18, 19, 22 and 23, in plane *p* the points 8, 10, and 12, and in plane *q* the triangle 2-4-6, and the lines are drawn as demonstrated in the perfect and in the degraded, the result will be the degraded of that body resting on a triangular surface, as seen in figure 26 (Fig. 3.57).

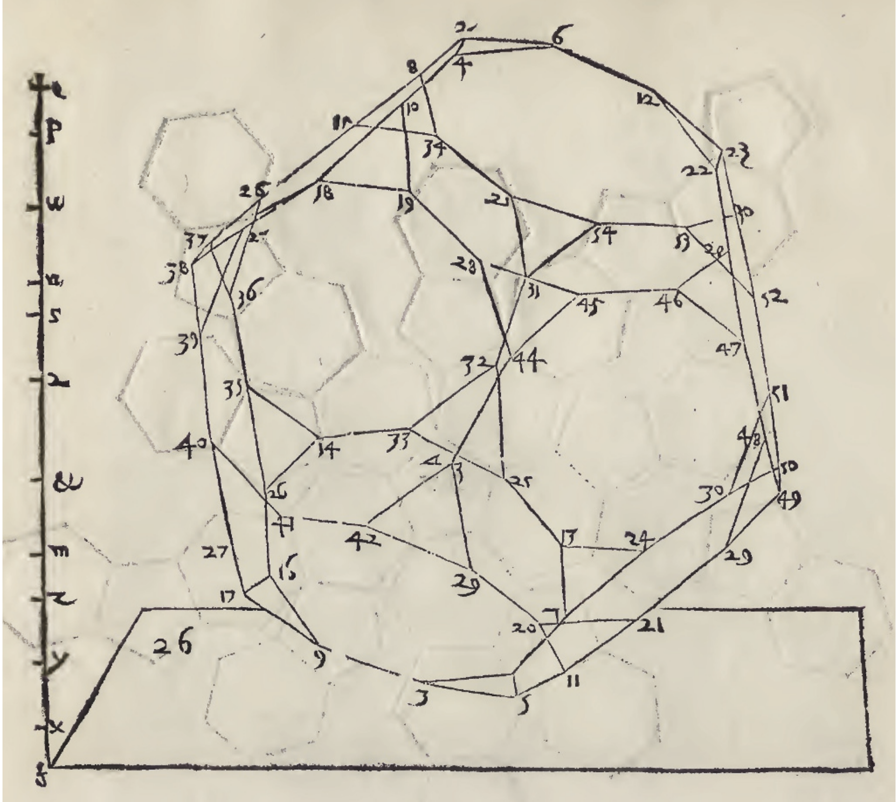
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Fig. 3.57 **[p. 81]**

[p. 81]

Chap. XIV. Unfolding and description of another body that is born from the icosahedron [truncated icosahedron]

Another body is born from the icosahedron [Chap. VI], which has 12 surfaces of 5 equal sides and 20 of 6 [equal sides] [truncated icosahedron]. It has 60 solid angles, 180 obtuse plane angles, and 90 edges. It is born by dividing the edges of the icosahedron into 3 equal parts and taking away the solid angles where the part in the middle terminates, so that the [solid] angles form hexagons, as can be clearly seen when the unfolding, shown in figure 27 (Fig. 3.58), is reduced to a body.

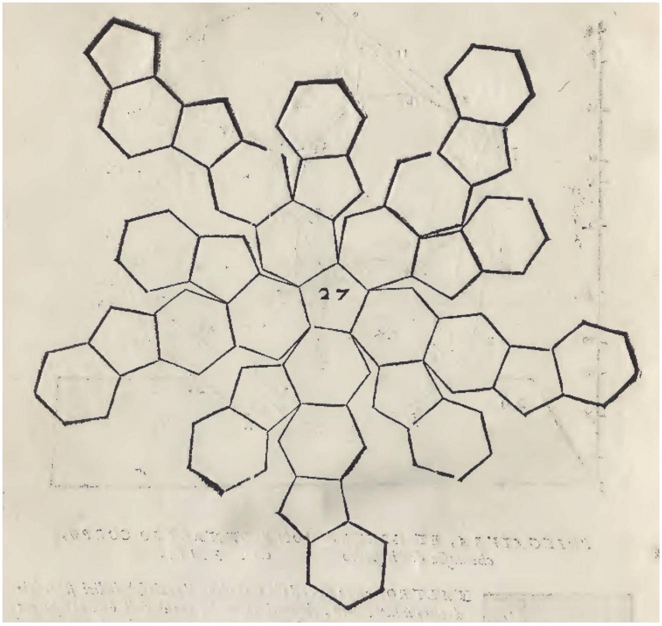

Fig. 3.58 **[p. 82]**

If this body rests on a surface with 5 sides, it is made in this manner. On centre *a* draw a circle in which are made 2 pentagons of equal sides, their vertices alternating so that they are equally distant from each other; one of these is *bcdef* and the other *ghikl*. In the first draw the line *bd*. Let there then be, in a separate drawing, the centre *m*, on which is made a circle with space *bc* of the pentagon. In that is described the hexagon of equal sides *nopqrs*, in which are drawn the lines *or* and *oq*. Let there be on the centre *a* of figure 28 (Fig. 3.59) a circle of a circumference such that inside it the line *or* of the hexagon will form 2 pentagons, the vertices of which are directly opposite those of the previous pentagons; these are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Let there be another circle of a circumference such that the lines *bc* and *bd* of the pentagon go into it 10 times, that is, that the line *bc* is equal to 11-12 and 15 [*recte* 13]-14 **[p. 82]** and *bd* is equal to the lines 12-15 and 14-17. Also on centre *a* make another circle of a circumference such that the lines *bc* of the pentagon and *or* of the hexagon each go 10 times in it, that is, that the line *bc* is equal to 31-32 and 33-34, and the line *or* is equal to the lines 32-25 and 34-37. Then draw the lines as seen in figure 28 (Fig. 3.59). The result will be the perfect of this body, whose surfaces and bases are seen in the plan. This is better understood when the unfolding is reduced into a body, and the resulting form is considered.

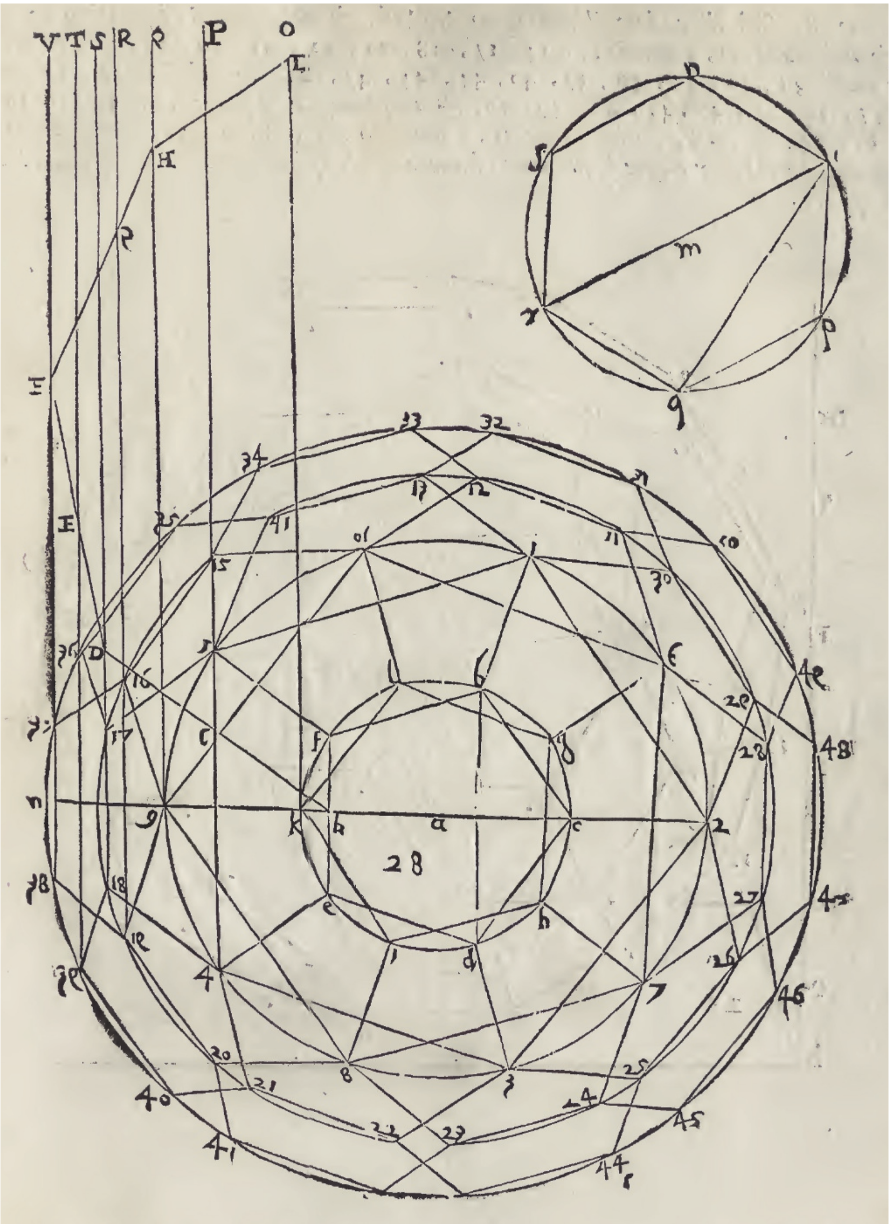
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Fig. 3.59 **[p. 83]**

[p. 83] The planes of this body are found in this manner. Draw the line *ck* in the pentagon *bcdef* [*recte* draw a line from the point *c* of the pentagon *abcde* to point *k* of the pentagon *ghikl*], which will cut *fe* in *B*. Draw the line 9*n* [*recte* 2*n*]. Then on *k* draw *Ok*; on line 4 the line 4-5-*P*; on 9 the line 9*Q*; on 19 the line 19-16-*R*; on 17 the line 17-S; on 39 the line 39-36-*T*; and on line 37 the line 37-*V*. With centre *B* and space *oq* of the hexagon drawn separately [that is, the hexagon with centre *m*] cut the line *S*-17 in *D*. Draw the line *BD*, which will cut the line 4-5 in *C*. With centre *B* [*recte D*] and space *BD*, cut *V*-37 in *F*. Draw the line *DF*, that cuts the line 36-*T* in point *E*. With centre *F* and space *CD* cut *Q*-35 [*recte* 9] in *H*. Draw line *FH*, which cuts *R*-16 in *G*. With centre *H* and space *cb* of the pentagon, cut line *Ok* in point *I*. All of these planes *B*, *C*, *D*, *E*, *F*, *G*, *H* and *I* are planes of this body, such that if **[p. 84]** in plane *B* is degraded the pentagon *bcdef*, in plane *C* the points 1, 2, 3, 4, and 5, in plane *D* the points 13, 14, 17, 18, 21, 22, 25, 26, 29 and 30, in plane *E* the points 31, 32, 35, 36, 39, 40, 43, 44, 47, and 48, in plane *F* the points 33, 34, 37, 38, 41, 42, 45, 46, 49, and 50, in plane *G* the points 11, 12, 15, 16, 19, 20, 23, 24, 27 and 28, in plane *H* the points 6, 7, 8, 9, and 10, and in plane *I* the pentagon *ghikl*, and its lines are drawn, you will have all the planes, as shown in the degraded figure 28 (Fig. 3.60).

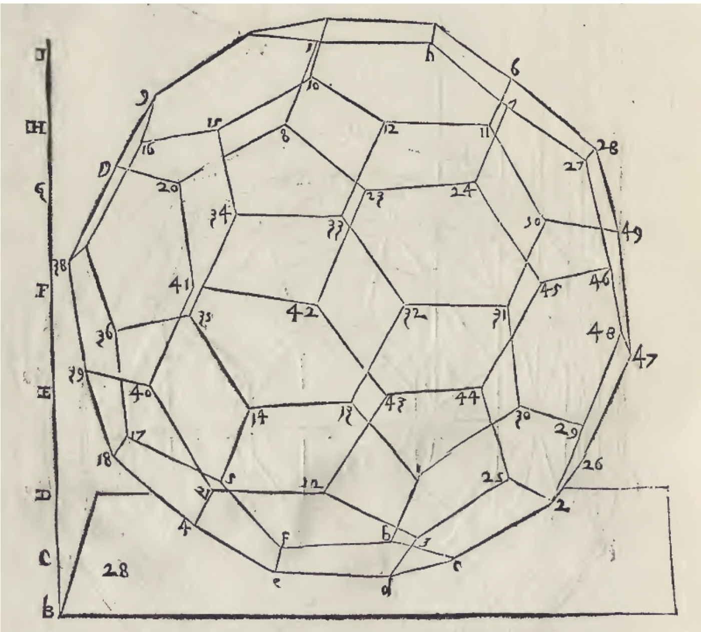

Fig. 3.60 **[p. 84]**

If this body rests on a base of 6 sides, the perfect is made in this manner. Let there be made in a separate drawing a line equal to *bc* of the preceding perfect; this will be *AB* (Fig. 3.61, right). With that space at centre *a* make a circle in which is described a surface of 6 equal sides *bcdefg*; draw lines *be* and *bd* (Fig. 3.62). Then, parallel to and alongside *AB*, make another [line] that is equal to *bd* in the pentagon of the perfect above, which is *cD* [*recte CD*]. With centre *a* make another circle of a circumference such that the line *be* of the hexagon and line *CD* each go into it 6 times, terminating in points *h*, *i*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s* and *t*. I say 6 times because *hi* is equal to *qr* and *kl* and also to the other parts that go around the circle, since each of said lines goes into the circumference 6 times. **[p. 85]** The line *hi* will be equal to the line *CD*, and the line *pq* will be equal to *be*. Then next to the 2 lines [*AB* and *CD*] place the line *EF* equal to the line drawn from point *h* to point *l*. On the same centre *a* draw another circle of a circumference such that line *EF* will make in it 2 triangles of equal sides whose vertices are equally distant; these are *uxy* and *z&2*. Then draw a line next to the 3 previous ones, which is *GH*, equal to the space between point 12 and point 27 of the perfect. On centre *a* draw another circle such that this *GH* and the *AB* from before each go into it 6 times, that is, that *AB* will make 1-2, 3-4, 5-6, 7-8, 9-10, and 11-12 and the line GH will be 2-5, 6-9, 10-1, 4-7, 8-11, and 12-3. On the same centre make another circle of a circumference such that the lines *GH* and *CD* go into it 6 times, that is, so that *CD* is equal to the lines 13-14, 15-16, 17-18, 19-20, 21-22, and 23-24, and *GH* is equal to the line 14-17, 18-21, 22-13, 16-19, 20-23 and 24-15. Then make a line *ik* [*recte IK*] next to the 4 already made, equal to the space between 18 and 29 of the perfect. On centre *a* make a circle of a circumference such that line *lk* [*recte IK*] makes in it 2 triangles of equal sides whose vertices are equally distant; these are 25-26-27 and 28-29-30. Then draw the lines 14-15, 16-17, 18-19, 20-21, 22-23, 24-13, &29, *x*27, 2-30, *y*26, *z*28, *u*25, and the other lines as seen in figure 29 (Fig. 3.62). The result is the perfect, the surfaces of which are clearly seen.

[p. 86] The planes are found in this way. Let there be a circle on centre *l* such that the line *AB* is the side of a pentagon of equal sides formed within it; this is *mnopq*. In it draw the line *no* and the line *m* through centre *l* to the line *pq* in point *R*. The line *no* will cut *mR* in *s*. Then in the perfect draw line *xa*, which will cut the line [*f*]*e* in *T* and, on *T*, draw the line 31*T* at right angles to *Tx*. Then draw the lines 33*q*, 34*x*, 35-3, 36-17, 37-27 all parallel to the previously drawn line 31*T*.[65] Carry the points *T*, *q*, *x*, 3, 17, and 27 onto a line in a separate drawing, as shown in the perfect, so that these planes can be more clearly indicated, since the circles and lines of the perfect are confusing. So with centre *T* and space *Rs* draw the arc that will cut line 33-9 [*recte* 33*q*] at point 38. With centre 38 and space *ms*, cut 34[*x*] at point 39. With centre 39 and space *np* cut line 37-27 at point 40. With centre 40 and space *ms* cut line 36-17 at point 41. With centre 41 and space *Rs* cut line 35-3 at point 42. With centre 42 and space *bd* cut line 31-*T* at point 43. With centre *T* and space *bd* cut line 35-3 at point 44. With centre 44 and space *kf* cut line 36-17 **[p. 87]** at point 45. With centre 45 and space *ms* cut line 37-27 in point 46. With centre 46 and space *np* cut line 34x in point 47. With centre 47 and space *ms* cut line 33-9 [*recte* 33*q*] in point 48. All of these points 3, 38, 39, 44, 40, 45, 41, 46, 42, 47, 48 and 43, with the exception of *T*, are the planes and heights of this body (Fig. 3.61).

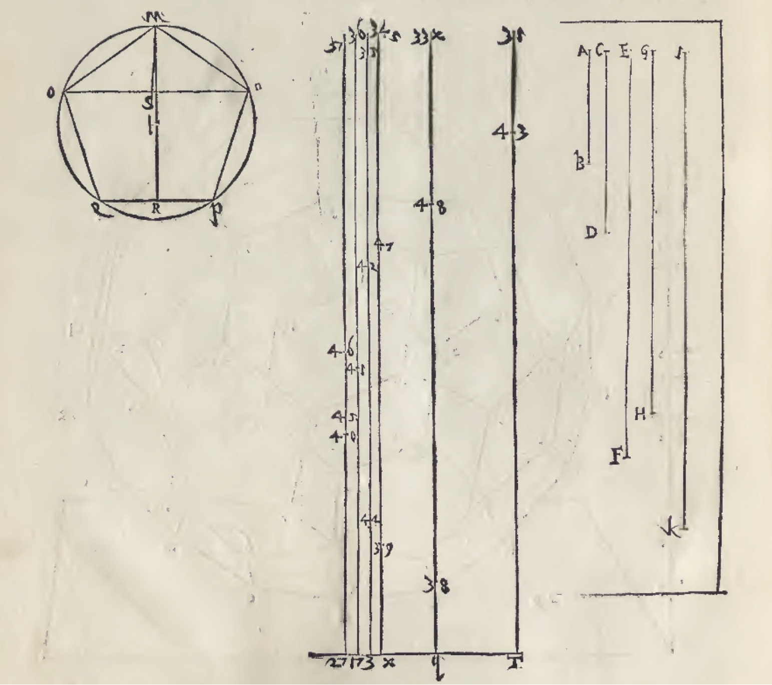

Fig. 3.61 **[p. 86]**

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[65] Points up to 30 are constructed within the perfect. Points 31, 33, 34, 35, 36 and 37 are the end points of the parallel lines whose other ends are *T*, *q*, *x*, 3, and 27. They have a fixed position on the perfect. Indeed, the points *T*, *q*, *x*, 3 and 27, which are in the perfect (Fig. 3.62), conserve their relative horizontal distances in the separated drawing (Fig. 3.61). Points 38 to 48 are constructed on the parallel lines.

So, if in plane *T* [*recte* 3] are located the points *b*, *c*, *d*, *e*, *f* and *g*, in plane 38 the points *h*, *i*, *k*, *l*, *m*, and *n*, and in plane 39 the points 7 and 9, in plane 44 the points 3, 4, 7, 8, 11, and 12, in plane 40 the points 28, 29, and 30, in plane 45 the points 13, 14, 17, 18, 21, and 22, in point [*recte* plane] 41 the points 15, 16, 19, 20, 23, and 24, in plane 46 the points 25, 26, and 27, in plane 42 the points 1, 2, 5, 6, 9, and 10, in plane 47 the points *u*, *x*, and *y*, in plane 48 the points *o*, *p*, *q*, *r*, *s*, and *t*, and in plane 43 the points *b*, *c*, *d*, *e*, *f* and *g*, and lines are drawn from point to point, according to what is clearly shown in the degraded figure 29 (Fig. 3.63), the result will be this body, which is extremely difficult to shown in perspective.

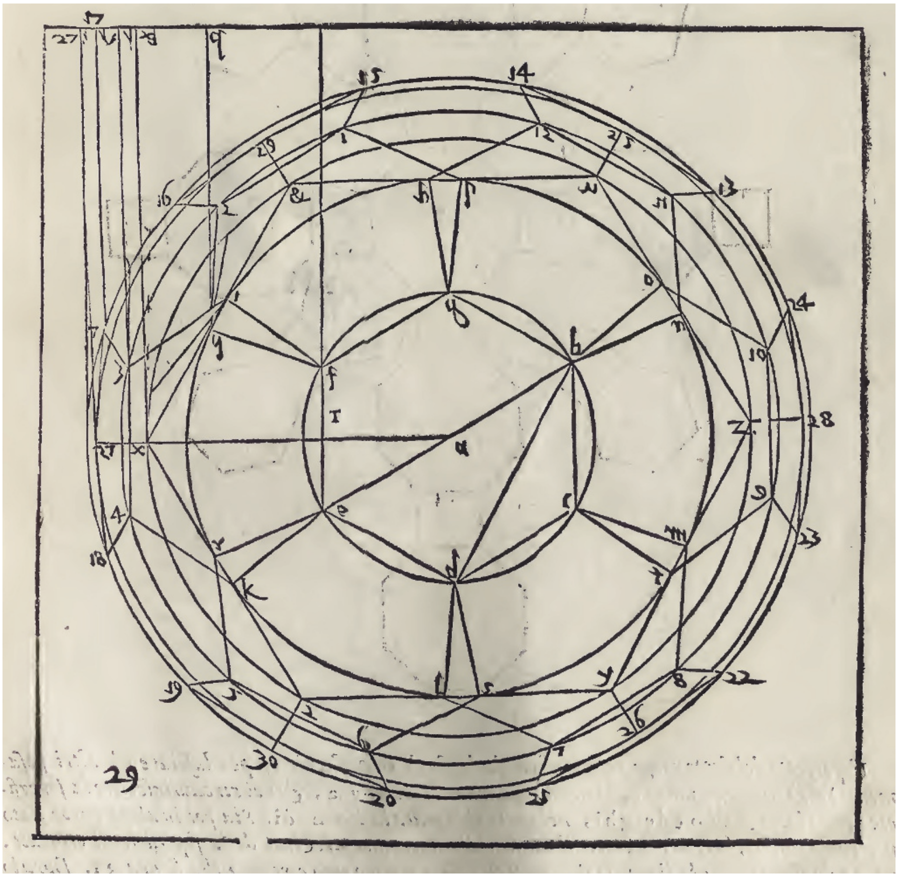
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Fig. 3.62 **[p. 87]**

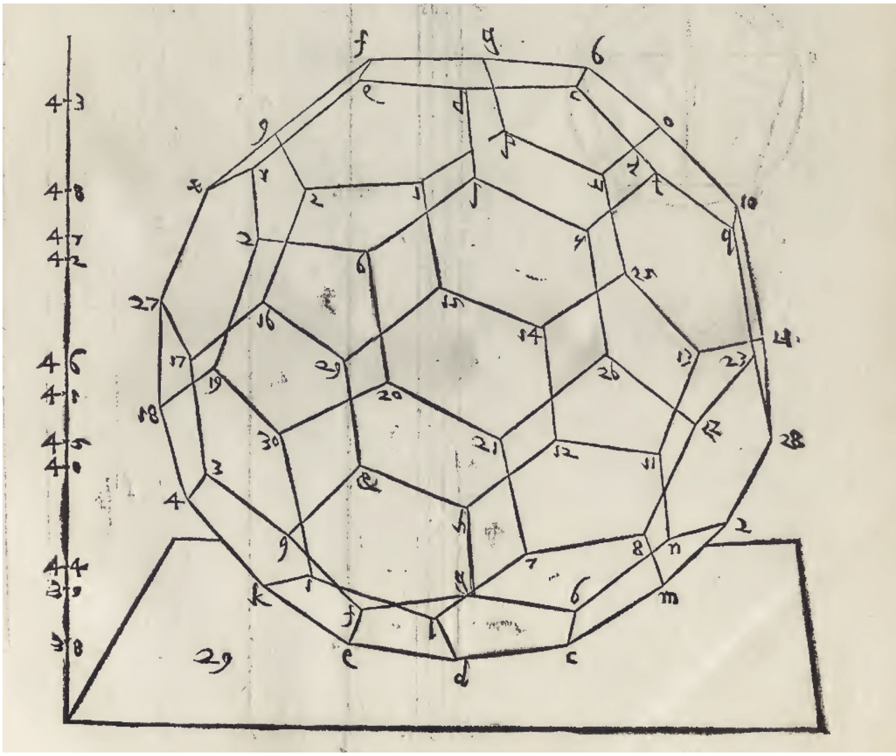
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Fig. 3.63 [p. 87]

[p. 88]

Chap. XV. Unfolding and description of another body that is born from a composite body [great rhombicuboctahedron]

Starting from the body formed of 8 triangles and 6 squares demonstrated above [cuboctahedron, Chap. VIII], dividing its edges into 3 equal [*recte* unequal[66]] parts, and taking away the solid angles where the parts terminate forms another body of 12 squares, 8 hexagons and 6 surfaces of 8 equal sides [great rhombicuboctahedron]. It has 72 edges, 48 solid angles, 48 right plane angles, and 96 obtuse angles. Its unfolding is shown in figure 30 (Fig. 3.64).

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[66] If the faces are to be regular octagons, the divisions of the edges will not be equal.

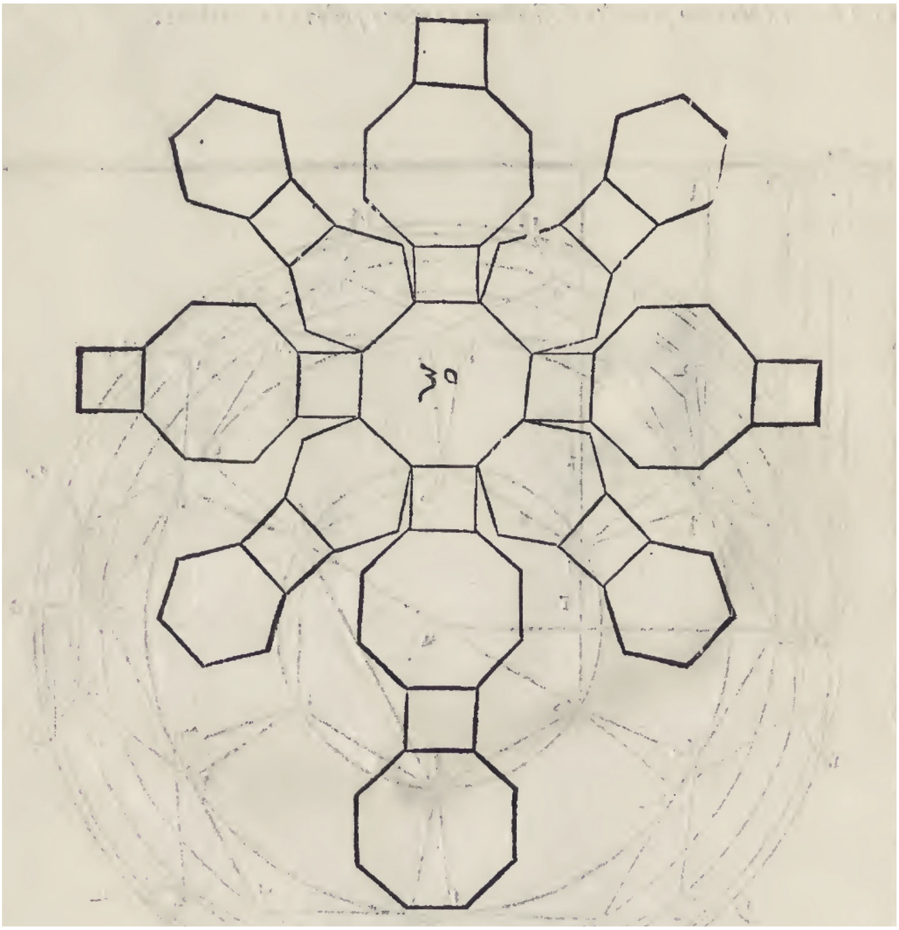

Fig. 3.64 **[p. 88]**

The perfect of this body resting on the base of 8 faces (leaving aside the other resting positions) is made in this way. Let there be a square with centre *a* and the circle in which is inscribed the surface of 8 sides, which is *bcdefghi*, in which is drawn line *di*. Then in a separate drawing draw the circle *BCDEFG*, whose semi-diameter is one side of the surface of 8 sides. Draw in this the lines *DE* and *BF*. Then let there be in figure 31 (Fig. 3.65) made a circle on centre *a* of a circumference such that the lines *bc* and *DE* each go into it 4 times,[67] form another octagon [of unequal sides] with the points *k*, *l*, *m*, *n*, *o*, *p*, *q*, and *r*. Then through centre *a* draw another circle of a circumference such that said lines *bc* and *DE* **[p. 89]** each go into it 4 times with the points *s*, *t*, *u*, *x*, *y*, *z*, &, and 2, forming another octagon. Then draw the lines *kb*, *lc*, *md*, *ne*, *of*, *pg*, *qh*, and *ri*. The result will be the perfect, as seen in figure 31 (Fig. 3.65).

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[67] That is, *bc*=*lm*=*no*=*pq*=*kr* and DE=*lk*=*mn*=*op*=*qr*.

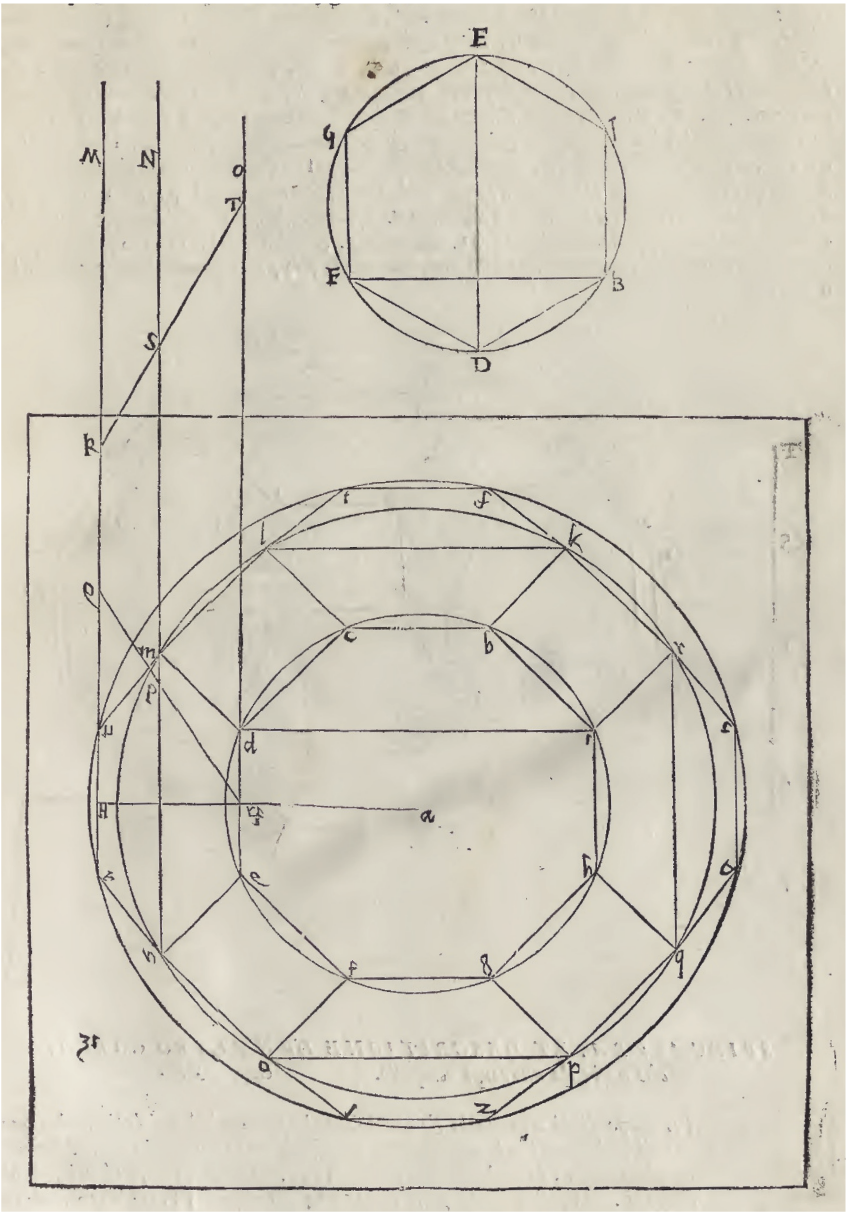
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Fig. 3.65 **[p. 89]**

[p. 90] If you want the planes and heights of this body, you will divide the line *ux* in *H* and draw the line *Ha*, which will cut *de* in ℞ [*recte* ℟]. Then draw the line *MH* at right angles to line *H*℞ [*recte M*℟]. On the points *m* and *d*, make the lines *Hm* [*recte mN*] and *od* [*recte Od*], parallel to the line *MH*. With centre ℞ [*recte* ℟] and space *BF* cut line *MH* in *Q*. Draw the line *Q*℞ [*recte Q*℟], which will cut *Mn* [*recte mn*] in *P*. With centre *Q* and space *bc*, cut *MQ* in *R*. With centre *R* and space *BF*, cut *Od* in *T*. Draw the line *RT*, which will cut *NM* [*recte Nm*] in *S*. All of these points ℟, *P*, *Q*, *R*, *S*, and *T* are the planes of this body, and *p* [*recte P*], *Q*, *R*, *S*, and *T* are the heights. Therefore, if in the planes ℞ [*recte* ℟] and *T* is described the octagon *bcdefghi*, in the planes *P* and *S* are made the points *k*, *l*, *m*, *n*, *o*, *p*, *q*, and *r*, and in the planes *Q* and *R* are set the lines *st*, *ux*, *zy* and &2, and then the lines from the vertices of those octagons to the points of the lines set in the planes *p* [*recte P*] and *s* [*recte S*], and from those lines are drawn the lines to the ends of the lines of the planes *Q* and *R*, the result will be the degraded body, as seen in the degraded figure 31 (Fig. 3.66). The lines *kl*, *mn*, *op* and *qr* in the perfect must be erased, because they were only located in order to find the octagon *klmnopqr*.

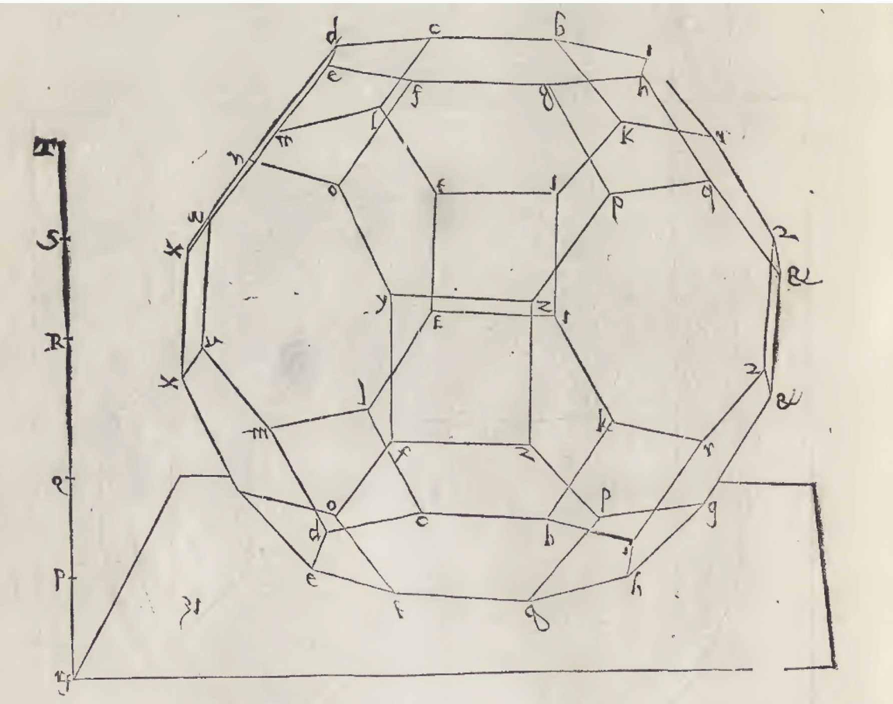

Fig. 3.66 **[p. 90]**

Chap. XVI. Unfolding and description of another body that is born from a composite body [rectified truncated octahedron]

Starting from the body of 6 squares and 8 hexagons [rectified truncated octahedron, Chap. XVI], dividing its edges into 2 equal parts, and taking away the solid angles where these parts finish, another body is formed whose surfaces are formed of 24 triangles, 6 squares and 8 hexagons [rectified truncated octahedron]. It has 72 acute [plane] angles, 24 right [plane] angles, 58 obtuse [plane] angles, 36 solid angles, and 72 edges, as seen in the unfolding in figure 32 (Fig. 3.67).

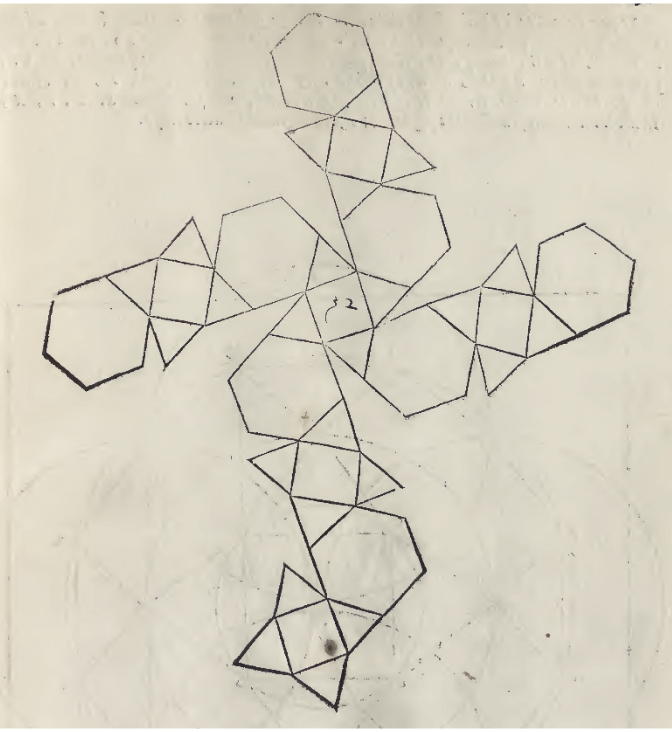

Fig. 3.67 **[p. 91]**

[p. 91] This body is very irregular, because it rests with 3 angles only of the hexagonal base, even if all of its sides are equal. I will demonstrate how it rests on a square base. On centre *a* let there be given a circle in which is described the hexagon *bcdefg* (Fig. 3.69). In this draw the line *bd*, make the triangle *abg*, divide the line *bg* equally in *h*, and draw the line *ah*. On a centre *A* make a circle of a circumference such that the line *bc* makes in it the square

BCDE (Fig. 3.68). On centre *A* make another circle in which is inscribed *FGHI*, and make the triangles *FBE* [*recte FBC*], *GBD*, *IDE* and *HCE*. Then make another circle of a circumference such that the lines *bd* and *bc* each go into it 4 times,[68] and draw only the lines *KL*, *MN*, *OP*, *QR*. Then make the triangles *FKL*, *GMN*, *iop* [*recte IOP*], and *HQR*. Then draw the line *KB* at right angles to points [*recte* line] *KB* [*recte BC*]. With centres *F*, *G*, *H* and *I* and space *KB* draw 4 circles which will intersect each other at points *S*, *T*, *V* [*recte* &], and *X*. Then draw the lines *SK*, *SM*, *VN* [*recte* &*N*], *VO* [*recte* &*O*], *XP*, *XQ*, *TR*, and *TL*. The result will be the perfect of this body (Fig. 3.67). That this is true is known by the fact that the lines *MS* and *GB* are equal to the line *ah*.

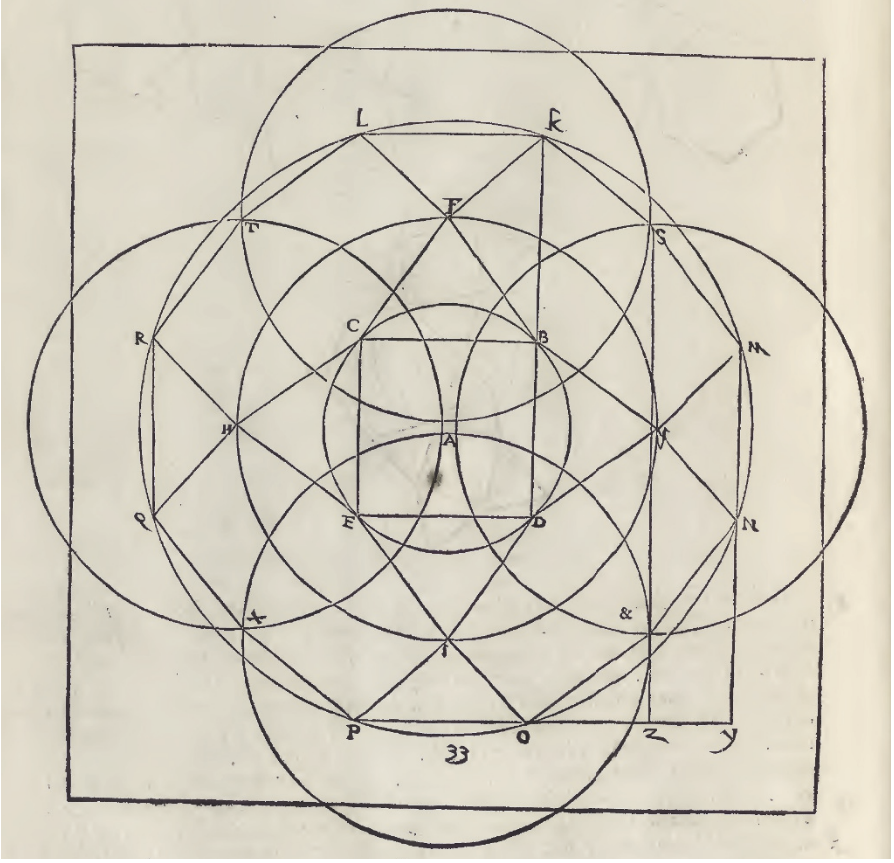

Fig. 3.68 **[p. 92]**

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[68] That is, *bd*=*LR*=*QP*=*ON*=*MK* and *bc*=*KL*=*RQ*=*PO*=*NM*.

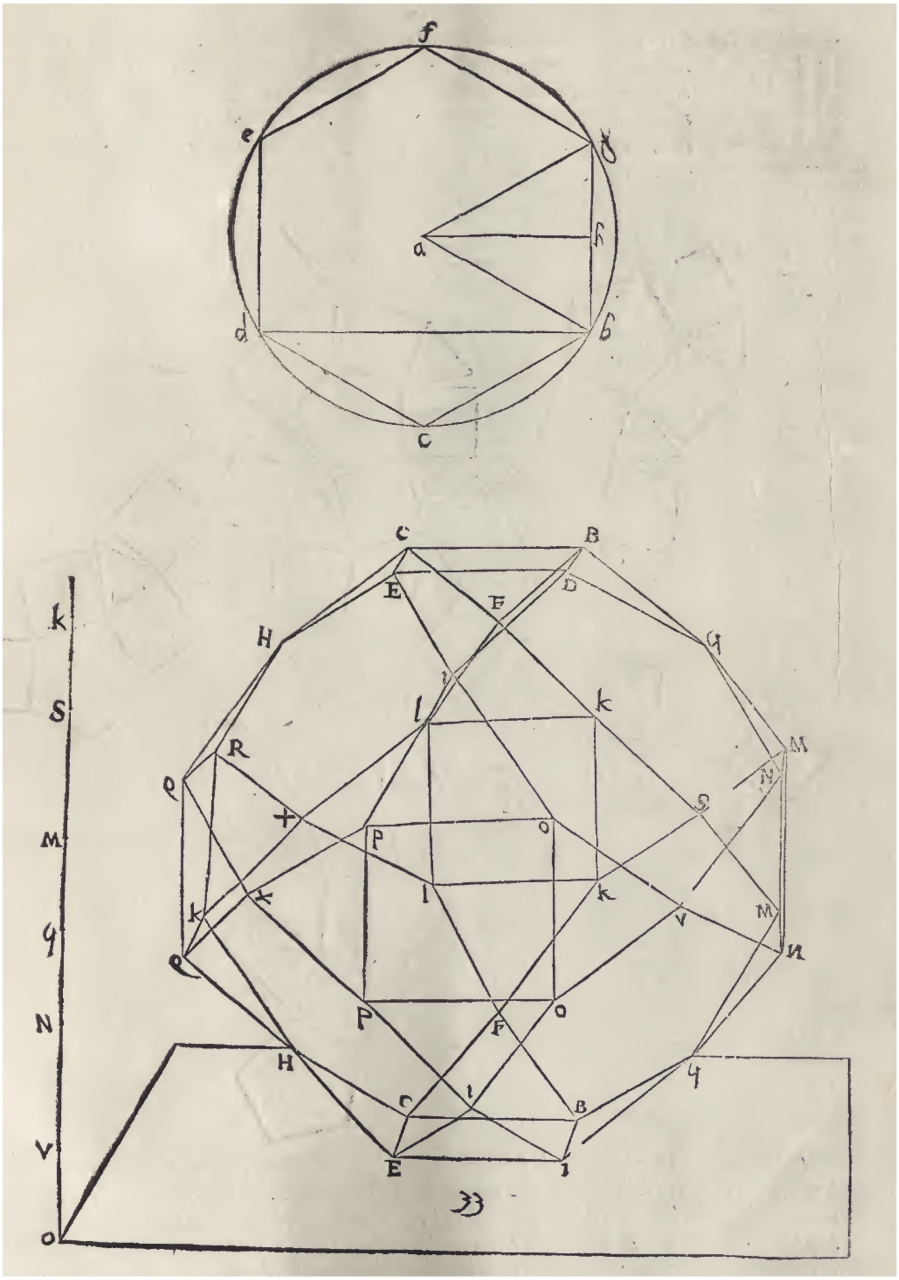
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Fig. 3.69 **[p. 93]**

[p. 92] To find the planes, from points *N* and *O* draw lines that meet at right angles in point *y*. From the points *S*, *G*, and *V* [*recte* &] draw a line to point *z* on *yo* [*recte yO*]. These points *O*, *V* [*recte* &], *N*, *G*, *M*, *S* and *K* are the planes of this body. Therefore, if in the planes *O* and *k* [*recte K*] is located the square *BCDE*, in the planes *V* [*recte* &] and *S* the points *F*, *G*, *H*, and *I*, in the planes *A* [*recte N*] and *M* the lines *KL*, *MN*, *OP* and *QR*, and in plane *G* the points *S*, *T*, *V* [*recte* &], and *X*, and lines are drawn as seen in figure 33 (Fig. 3.69), the result will be the degraded figure of that body.

[p. 94]

Chap. XVII. Unfolding and description of another composite body [small rhombicosidodecahedron]

Starting from the body formed of 20 triangles and 12 pentagons [icosidodecahedron, Chap. XII], dividing its edges into 2 equal parts, and taking away the solid angles where those parts finish is born the body formed of 12 pentagons, 20 triangles and 30 squares [small rhombicosidodecahedron]. It has 60 acute plane angles, 120 right angles, 60 obtuse angles, 120 edges, and 60 solid angles. Its unfolding is shown in figure 34 (Fig. 3.70).

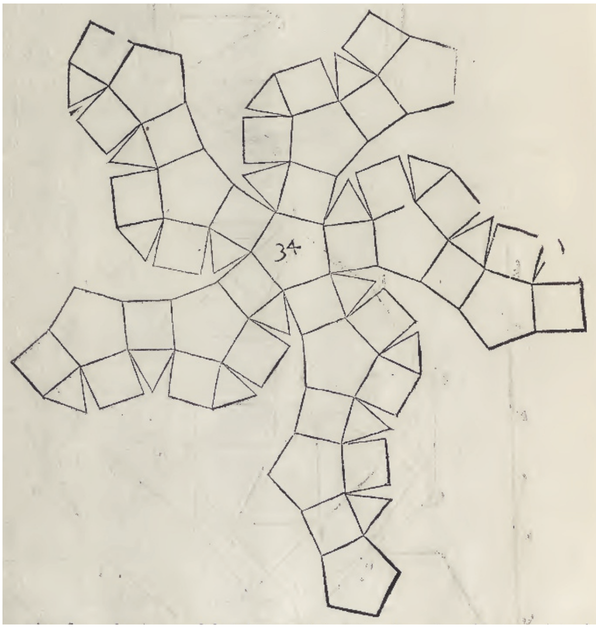

Fig. 3.70 **[p. 94]**

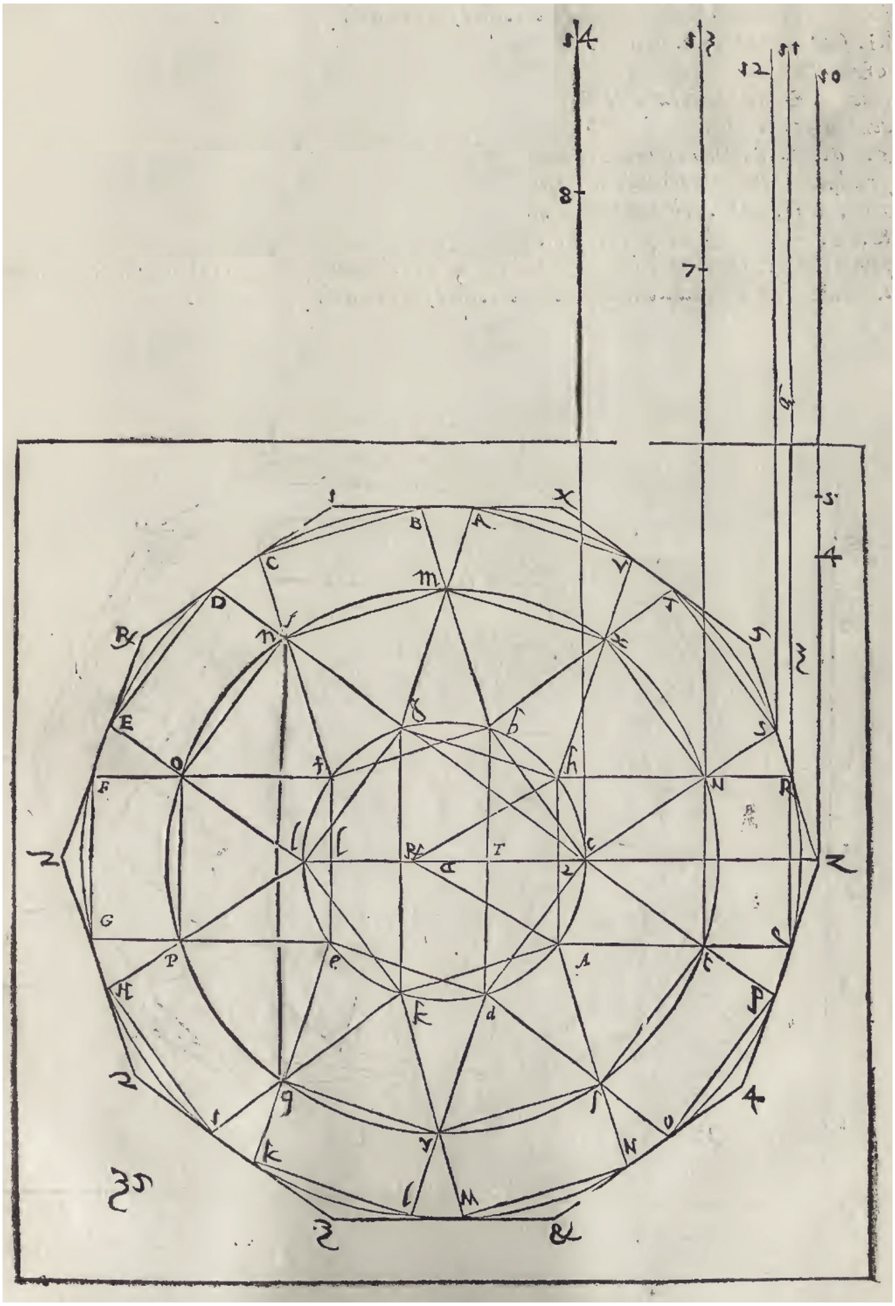
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Fig. 3.71 **[p. 95]**

The perfect, when this rests on a base of 5 sides, is made in this manner. In a circle on centre *a* inscribe 2 pentagons of equal sides whose vertices are equally distant from each other; these are *bcdef* and *ghikl* [*recte ghAkl*[69]]. Draw the line *bd* and make another circle on centre *a* of a circumference such that the line *bc* goes into it 10 times, forming a surface of 10 equal sides whose vertices are *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, and *x*. On the same centre [*a*] draw a circle of a circumference such that **[p. 95]** the lines *bc* [a side of pentagon *bcdef*] and *bd* [a diagonal of pentagon *bcdef*] each go into it 5 times, making the points *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *K*, *L*, *M*, *N*, *O*, *P*, *Q*, *R*, *S*, *T*, and *V*. That is, it is seen that the spaces [occult lines] *AD*, *CF*, *EH*, *GK*, *IM*, *LO*, *NQ*, *PS*, *RV* and *TA* [*recte TB*] are each equal to the space *bd*, and the spaces *BC*, *DE*, *FG*, *HI*, *KL*, *MN*, *OP*, *QR*, *ST* and *VA* are all equal to the line *bc*. Then draw *nb* [*recte nq*], and on that same centre *a* make a circle of a circumference such that the line nq forms the points of 2 pentagons of equal sides, *yz*&2℞ and 1-2-3-4-5, whose vertices are equally distant from each other. Then draw the lines as are seen in figure 35 (Fig. 3.71). The result is the perfect of this body.

[p. 96] The planes are made in this way.[70] Divide the line *hi* [*recte hA*] equally in point 2. Draw the line *cz*2*z* [*recte cz*2℞]. On *hf* [*recte hA*] make the triangle of equal sides *hi*℞ [*recte hA*℞], and draw the line *z*℞ [*recte z*℞] which cuts *bd* in the point ✠ [*recte T*]. Draw the line ℞-1 [*recte* ℞-1], which cuts *fe* in *l*. From *z* draw line 10*z* right angles to *zc*, and from points *R*, *S*, *u*, and *c* draw lines parallel to 10*z*, that is, 11*R*, 12*S*, 13*u*, and 14*c*. With centre 2 and space *hi* [*recte hA*] cut 13*u* in *u*. With centre *u* and space *l*✠ [*recte lT*] cut 11*R* in 3. With centre 3 and space *c*✠ [*recte cT*] cut 10*z* in 4. With centre 3 and space 2℞ [*recte* 2℞] cut 10*z* in *S* [*recte* 5]. With centre *S* [*recte* 5] and space *c*✠ [*recte cT*] cut 11*R* in point 6. With centre 6 and space *hi* [*recte hA*], cut 13*u* in point 7. With centre 7 and space 2℞ [*recte* 2℞] cut 14*c* in point 8. All of these points 2, *u*, 3, 4, 5, 6, 7, and 8 are the planes and heights of this body. Therefore, if in plane 2 is described the pentagon *bcdef*, in *u* the decangle *mnopqrstux*, in plane 3 *RQ*, *NM*, *IN*, *ED*, *AV*, in plane 4 the points *x* [*recte* ℞], 9 [*recte* 2], &, *z* and *y*, in plane *S* [*recte* 5] the points *z*, &, 2, ℞, and *y*, in plane 6 the points *B*, *C*, *F*, *G*, *K*, *L*, *O*, *P*, *S*, and *T*, in plane 7 the points of the decangle just mentioned [*mnopqrstux*], and in plane 8 the pentagon *ghikl* [*recte ghAkl*], and the lines are drawn as they must be, the result will be the degraded of this body, as shown in figure 35 (Fig. 3.72).

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[69] Fig. 3.71 is marked with *A* in the place of *i*.

[70] The corrections made in this paragraph make the text match the figure.

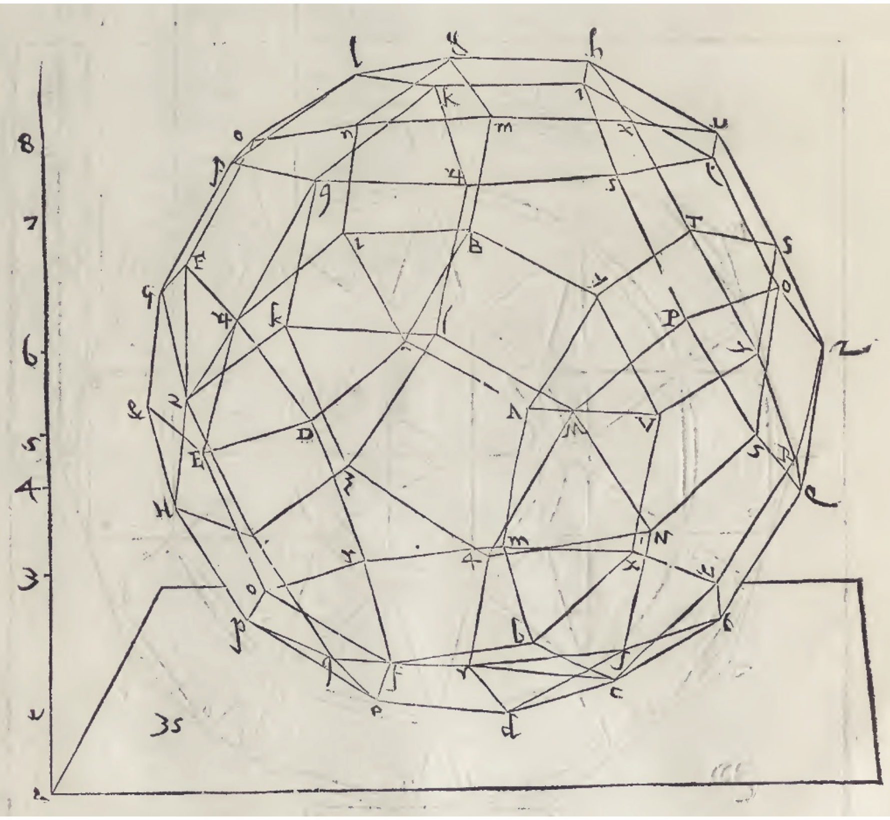
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Fig. 3.72 **[p. 96]**

[p. 97]

Chap. XVIII. Unfolding and description of another body [rectified truncated icosahedron]

Starting from the body of 20 hexagons and 12 pentagons [truncated icosahedron, Chap. XIV], dividing its edges equally [into two] and taking away the solid angles at the terminations of those parts forms the body of 60 triangles, 12 pentagons, and 20 hexagons [rectified truncated icosahedron]. It has 180 edges, 90 solid angles, 180 acute plane angles. Its unfolding is in figure 36 (Fig. 3.73). It is a very beautiful shape although quite irregular as regards its resting position. The unfolding shows only a third of it, so if to triangle *ab* is added the pentagon added to triangle *e* with the other 3 pentagons, hexagons and triangles, except the triangles *d*, *e* and *f*, and the hexagons *g*, the result will be the entire unfolding.

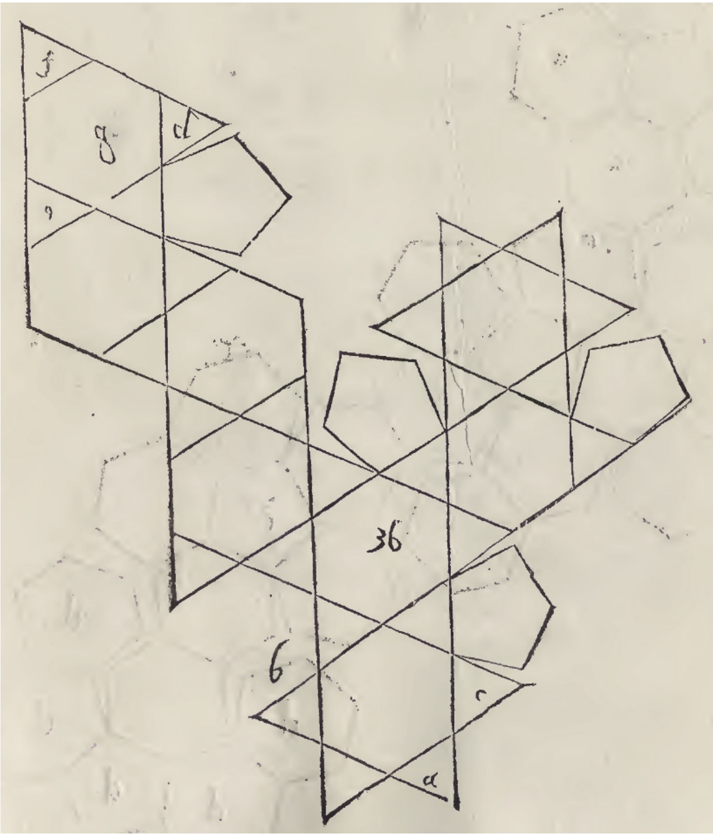

Fig. 3.73 [p. 97]

[p. 98]

Chap. XIX. Unfolding of some other bodies [truncated pentakis dodecahedron]

Starting from the body of 60 triangles, 20 hexagons and 12 pentagons [rectified truncated icosahedron, Chap. XVIII] and changing the triangles into hexagons, the body with 12 pentagons and 80 hexagons is created [truncated pentakis dodecahedron]. It has 180 solid angles, 270 edges, 540 obtuse angles. The unfolding shown in figure 37 (Fig. 3.74) is a fifth of the entire surface, so adding to the hexagons marked *d* the hexagon *b* with the other hexagons and pentagons, except for the pentagon *c* [not labelled in the figure] and all of the hexagons *d*, the result will be the entire unfolding.

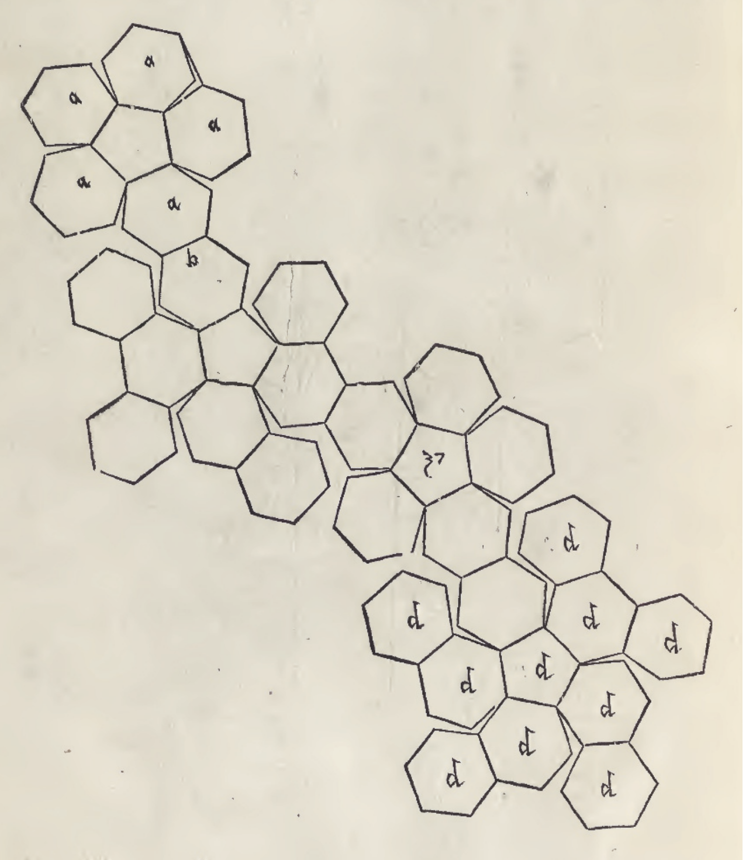

Fig. 3.74 [p. 98]

[p. 99]

Chap. XX. Unfolding of another irregular body [truncated truncated icosahedron]

Starting from the previous body [truncated pentakis dodecahedron, Chap. XIX], dividing its edges into 3 equal parts and taking away the solid angles where the middle part terminates gives birth to the body of 60 triangles, 12 decagons, and 20 surfaces of 12 equal sides [truncated truncated icosahedron]. It has 180 solid angles, as many acute plane angles, 360 obtuse plane angles, and 220 edges. Its unfolding in figure 38 (Fig. 3.75) shows a fifth of it, so if to the triangles marked *a* is added the dodecagon *b*, with the other dodecagons, pentagons and triangles, except for the dodecagon marked *c* and the triangles *d*, the result will be the entire unfolding.

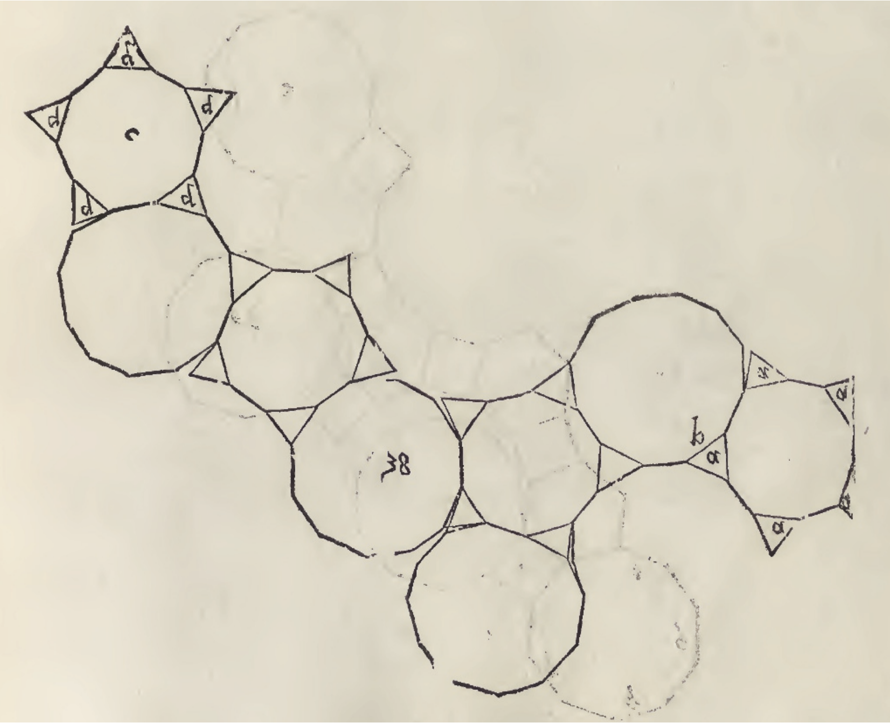

Fig. 3.75 [p. 99]

[p. 100]

Chap. XXI. Unfolding of another irregular body [great rhombicosidodecahedron]

From the body made of 20 triangles and 12 pentagons [icosidodecahedron, Chap. XII], dividing its edges into 3 equal parts and taking away the solid angles where the part in the middle terminates gives birth to the body with 30 squares, 20 hexagons and 12 decagons [great rhombicosidodecahedron].[71] It has 120 solid angles, again as many right [plane] angles, 240 obtuse [plane angles], and 180 edges. The unfolding shown in figure 39 (Fig. 3.76) is a fifth of it, so if to the other 4 sides of the decagon *b* are added the square *a* with all the remaining shapes of that form, except for the decagon *c*, the result will be the entire unfolding.

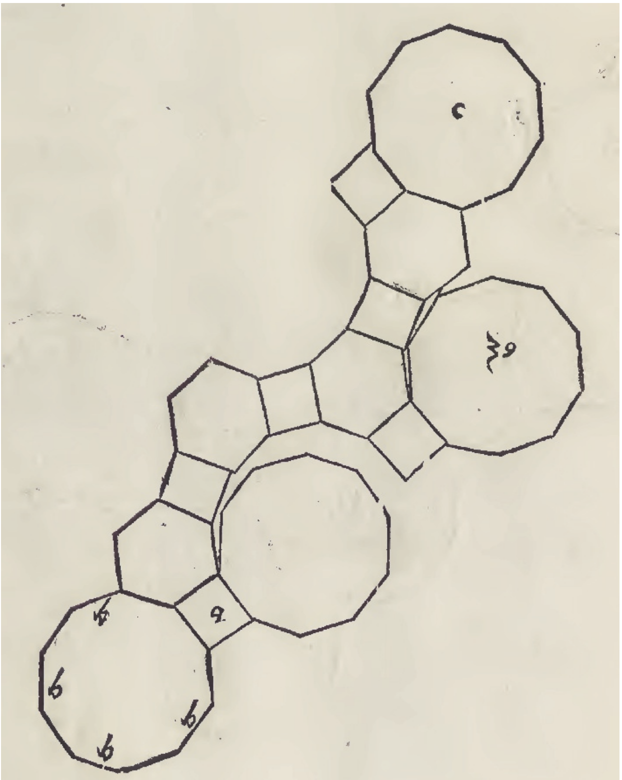

Fig. 3.76 **[p. 100]**

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[71] This polyhedron is mentioned by René Descartes in 'De solidorum elementis' (1860, vol. II, p. 226).

[p. 101]

Chap. XXII. Unfolding of another irregular body [truncated truncated octahedron]

Starting from the body of 6 squares and 8 hexagons [truncated octahedron, Chap. XI], dividing its edges into 3 equal parts, and taking away the solid angles where the middle part terminates gives birth to the body of 24 triangles, 6 octagons, and 8 dodecagons [truncated truncated octahedron]. It has 180 edges, 72 solid angles, as many again acute plane angles, and 140 obtuse plane angles. Its unfolding is shown in figure 40 (Fig. 3.77). It is the fourth part of it, but if to the sides of the triangles marked *b* you add the side of the decagon *a* with the rest of this form, except the octagon *c* and the triangles marked *d*, you will form the entire surface of this body.

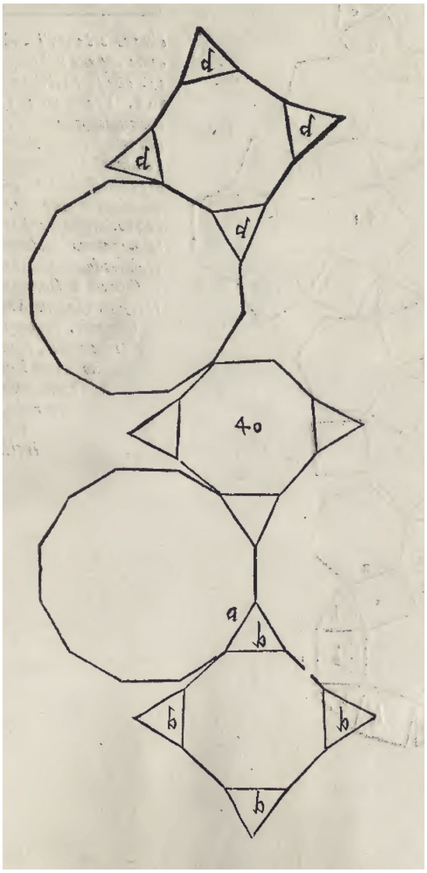

Fig. 3.77 [p. 101]

[p. 102]

Chap. XXIII. Unfolding of another irregular body [great rhombicosidodecahedron augmented with pentagonal cupolas]

If the previous body of 30 squares, 20 hexagons and 12 decagons [great rhombicosidodecahedron, Chap. XXI] taking on each of its decagonal bases 5 squares, 5 triangles, and one pentagon, as seen in the body described in Chap. XVII, where on the decagon *mnopqrstux* was formed the pentagon *ghikl*, with 5 squares[72] and 5 pentagons [*recte* triangles], this will form the body with 60 triangles, 90 squares, 12 pentagons, and 20 hexagons [great rhombicosidodecahedron augmented with pentagonal cupolas]. It has 360 edges, 180 solid angles, 180 acute plane angles, 360 right plane angles, 180 obtuse plane angles. The unfolding shown in figure 41 (Fig. 3.78) is a fifth of it, such that if to the triangles and squares marked *b* are added the squares and hexagons marked *d* with all the rest of the hexagons, pentagons, squares, and triangles, except for the pentagon *c* and the squares marked *d*, and the triangles marked *e*, the result will be the entire unfolding.

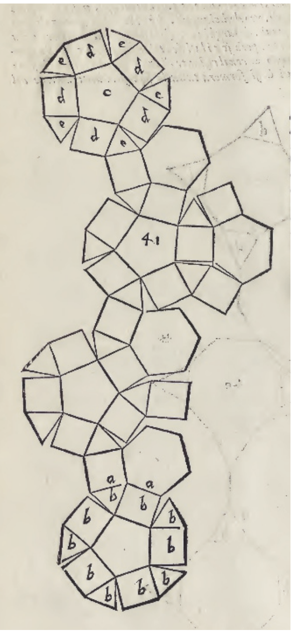

Fig. 3.78 **[p. 102]**

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[72] The 'squares' are not actually square, but are rectangles whose sides are approximately in the ratio 17:20.

[p. 103]

Chap. XXIV. Unfolding of another irregular body [great rhobicuboctahedron augmented with square cupolas]

The unfolding shown is a fourth of the entire surface of the body formed of 8 hexagons, 42 squares, and 24 triangles [great rhombicuboctahedron augmented with square cupolas], such that if to the sides of the other 3 triangles marked *b* are added the square *a* with the rest of that form, except for all the squares marked *c* and the triangles marked *d*, the result will be the entire surface of this body. It is born from the body of 6 octagons, 8 hexagons and 12 squares [great rhombicuboctahedron, Chap. XV]. So, if over an octangular base of this body are elevated 5 squares and 4 triangles, the whole body will have 8 hexagons, just as the original body has, but the squares increase in number to 24. This occurs by the elevation of the octangular base. It has 24 triangles, 72 solid angles, 144 edges, 72 acute plane angles, 168 right plane angles, and 48 obtuse plane angles, as seen in figure 42 (Fig. 3.79).

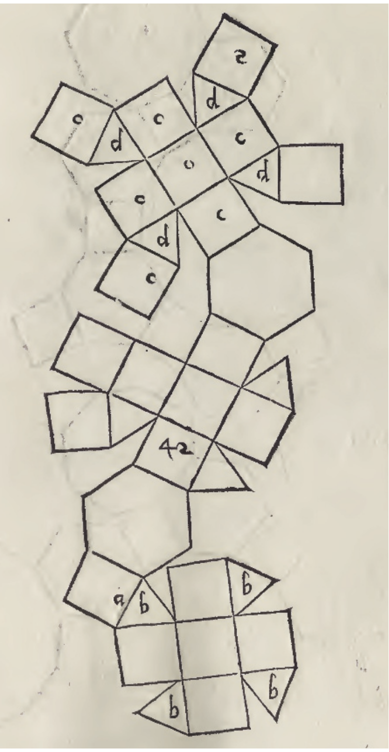

Fig. 3.79 **[p. 103]**

[p. 104]

Chap. XXV. Unfolding of another irregular body

Starting from the body described above of 24 triangles, 6 octagons, and 8 dodecagons [truncated truncated octahedron, Chap. XXII], changing the 24 triangles into 24 hexagons, and placing between one of them a dodecagon and between another a square, the result is a body of 36 squares, 24 hexagons, 6 octagons, and 8 dodecagons.[73] It has 216 edges, 144 solid angles, 144 acute plane angles, 288 obtuse [plane angles]. The unfolding is in figure 43 (Fig. 3.80), which is a fourth of the entire surface. So if to the 3 sides marked *b* of the octagon are added the square *a* with the dodecagon added to that, and to the 3 sides marked *d* are added the hexagon *c* with all of the other hexagons, squares, octagons and dodecagons, the result will be the entire surface, except the octagon.

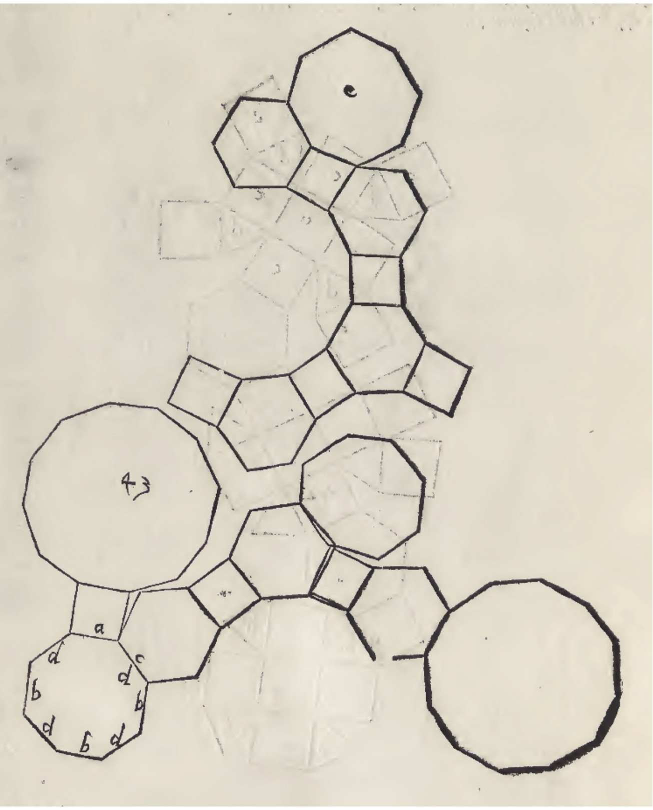

Fig. 3.80 **[p. 104]**

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[73] This solid remains unidentified; see the essay by Kim Williams in this present volume.

[p. 105]

Chap. XXVI. Unfolding of some bodies based on the surfaces of bodies both regular and irregular, and first that which is based on the pyramid [elevated tetrahedron]

The following practice is very delightful, and gives rise to fine considerations, because it reveals the manner in which on the plane surfaces of the regular and irregular bodies are made pyramids of many edges, as seen when the unfolding of 12 triangles of equal sides, closed and glued together, forms a body of many points based on the pyramid [elevated tetrahedron]. These must be put together according to the numbers marked on the triangular surfaces, as seen in figure 44 (Fig. 3.81).

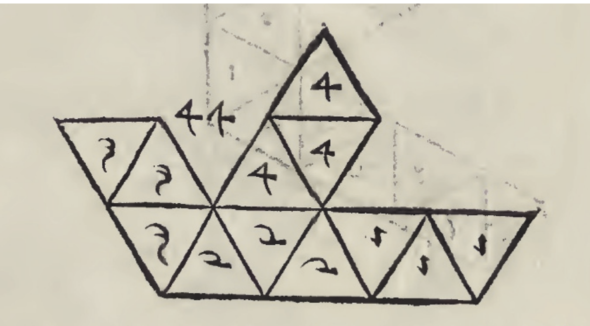

Fig. 3.81 **[p. 105]**

Chap. XXVII. Unfolding of a body based on the cube [elevated cube]

The triangles put together here, which are 24, are the surfaces of a body formed of 6 quadrilateral pyramids [elevated cube], and their basis is the cube. They are to be put together according to the numbers marked as in figure 45 (Fig. 3.82).

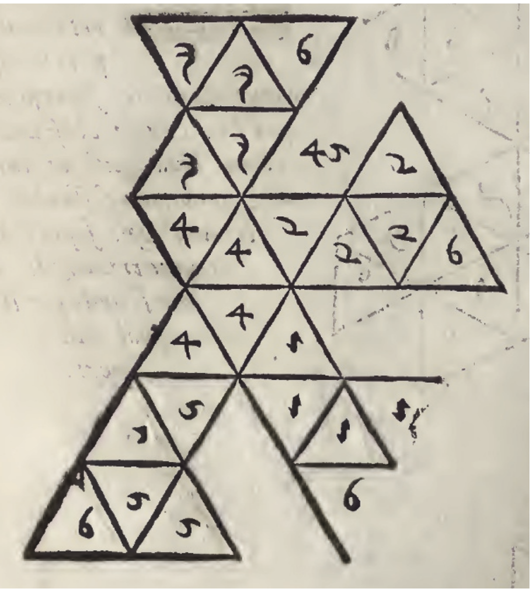

Fig. 3.82 [p. 105]

[p. 106]

Chap. XXVIII. Unfolding of a body based on the octahedron [elevated octahedron]

The unfolding in figure 46 (Fig. 3.83) has 24 triangles, and is a body formed of 8 3-sided pyramids [elevated octahedron]. The basis for it is the octahedron.

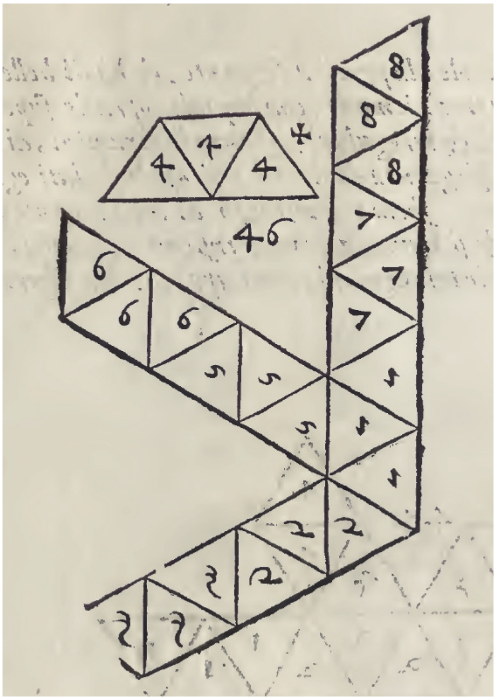

Fig. 3.83 **[p. 106]**

Chap. XXIX. Unfolding of a body based on the dodecahedron [elevated dodecahedron]

The unfolding in figure 47 (Fig. 3.84) is that of a body based on the dodecahedron [elevated dodecahedron]. It is formed of 60 triangles, because this body being of 12 surfaces of 5 equal sides, each supports 5 triangles added together, making 60. Each surface will have 5 triangles, as seen by their numbering.

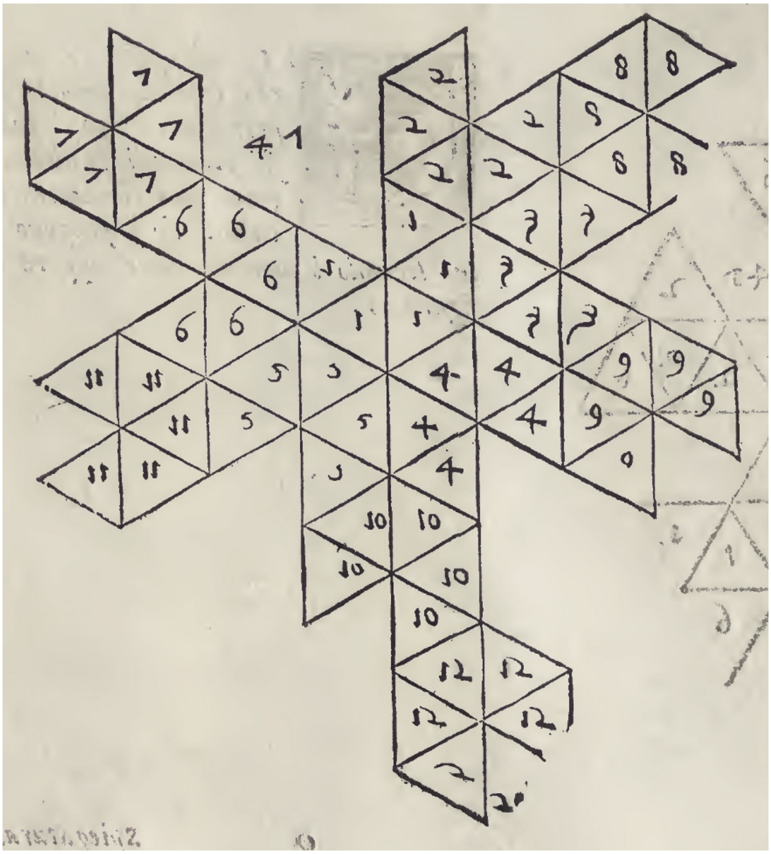
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Fig. 3.84 **[p. 106]**

[p. 107]

Chap. XXX. Unfolding of a body based on the icosahedron [elevated icosahedron]

The body called icosahedron being formed of 20 triangles, and each triangle having on it a triangular pyramid, this body [elevated icosahedron] necessarily has 60 triangles on its surface. This is shown in figure 48 (Fig. 3.85), with their numbers so that they can be put together as required.

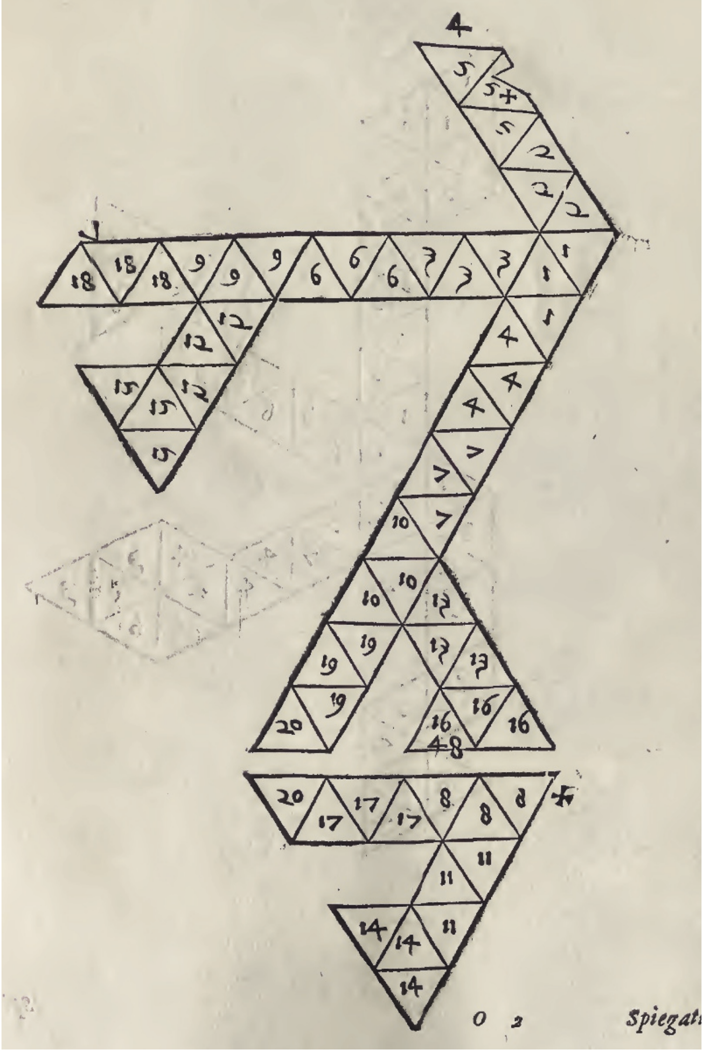
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Fig. 3.85 [p. 107]

[p. 108]

Chap. XXXI. Unfolding of a body based on an irregular body [elevated cuboctahedron]

The figure 49 (Fig. 3.86) is the unfolding of a body based on the body of 6 squares and 8 triangles, so it contains 48 triangles, that is, 24 for the triangular bases, and 24 for the square bases [elevated cuboctahedron]. So all of the triangles numbered 4 are similar and located on square bases. The numbers show how it is to be put together.

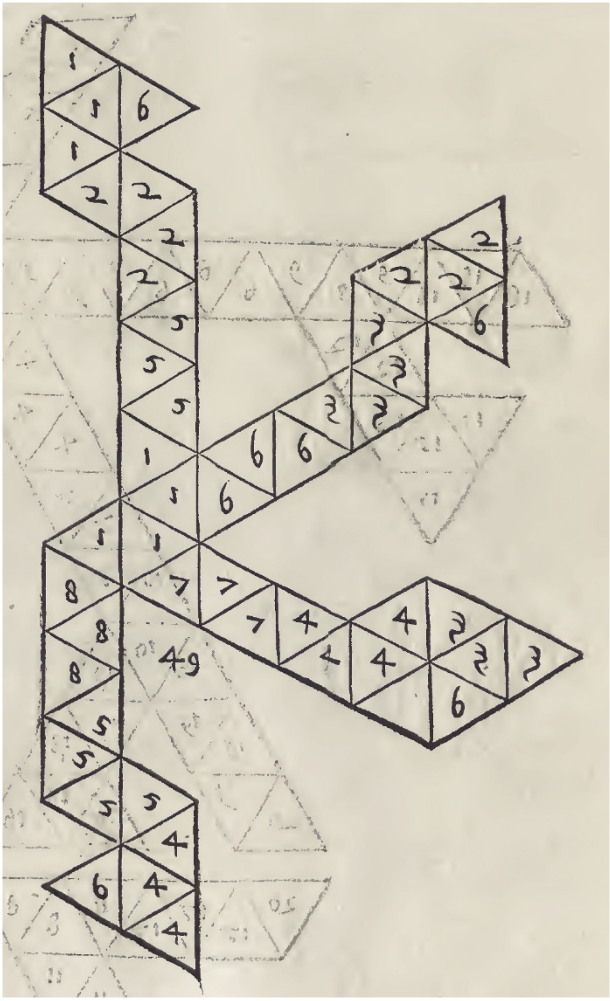
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Fig. 3.86 [p. 108]

[p. 109]

Chap. XXXII. Unfolding of a body resting on an underlying body [elevated small rhombicuboctahedron]

If on top of the body of 18 squares and 8 triangles [small rhombicuboctahedron] are placed the triangles of the unfolding in figure 50 (Fig. 3.87), this will form the body of 96 triangles [elevated small rhombicuboctahedron]. It should be noted that the triangles numbered 4 go on top of the square surfaces.

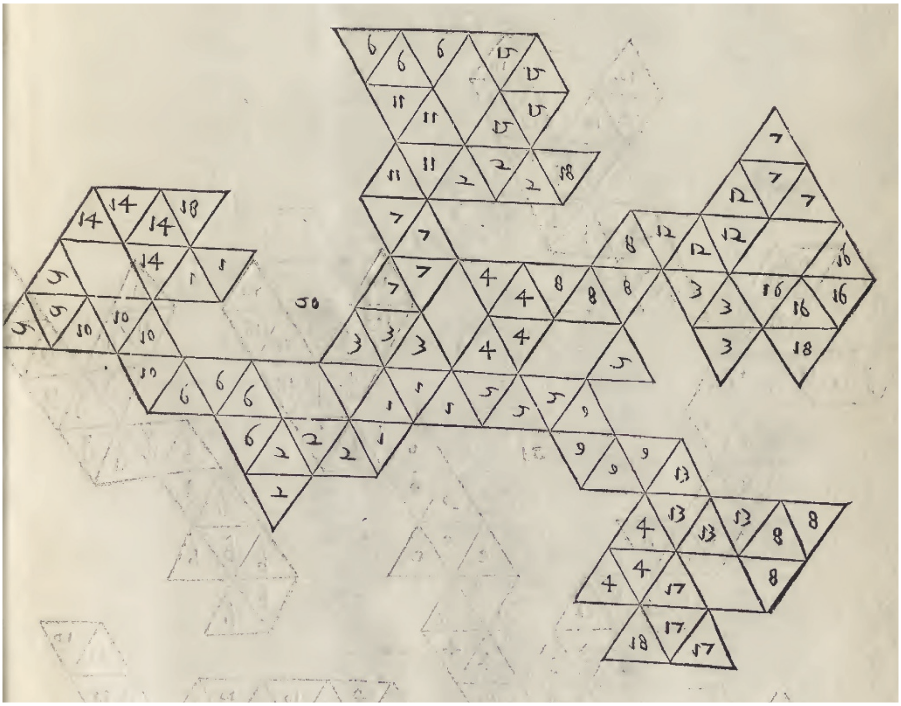
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Fig. 3.87 **[p. 109]**

[p. 110]

Chap. XXXIII. Unfolding of another body [elevated icosidodecahedron]

Putting together the unfolding shown in figure 51 (Fig. 3.88) according to its numbering, the result is a body based on a body of 20 triangles and 12 pentagons [elevated icosidodecahedron]. It is made of 120 triangles. The pentagons are marked with letters, and the triangles with numbers.

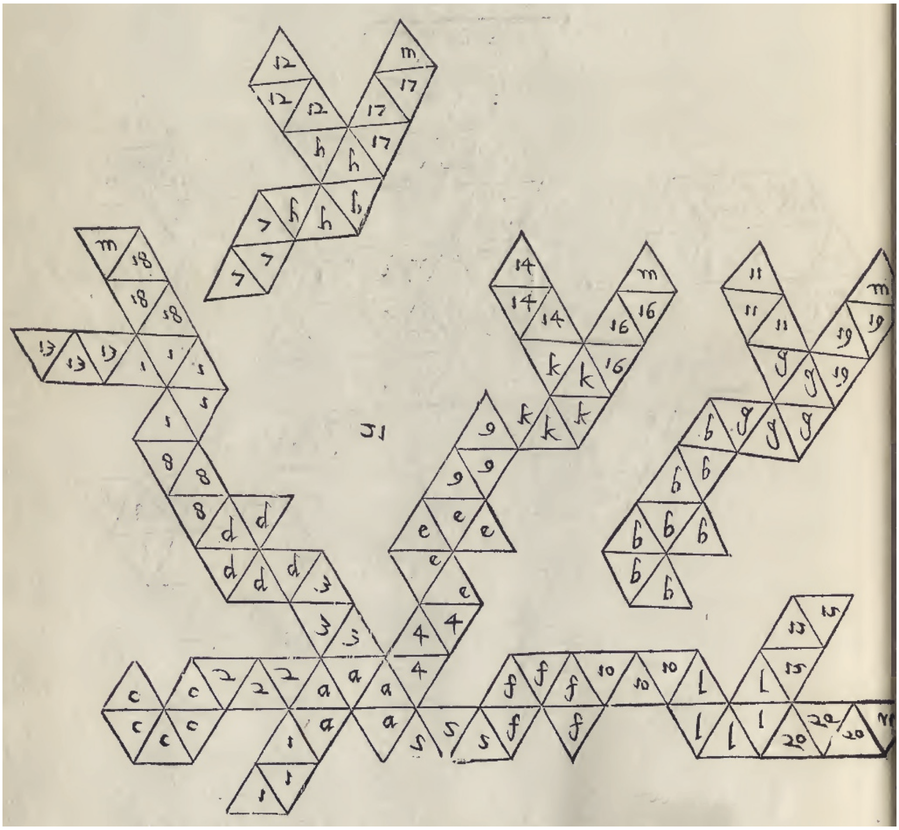
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Fig. 3.88 **[p. 110]**

[p. 111]

Chap. XXXIV. Description of some of the previous bodies with pyramids

It is possible to form many other similar bodies, such as one with 6 squares and 12 hexagons, and another with 32 hexagons and 6 squares, and another with 90 squares, 60 hexagons, 12 decagons, and 20 dodecagons, but this could go on forever, so let us leave aside the boredom of writing a lot, as it unfortunately appears to me to have done up to now. I will only set forth some of the pyramidal bodies for the notice of the reader. The first are those based on the dodecahedron, in figure 52 (Fig. 3.89), and that based on the icosahedron in figure 53 (Fig. 3.90). Then follow the others made with various intentions, as in the figures 54 (Fig. 3.91) and 55 (Fig. 3.92).

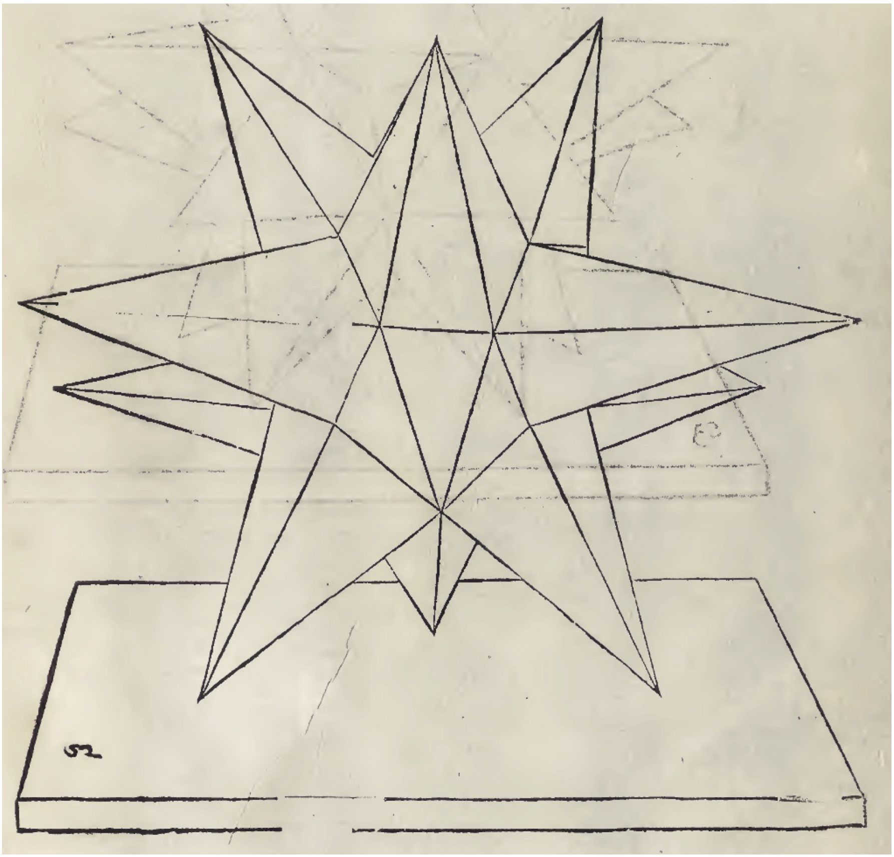

Fig. 3.89 **[p. 111]**

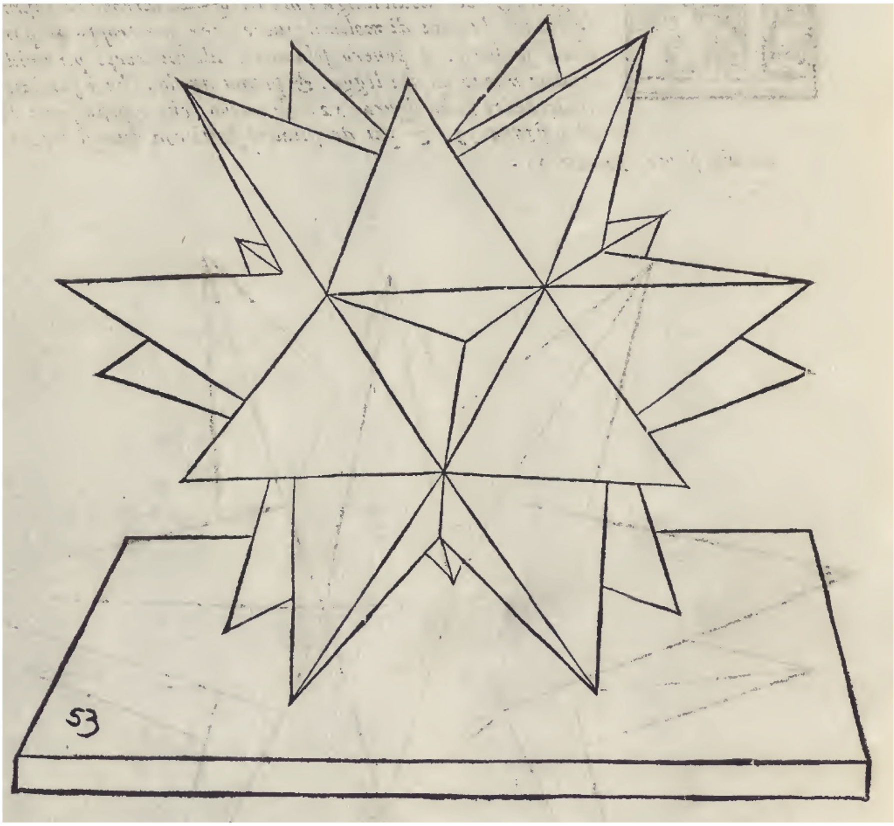

Fig. 3.90 **[p. 112]**

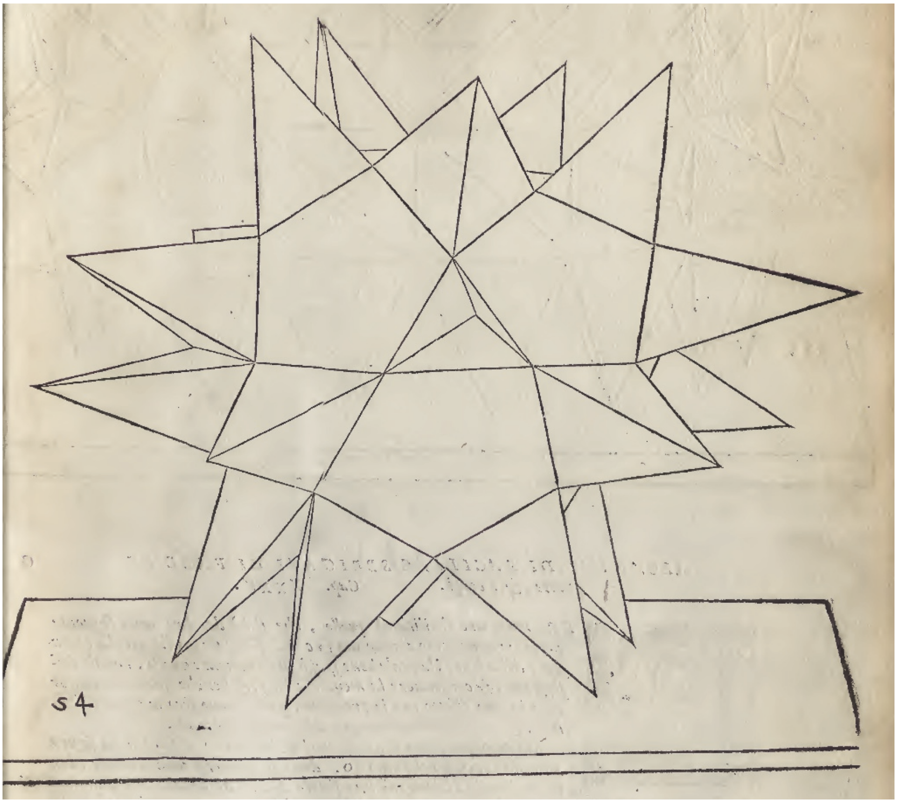

Fig. 3.91 **[p. 113]**

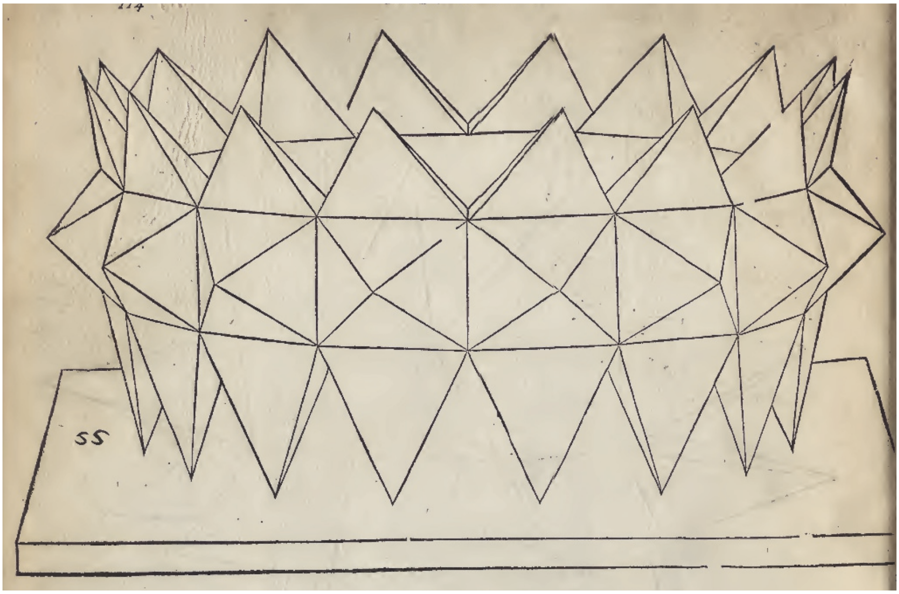

Fig. 3.92 **[p. 114]**

[p. 114]

Chap. XXXV. Some easy and quick ways to draw plans and bodies[74]

For the greater ease of that which must be said in the following part, I will demonstrate some easy and quick ways to avoid the confusion of there being so many lines, which quite often wearies those who draw and confounds their memory. Although these ways produce the same results as the previous ones, they are much easier, and consequently more delightful to work with.
So, make a square *bced*, as in figure 56 (Fig. 3.93a, above). Let the eye be at point *o*, at which a thin needle is fixed, around which will be tied a horsehair that is as long as it needs to be. Then draw a line parallel to side *bc* of the square, which will be *fg*. This line is set as the termination (*termine*) between the eye and the square. By 'termination' I mean the plane on which are to be located the points of the square and the surface described in it [i.e., the pictorial plane]. On that line *fg* place point *m* wherever you wish; in our example it is placed in the middle. This point *m* will serve us (as will be seen when practicing) in

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[74] This concluding section of Barbaro's Part III refers to Piero della Francesca's third book of *De prospectiva pingendi* (Ms. Rgg. 42/A, fol 40r for the square; fol. 41v for the octagon; fols. 42v and 45r for the concentric circles; fols. 46v, 49r and 51r for the mazzocchio).

each plan as a point of reference. Next cut some rulers of paper that are as long as *fg*, which is as long as one of the sides of the square. With a little bit of wax fit one of those rulers on line *fg* so that it doesn't move while you are working. Now stretch the horsehair over to point *b* of the square, and where it passes over the paper ruler, mark *h* [*recte b*[75]]. Then stretch it over point *c* of the square **[p. 115]**, and where it crosses the ruler, mark *c*. Do the same with points *d*, *e*, and *m*, marking the paper ruler where they go. Having located all the points of the square on the ruler, set it aside until you need it, because on it are the points of the width of the square. You will call it the ruler of widths, and you can make some kind of mark on it so in order to recognize it later.

Now it is necessary to see how much higher *de* is than *ab* on this surface *bcde*. So, place eye *o* as high is it should be to see over line *ce*, without moving it nearer or farther away from line of termination *fg* than it was in the first position. Fix the needle in the second *o*, tie the horsehair to it, and set another paper ruler over *fg*. Stretch the horsehair from the eye to point *e* of the square *bcde* in the second figure in 56 (Fig. 3.93a, below), and where it crosses the paper ruler mark points *e* and *d* in the same place. Then stretch the horsehair over point *c*, and where it crosses the paper ruler mark points *c* and *b* in the same place. Remove the paper ruler, which you will call the ruler of heights, and mark it with the letter A so that you will recognize it when you are practicing. Make another ruler identical to this line of heights with the same points *de* and *bc*; you can also do this in a single move by placing two rulers onto *fg* such that the edge of one of them touches *fg*, and the edge of the other is right next to it. Then, wanting to degrade this surface *bcde*, draw a line *fg* crosswise, the length of which is the same as *fg*, the termination, which is also the same as the side of the square. In the middle of that mark *m*. On points f and g place the two rulers A of the heights at right angles. From *m* draw a line up to point *n*, the height of which is equal to that of point *o* in figure 56 (Fig. 3.93a) over plane *ce*. Then take the ruler of widths and make the point *m* of the ruler align with the point *m* of *fg*, and raise that ruler next to the line *fg* such that in raising it to the points *b* and *c* of the ruler of heights, it is always parallel to line *fg*. So, raising it to those points *b* and *c*, mark on the plane [of representation] the points *b* and *c* from the ruler of widths. Then raising that ruler to the points *d* and *e*, mark the points *d* and *e* below the points *d* and *e* marked on the ruler of widths. Thus by drawing the lines as in the perfect, you will have degraded the plane *bcde*, as seen in figure 56 (Fig. 3.93b).[76]

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[75] This point is not shown in the figure.

[76] This correction and the correct image are noted in the Errata on an unnumbered page at the end of the book.

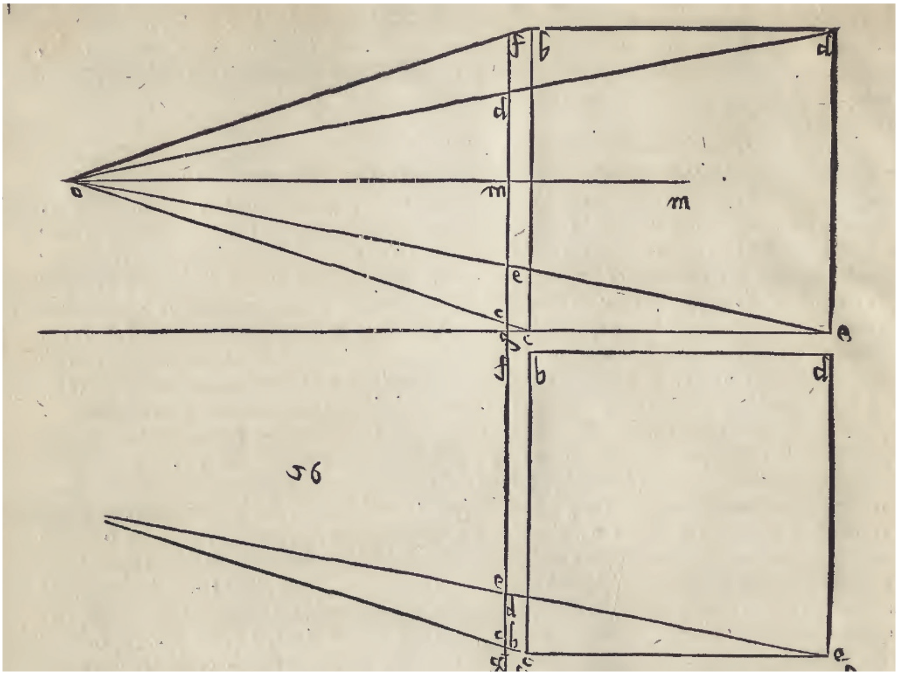
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Fig. 3.93a **[p. 116]**

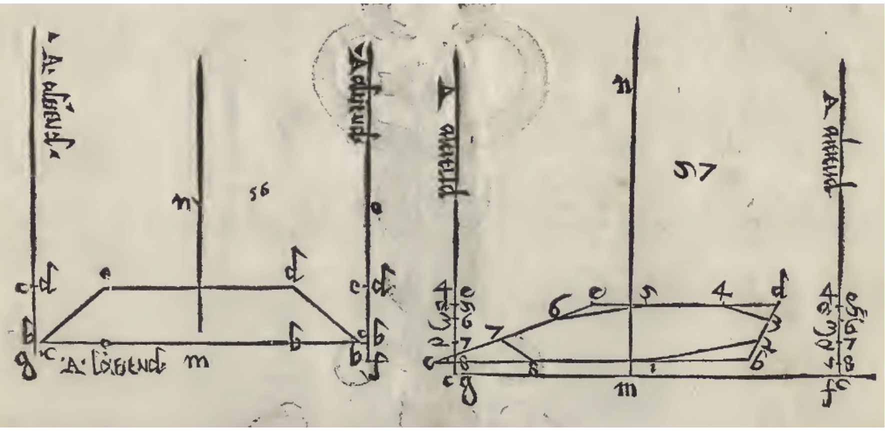
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Fig. 3.93b, from the Errata at the end of the book

If you were to ask me the reason why points *b* and *c* are marked in one place on the rulers of heights, and points *d* and *e* are similarly marked in a place on the same ruler, I would reply that this occurs on surfaces that have points that are seen straight on, since all points which are equally distant from the line of the termination and none of which lie on top of another are placed on the rulers of heights in a single point. That is, that no vertex lies on top of another, as it is understood that *b* is even with *c*, and *d* is even with *e*, and the line *pg* [*recte fg*], which is the termination, is parallel to *bc* and to *de*.

Other surfaces are degraded by means of the same rationales, so it is sufficient for us to take another example of degrading a plane of 8 faces of equal sides. So, let that plane 1-2-3-4-5-6-7-8 lie in the perfect *bcde*. Set the termination *fg* as above, and the eye at *a* [*recte o*]. Place and fix the ruler on *fg*, and having tied the horsehair to the needle, stretch it over the points of the plane one by one, and mark them on the ruler of widths, along with point *m* for alignment, as seen in the lower image in figure 57 (Fig. 3.94, bottom[77]).

Then make the plane *bcde* in a separate drawing, with that octagonal surface and raise [draw vertically] the line of the termination *fg*. On this place the rulers of heights, stretching the horsehair from the eye to the points *e*, 6, 7, and *c*, as these are sufficient for the reason stated above. Mark the rulers with these points. It will come out that 4 and 5 are marked in a single point [with *e*], 3 and 6 in another, 2 [with point 7], [1 with] 8 and *c* in another, as seen in the lower image in figure 57 (Fig. 3.94, top).

To degrade this surface, fix the rulers of heights on the points of line *fg* drawn crosswise so that they remain still, and align the ruler of widths with its point *m* over the *m* of the crosswise *fg*, raising it to the points marked in the rulers of heights, keeping it parallel to *fg*, and marking the corresponding points of the line of widths to the lines of heights. By connecting all with lines in the degraded as they are in the perfect, you will have degraded that octagonal surfaces, as seen in the degraded in figure 57 (Fig. 3.93b).

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[77] Figures 56 and 57 are combined into a single figure; 57 is not marked.

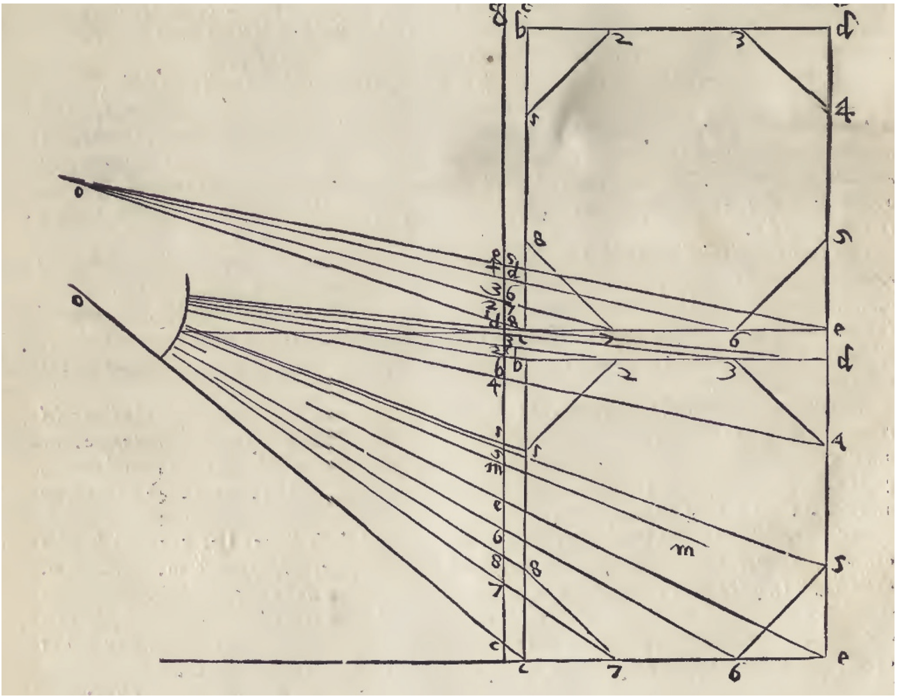

Fig. 3.94 [p. 116]

[p. 117]

Chap. XXXVI. How to degrade many circles drawn on the same centre and divided into several parts

It is necessary with the above manners to know how to degrade many circles that are made over the same centre and divided into several parts, since these manners make it easier to describe bases, capitals and other circular parts as need arises. So, whereas for degrading a simple surface one ruler for width and two for height are necessary, where there are many surfaces one within another it is necessary to increase the number of rulers for both width and height. Therefore, if we want, for example, to describe 4 circular surfaces that are one inside another, it is necessary to have 4 rulers of width and 8 for height. So, let the 4 circles *a*, *b*, *c*, and *d* made on the same centre *m* be divided into 12 parts at 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. The largest circle is *a*, and the smallest is *d*; the diameters are 10-4 and 1-7. Draw the line of the termination *kl* parallel to diameter 10-4, and place the eye wherever you want it; here it is at point *o*, with the needle and horsehair. [Starting] from *k* place the ruler of width, which is marked A, along line *kl* of the termination. Stretching the horsehair, you will mark all the points of the largest circle *a*, together with the point *m*. Then laying aside the ruler of widths, place another one, marked B, on *kl*, and stretching the horsehair you will mark on it all the points of circle *b*. The taking away ruler B, place ruler C on the same termination, and mark on it the points of circle *C* [*recte c*], and do likewise with ruler D and the points of circle *d*. In this manner you will have marked the rulers of width of all 4 circles, as seen in the first image of figure 58 (Fig. 3.95, left). Now, in another perfect draw the same circles, divided in the same manner, as seen in the second image of figure 58 (Fig. 3.95, middle). Draw the line *kl* of the termination, placing the eye in the same distance as above. Draw from point *a* of circle *a* a diameter line that passes through *m*, which will cut circle *a* in point *f* on the opposite side, and *b* in point *g*, *c* in point *h*, and *d* in point *i*.[78] Extended, it will cut line *kl* in *p*. Therefore point *a* will correspond to point 1 and point *f* to point 7. Then draw a line from the number 2 to 12, and where this crosses the diameter *af*, mark 2 and 12. Similarly, draw a line from 3 to 11, and where it cuts *af* mark 3 and 11. Similarly, where the diameter 10-4 cuts the diameter *af*, mark 10 and 4. In the same manner draw the lines 5-9 and 6-8 and mark the points at which they cross line *af*, as seen in the second image of figure 58 (Fig. 3.95, middle). Then from point *p* you will extend the line *fap* out to point *q*, which will be the distance of the eye, so that a line falls at right angles through *q* from the eye *o*. It will show the height from *q* to *o*.

Having arranged things in this way, take the rulers of heights marked A made for the largest circle and place them on *kl* at point *p*. Stretch the horsehair to point *a*, or to point 1, which is the same thing, and where the horsehair crosses the ruler A, mark 1. Then stretch the horsehair over the points 2 and 12 marked on the diameter *af*, and where it crosses the rulers mark 2 and 12. You will do the same for the points 3 and 11, 14 and 10, 5 and 9, and 6 and 8, and point 7, marking them on the rulers of heights. Having thus marked all the points of the largest circle *a*, you can degrade it in keeping with the rule of the previous chapter, setting at right angles the rulers of height A to the ends of line *kl* crosswise and raising the rulers of width A to the points of the rulers of height, and marking those on the degraded plane. With similar manners you will make the rulers of the width of the circles *b*, *c*,

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[78] These letters are not marked in the figure.

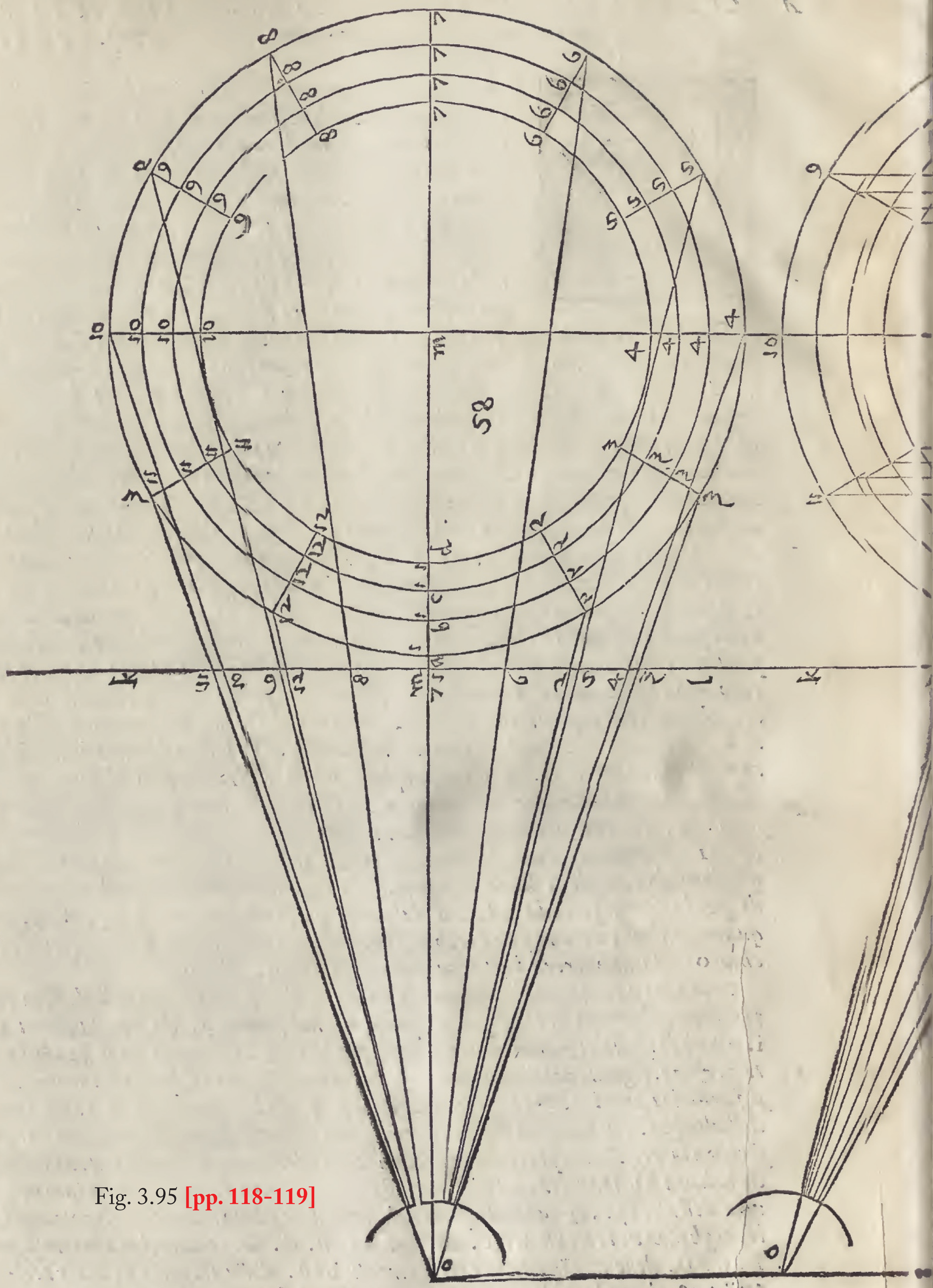
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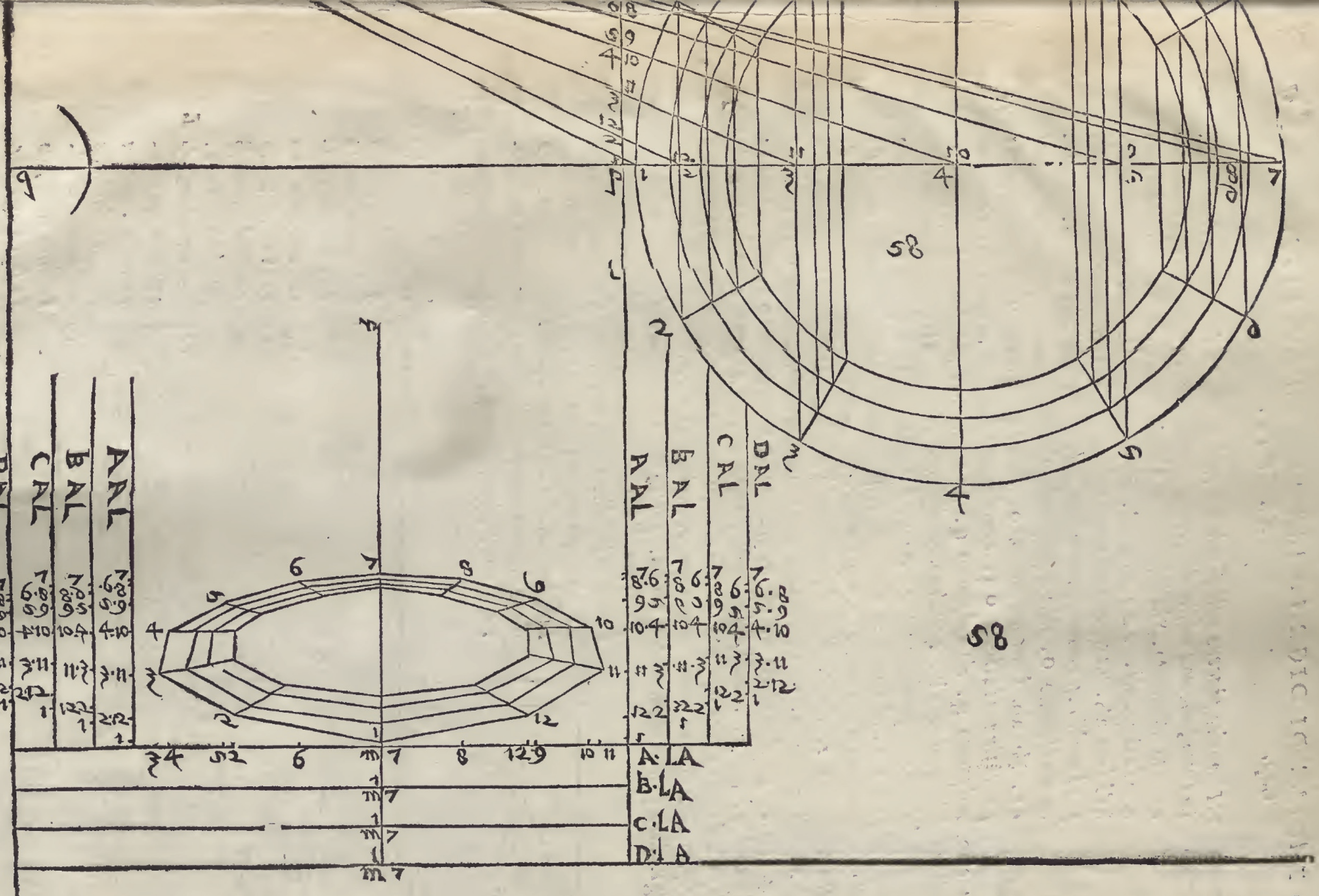
Fig. 3.95 **[pp. 118-119]**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58
A AL
B AL
C AL
D AL
A·LA
B·LA
C·LA
D·LA
AAL
BAL
CAL
DAL

and *d*, marked respectively B, C, and D, taken from the first image of figure 58, and the rulers B, C, and D of the heights taken from the second image of figure 58. Carry them to the degraded plane in the third image of figure 58 (Fig. 3.95, right). These things are easier to do in practice than to describe in writing.

[p. 120]

Chap. XXXVII. An easy way to degrade without rulers

With much pleasure it is possible to degrade without the use of rulers, and there is no difference between the following manner and the previous one in terms of its results. Let there be the perfect *abcd*, in which lies the surface of 8 equal sides 1-2-3-4-5-6-7-8. From the middle of side *cd* to the middle of side *ab* draw a line to point *o*, which will serve as the eye. Also extend the side *cb* out to point *e*, which is perpendicularly below point *o*.[79] Then extend the sides *dc* and *ab*, one to point *o*, the other to point *q*. From these points draw line *oq* parallel to side *cb*, and equal in length to line *cbe* to point *g*. From point *g* to the eye *O* draw the line *geo*. Draw also the line of the termination *klr*, parallel to side *abq*. On the surface of 8 sides draw the diagonal through points 3 and 7 out to the line *oq* in point *m*, and also draw the lines from point 2 to point 4 of the 8-sided surface, as well as from 8 to 6, and where those cut the diagonal 3-7 mark 3 [*recte* 2] and 4 on top of the line and 8 and 6 below the line, and in the middle mark 1 and 5. Then from the diagonal where the point 2-4 is marked, take the distance from this point to the vertices 2 and 4 and carry it to either side of point *m* of line *oq*, marking 4 and 6 on one side [to the left] and 2 and 8 on the other [to the right]. Also carry to line *oq* the distance between points 1 and 5, and note on the diagonal the points and vertices 1 and 5 of that surface, locating point 1 on one side [to the right] and point 5 on the other [to the left]. In this manner you will have arranged the perfect for the degradation, as seen in figure 59 (Fig. 3.96).

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[79] It is confusing to have point *O* as the eye, and a second point *o* on line *qg*. In any case, the corresponding figure in the Italian menabò manuscript (It. IV, 40=5447, fol. 65r) has both points marked *o*, so to distinguish them in this exercise we have marked the eye as *O* and the point on line *qg* as *o*.

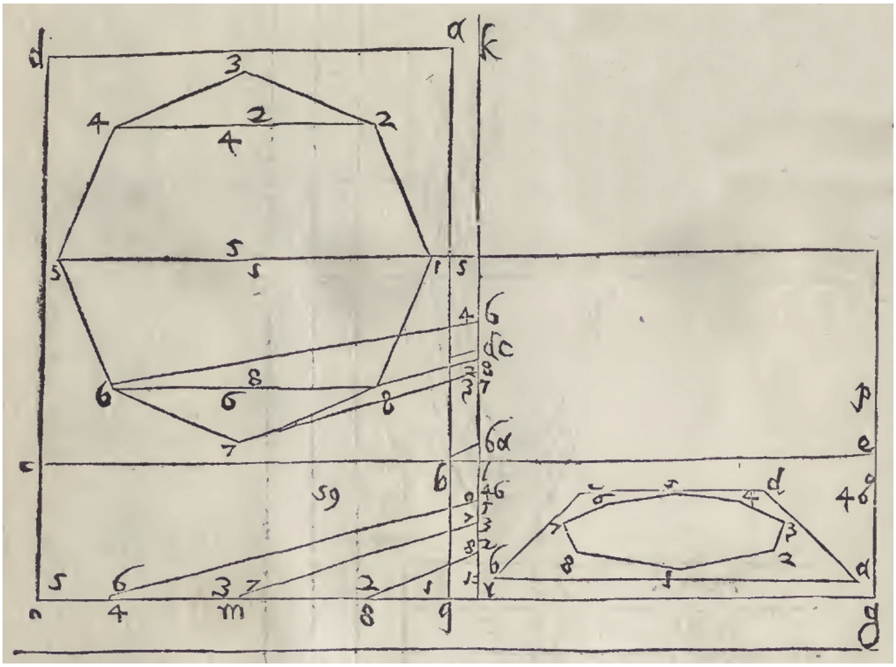

Fig. 3.96[80] **[p. 120]**

Now, first the square *abcd* in the perfect must be degraded, and then the surface of 8 sides that lies in it. So set up two good, well-made compasses, because the making lies in the exactitude of the instruments. One will be for the points of the width, and the other for the points of the height, so they should be distinguished by some kind of sign so that in working you do not take one instead of the other. So, placing the horsehair in eye *O*, stretch it to the point *b* of the square, and with the compass of width measure the space that is from point *r* to the cut that the horsehair makes on line *kr* of the termination, and keep that opening. Then measure the distance from point *e* to the eye *O* and place it over *g* towards *O*, at point *p*. That point *p* will be the eye for the length [*recte* height]. Stretch the horsehair from point *p* to point *q*, and where it cuts the line of the termination [*kr*] make a point, and take the compass of height put it in point *r*. Measure the space between *r* and the [last] point marked on the termination and carry it over *g* towards *p*. Placing the ruler over those points, one on the line of the termination and the other on line *gp*, take the compass of the width, which was kept open, and carry that **[p. 121]** space of the opening of the compass into the surface that you are using for degradation, and mark *a* to the left and *b* to the right. Then stretch the horsehair from the eye *O* to the point *c* in the perfect, and placing the compass of width on 1, measure the space from point 1 to the cut that it makes on the termination, and keep the compass at that opening. Then stretch the horsehair from point *p* to point *o* and with the compass of height placed on *r* out to the cut that the horsehair makes on the termination, carry it onto *g*, on the line *gp*. Placing the ruler on the points – one on the termination, the other on line *gp* – with the compass of width placed on those points mark *c* on one side and *d* on the other along the ruler. Then connecting the points *a*, *b*, *c* and *d*, you will have degraded the perfect [of the square].

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[80] In the Errata on an unnumbered page at the end of the book is the following remark: In the figure it is necessary to draw a line from 3 to 7 out to *m*.

If you want to degrade the surface of 8 sides, stretch the horsehair from the eye *O* to point 7, and where it cuts the termination make a point. Placing the compass of width on 1, measure the space from point 1 to that point, and keep the compass open like that; it will serve to carry two points – that is, 3 and 7 – onto the degraded. Then stretch the horsehair from point *p* to points marked 3 and 7 on the line *op*, and where it cuts the line of the termination make a point. Carry the space between *r* and that point onto line *gp*. Placing the ruler on these two points, carry these points with the compass of width along the ruler, marking 3 on one side and 7 on the other. Then with the compass of width measure the points 8 and 2 made by the horsehair on the line of the termination stretched from point *O* to point 8 on the perfect and keep that compass opening. Stretching the horsehair from *p* to the point marked 2 and 8 on line *oq*, measure the space from point *r* to where the horsehair cuts the termination and carry it onto line *gp*. Placing the ruler over the two points, carry it with the compass of width placed over the two points, marking 2 on one side and 8 on the other. Then with the compass of width, measure the distance from the point *l* to point *p* and carry it from point *r* towards point *g*; this will arrive to the midpoint of line *rg*. So at that midpoint draw an occult line upwards on which are placed points 1 and 5. This is done by stretching the horsehair from point *p* to the points 1 and 5 on line *oq* and measuring the space from *r* to where the horsehair cuts line *kr*, and carrying it onto line *gp*. Then setting the ruler over those points, look to see where the ruler cuts the occult line, and mark 5 above and 1 below. If in the degraded square you connect the points of the octagon, you will have degraded both the square and the octagon drawn inside it, as seen in figure 59 (Fig. 3.96).

Chap. XXXVIII. Description of the *torchio*, or mazzocchio

Extremely difficult is the description of the mazzocchio, or *torchio* as it is called, but with all its difficulties it is delightful to discover the way to make it and to understand its plan and its perfect, because it provides us with the occasion to make many handsome shapes, and makes the rest of perspective easy for us. This is a body made either of circular surfaces or those of many angles, like a garland, and as I understand it, it was used as an ornament for women's heads. So, let there be a surface of 8 equal sides of whatever height that you want the body to be; that will be 13-1, 1-13, *an*, *na*, of which sides 13*a* are one on top and one below. Through the centre of this surface passes line *CD*, of which take whatever part you wish for the width, that is, for the largest diameter of the mazzocchio. Divide it into two equal parts by the line *AB*, which goes below line *CD* and is extended above it as needs be. Then on line *CD* make another surface of 8 sides similar to the first and at the same distance from line *AB*, and let that be 24-12, 12-24, *m*&, &*m*. Draw the lines 1-12 above and below the line *CD*. These will pass through the points *n* and &. Similarly draw the lines 13-24, which will pass through the points *a* and *m* [above and below line *CD*]. In this way will be formed the height of the mazzocchio. Then draw the line 13-24, making it the diameter of a semicircle whose centre is point *A* of the line *AB*. Also draw the line *am* as the diameter of another semicircle which is likewise drawn over centre *A*. Then draw line 1-12 as the diameter of another semicircle which you will draw over point *A*. Finally the line *n*& **[p. 122]** will give you a diameter according to which on *A* you will make another semicircle. These 4 semicircles, which must be understood as whole circles, are the planes of the width of the mazzocchio. We must thoroughly understand how this body rests in order to degrade it. So, since 4 lines are described in the surfaces of 8 sides, so too are described 4 semicircles in the width, and since in the height over each line rests another one, so too in the width each semicircle signifies two. Therefore, think of this body as resting on the plane with that semicircle whose diameter is the line 13-24. Similarly in the same plane rests the semicircle whose diameter is *am*, so this diameter *am* is on the same line 13-24. Then the body rising to the first plane and widening, the semicircle that is born from the diameter 1-2 will be located in the second plane and in the first height, in which plane and at which height will also be located the semicircle that is born of diameter *n*&. And since the second height has the same width as the first, the semicircle that is born from diameter 1-12 will serve for the same width but on the plane above. And since the diameter *n*& that is above is in the same plane, so the semicircle that is born from that diameter *n*& will serve us for the width. Finally, since this body has its upper surfaces similar to its lower ones, the semicircle that is born from diameter 13-24 on top and the semicircle that is born from diameter *am* on top serve for the width of the third height, which is the fourth plane and the upper surface of this body. So, through the points *C* and *D* draw lines at right angles up to the diameters of the semicircles to the corresponding points *C* and *D* and down as far as necessary to points *I* and *A* [*recte H*], which will thus enclose a quadrangular surface in points *C*, *D*, *I* and *H*. Then carry the distance *AD* from point *D* above towards the point *I* to point ✠ and from point *C* towards point *H* in point *O*, and by connecting the points ✠ and *O* with a line you will have half of the perfect *D*✠*OC*. Divide the semicircles into 12 equal parts such that the half of one part is near one end of the diameter and the other half is near the other end, and that one whole part corresponds to the middle, as seen in figure 60 (Fig. 3.97).

In the semicircle 1-12 mark the points 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11; in the semicircle 13-24 mark the points 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, and 23; in the semicircle *am* mark the points *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, and *l*; and in the semicircle *n*& mark the points *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *y*, *z* and &. With these points, we have made the plan of the mazzocchio in the perfect.

Now it is necessary to carry all of the points of the semicircles onto their diameters located in the heights. This is done easily in the following manner. Carry onto diameter 13-24 the points of the semicircle 13-24 in this way: place the ruler on points 14 and 23 of the semicircle, and taking the distance of the diameter *AB* at those points, carry it onto the diameter 13-24, putting the compass where line *AB* cuts that diameter, locating 14 on one side and 23 on the other. In a similar fashion you will carry points 15 and 22, 16 and 21, 17 and 20, and 18 and 19 onto diameter 13-24. On this same diameter are also marked the points of the semicircle *am*, since they are in the same plane. In order not to be tedious in something so easy, use the same rationale to carry the points of the semicircles 1-12 and *n*& onto the diameters 1-12 and *n*&, as is seen clearly in the drawing of the height. To finish the preparation of the plans, extend the line 13-24 down to points ✠ 2° [the second ✠] and *O* 2° [the second *O*],[81] and draw the line *EFG*[82] of the termination, parallel to the line *CO*-*O* 2°, which will be next to that line and of equal length.

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[81] Today we would say ✠' and O'.

[82] In Fig. 3.96 the point is marked *h* instead of *G*. In the Errata on an unnumbered page at the end of the book is the laconic remark: 'Make on your own some letters in the figure and in the text'.

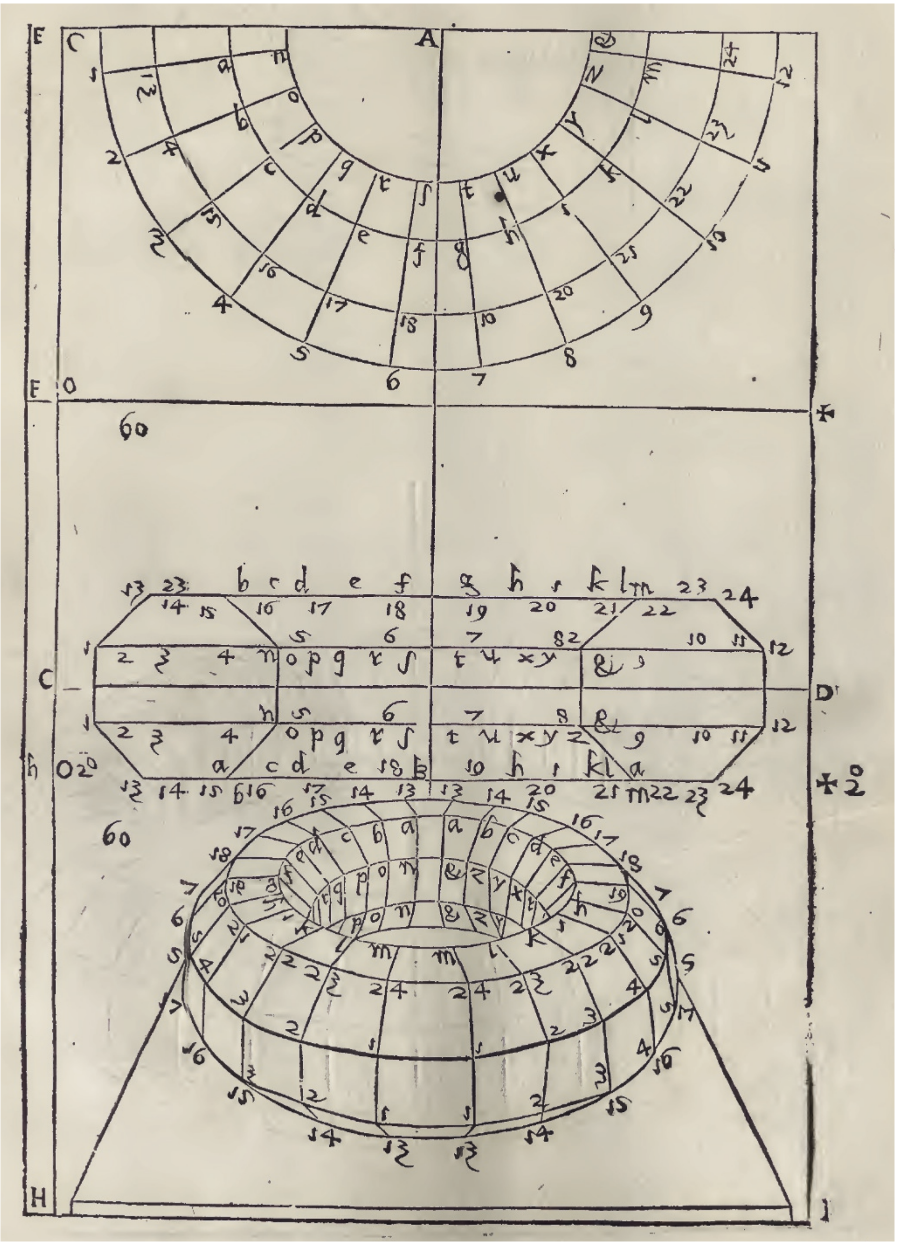
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Fig. 3.97 **[p. 123]**

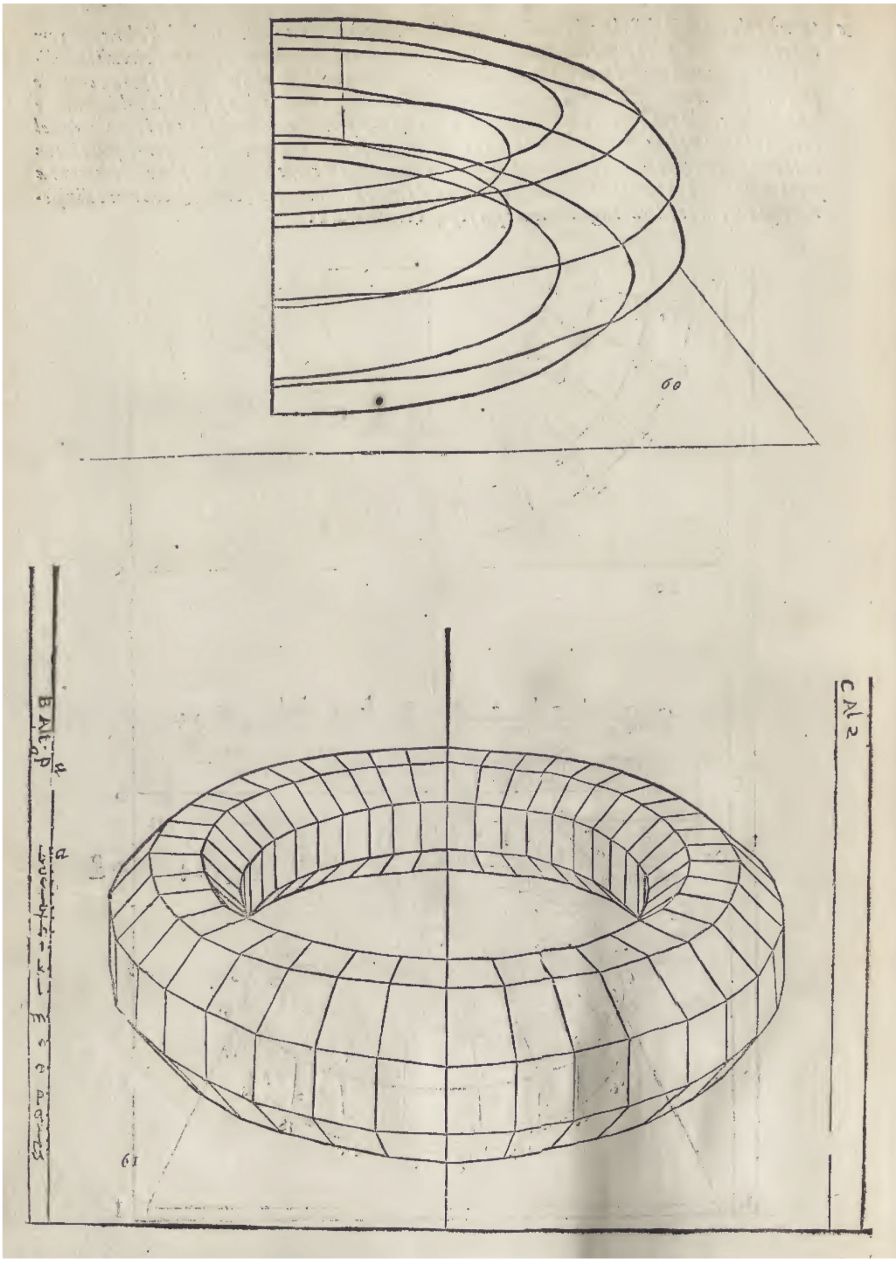

Fig. 3.98 [p. 124]

Now to degrade the mazzocchio it is necessary to begin to degrade the semicircle whose diameter is 13-24 below, but because I do not want to confuse the degraded plane with a multitude of points, I will begin from the upper plane of the diameter 13-24, and will then go on by locating the points of the line *am*, since the circles of those diameters are seen in their entirety in the degradation and shadowing of the mazzocchio. So, to take the width of the points of the semicircle 13-24, I will take the compass of widths, and stretching the horsehair from the eye to point 13, I look to see where the horsehair crosses the line *EF* of the termination. Then placing the compass at *E*, I take the space between *E* and where the horsehair cuts the line *EF*, and I set that aside until I need it. Then I will come to the diameter 13-24, and I take the difference between point *G* [*recte h*] to where the horsehair cuts the line *FG* [*recte Fh*], and carry that onto the points *H* and *I*. Placing the ruler parallel to the line *HI*, on those points **[p. 123]** I will locate those points with the compass of widths, marking 13 on one side and 13 on the other, since each of the semicircles is valid for two, owing to the fact that the semicircles stand for whole circles. With similar procedures are located all the other points of the semicircle 13-24 as well as the other semicircles, as seen in figure 60 (Fig. 3.97). I have no doubt that what has been said will be understood by those who have understood the practices of the bodies discussed previously. This way of the compass and ruler is the quickest and easiest that I know of. So that greater pleasure can be had from understanding not only the simple lines of the mazzocchio but the entire body, as well as other forms that are born from the same plans, I have degraded the figures 61 and 62 (Figs. 3.98 and 3.99) at a larger scale and with other divisions, which can be easily understood by simply looking at them.

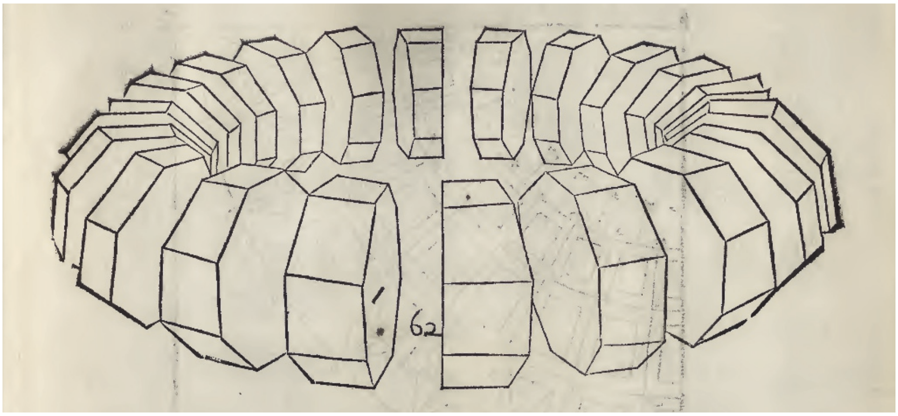

Fig. 3.99 **[p. 125]**

[p. 125]

Chap. XXIX. The perfect description of the mazzocchio[83]

Starting from the perfect of the mazzocchio, the next step is to place some pyramids or points among the squares of the planes, as these make a fine sight. These pyramids are usually set into the squares [intagliato], and they are also usually set superficially. I will set forth the superficial manner and leave to the scholars to manner of setting the pyramids into the thickness of the body.

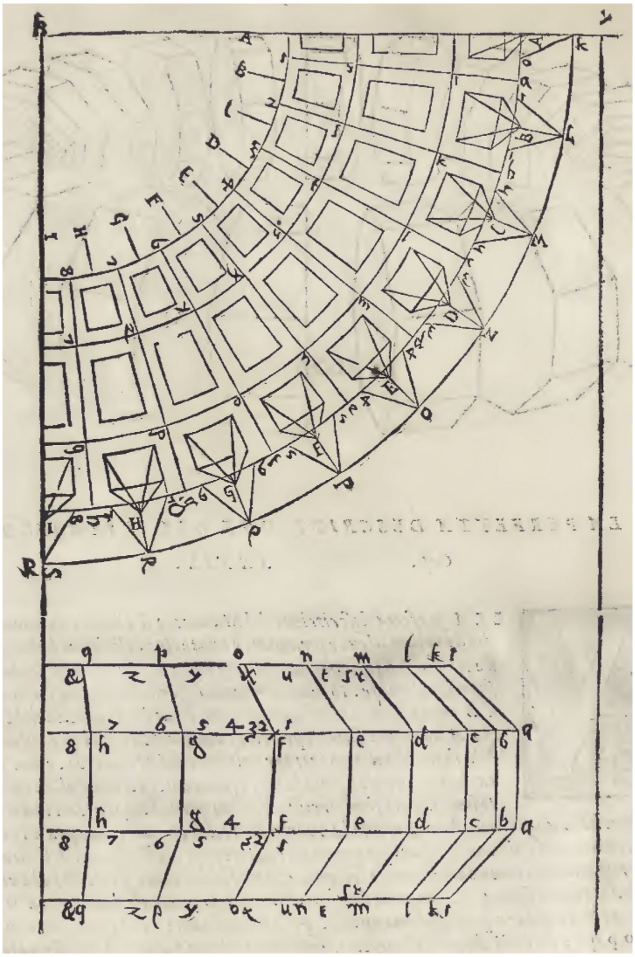

Fig. 3.100 [p. 126]

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[83] A better title would be ‘Elaborating the mazzocchio’.

So, let the square [*recte* quadrant] be made as in the previous chapter, making first of the surface of 8 sides for the height of the mazzocchio, as seen in figure O (Fig. 3.100, bottom), in which is placed only the half, since the other half is completely similar, and then the semicircles, which are divided into 16 parts, although we have only shown the fourth of the circle, to which all the others are similar, as in figure P (Fig. 3.100, top). Carry these parts into the figure of the heights onto the corresponding diameters, as the letters show, noting (as said) that each turn of the quadrant in the figure P counts for two, and each diameter line in figure O is for two, as will be clear to anyone who thinks about it. So the quadrant *I-I*, divided with the letters *i*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, is meant double in the perfect. Likewise the quadrant *Z-Z*, marked with letters [*a*,] *b*, *c*, *d*, *e*, *f*, *g*, and *h*, the quadrant 3-3, marked with the letters *r*, *s*, *t*, *u*, *x*, *y*, *z*, and 7, and finally quadrant 4-4, marked with the numbers 1, 2, 3, 4, 5, 6, 7, and 8, are all double in the plan such that if this body were flattened and set on the plane, the circles above would meet with those below, as we imagine in the figure P; for the same reason in the figure O of the height, the semi-diameter *ZB* is placed two times, and on both is marked the semi-diameter 4-4 with its parts 1, 2, 3, 4, 5, 6, 7, and 8. The semi-diameter is placed two times, one above and the other one below, and both have double divisions, one of the semi-diameter *IB*, and the other of the semi-diameter 3*B*, with the letters *r*, *s*, *t*, *u*, *x*, *y*, *z*, and 7, as seen in the figure O, exactly as was demonstrated in the previous chapter. **[p. 126]** Now it is necessary to place in the plan and in the perfect the squares and points of the pyramids, and understand clearly how they go. So, the squares are placed in this manner. Take as much space as you want for the width of the squares on the semicircle, or quadrant, *z*2 [*recte Z-Z*], between the points *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h*, such that between points *a* and *b* are points 1 and 1, between point *b* and *c* are points 2 and 2, and continuing on, between points *c* and *d* are points 3 and 3, between points *d* and *e* are points 4 and 4, between points *e* and *f* are *f* and *f* [*recte* 5 and 5], between points *f* and *g* are 6 and 6, and between points *g* and *h* the points 7 and 7. Let *o* be near *A* above, and 8[84] be near *I* below. All of these points marked by numbers are equally distant from the points marked by letters. So, draw occult lines to the centre *B*; these **[p. 127]** will serve for the two sides of all the squares that go around the semicircle. If you want to close these squares, you will draw, with spaces as large as you want, the occult quadrants which cut the occult lines that go from the points marked with numbers to the centre *B*. In this manner you will have placed in the plan P all the squares on which are fixed the pyramids. These you will carry into the figure of the heights, marked L (Fig. 3.101), in the same way that you carried all the points of the divisions of the semicircles in figure O, which will be manifest to you by the agreements of the letters and numbers marked in plan P and figure O. Here it is necessary for firm imagination and knowledge of the plan to serve you. I have placed in diverse figures the whole in order to clear up any confusion, always presupposing previous figures with their points. Up to here you have the plan and the figure of the widths and simple heights with its divisions, and the height with the division of the squares, as seen in figure L.

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[84] The printed text at this point includes the letter B, a printing error which does not appear in the Italian menabò manuscript (It. IV=5447, fol. 69r). It does not therefore appear in the translation.

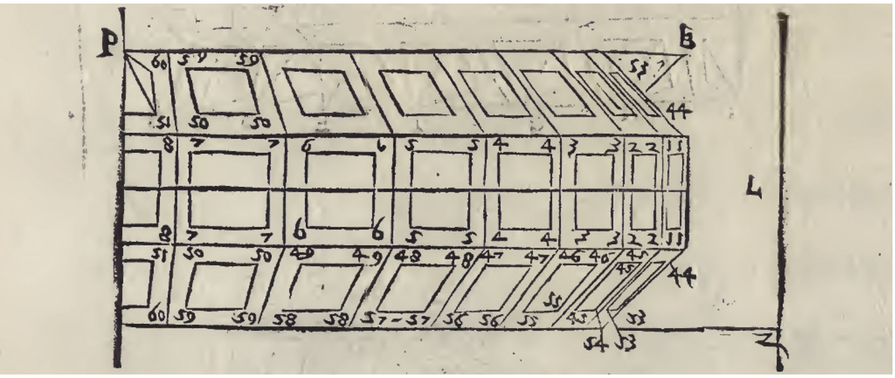
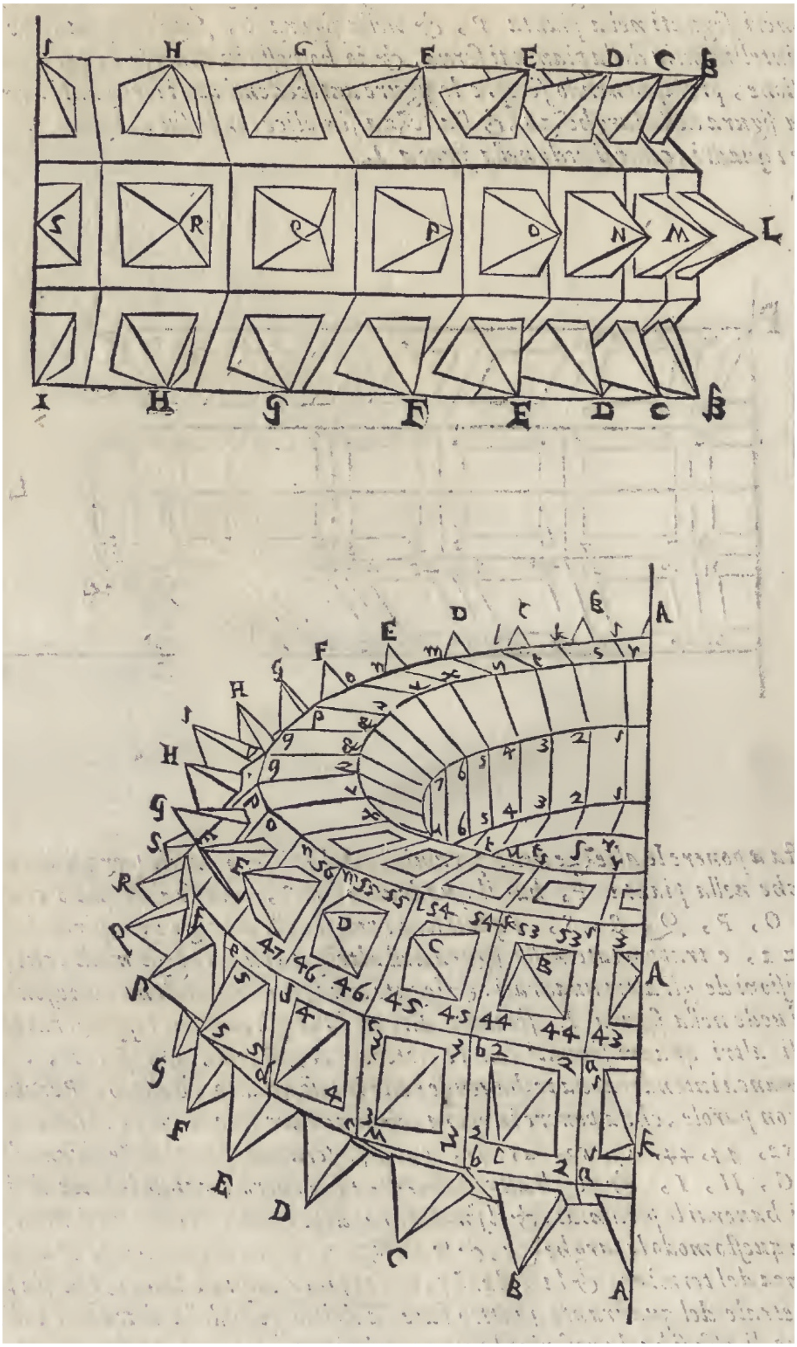


Fig. 3.101 **[p. 127]**

Now it remains to us to set the heights of the pyramids, which will be easy for you thanks to the agreement of the letters. So in plan P you have the quadrant *KS* divided and marked with the points *K*, *L*, *M*, *N*, *O*, *P*, *Q*, *R* and *S*, which are the points of the heights of the pyramids made in quadrant *z*2, and carried into the figure of the heights with the same procedures used to carry the divisions of the other squares, and connected with the corresponding points that are in plan P, as seen in the second figure L (Fig. 3.102). With the same rationale you will carry the points of the pyramids of the other quadrants marked with the letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* and *I*. What is missing you will understand on your own, having well understood what has been said. It is more difficult to explicate with words than it is to put it into in practice and make the body. Now these points and the squares 43-52, 44-44-53-53, 45-45-54-54 [etc.] will be the points of the pyramids *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* and *I*, and the others in their order. So by drawing the lines of these numbers to the points you will have the pyramids. You'll do the same with the rest, as seen in figure M (Fig. 3.102).


I
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1
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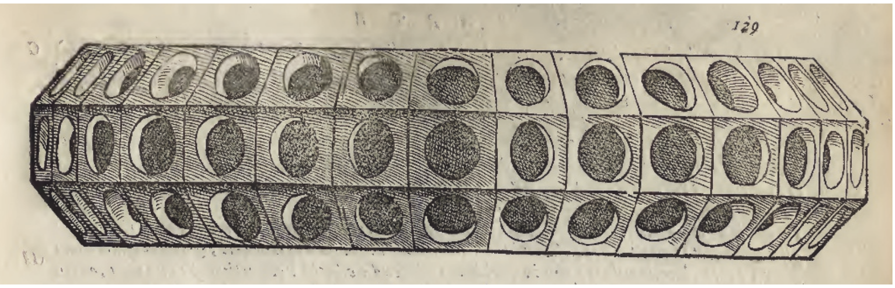
6
7

Fig. 3.102 **[p. 128]**

Having ordered in this manner the widths and heights, it remains to you to degrade the mazzocchio, which you will do by setting the line of the termination and the distance that you like; I think it is best when this distance is equal to the diameter line of the quadrant taken 4 times, but drawn orthogonally to this diameter line and to the point and end of it must be referred to the line of the plane *BI* of the figure M. Using the horsehair and two compasses you will happily make short work of it. To avoid confusion, first locate the faces that hide the others either partially or altogether, and thus you will **[p. 128]** avoid placing things that are superfluous and won't hinder yourself with many lines that are not necessary.

What has been said regarding the degradation of plans and the raising of bodies, and the different manners of degrading will suffice for present needs. Let us conclude the Part III.



Part IV, in which will be treated *scenographia*, that is, the description of scenes

Chap. I. The manners of scenes

[p. 129] I hope that the difficulty of the things before this will make *scenographia* appear easy, for all that I have said in the three previous parts will be of use for it. Even so, it will be necessary to point out many things, as will be seen in their proper places. Therefore, to that aim it is fitting to do that which Vitruvius says in the fifth book of architecture: I will adduce the steps necessary to the understanding of this part, reiterating my opinion of that term that Vitruvius places in the first book in chapter 2, where he speaks of the ideas of disposition,[1] and says that the third idea, or species, or manner, of disposition is called *sciographia*, which some understand to mean perspective, confirming their opinion with that part of the eighth chapter of the fifth book, where there are found three manners of scenes, or three sorts of apparatuses and appearances, according to the three manners of subjects that are usually represented in the scenes.[2] These treating the things and actions of men, some of whom are of a condition that is high and signal, others of one that is mediocre, and others still of one that is low, they would have it that for the important actions, in which are involved persons who are great and of a high state, be made an apparatus of buildings that are sumptuous and a magnificent and royal perspective. Together with Vitruvius they give this appearance to the tragedies, and this set-up is called the 'Tragic scene'.

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[1] Cfr. Barbaro (1567, pp. 29-30), Williams (2019: 60-61).

[2] Cfr. Barbaro (1567, p. 256), Williams (2019: 420-421).

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0_7

Where there are domestic dealings among persons of mediocre and ordinary conditions, they make another sort of scene, that is called 'Comic', and this is the apparatus for the comedies, which are actions of private persons. To the base, coarse and simple persons, such as shepherds and other inhabitants of country houses (*ville*), for what occurs among them is made a display of towns, trees, water, and rustic dwellings. The display represented in that scene is called 'Satiric'. Having thus divided into three manners all of the apparatuses of the fables, they saw that perspective was very necessary to the architect, and thus have interpreted the word *sciographia* as perspective, which is like a shading (*adombratione*).

[p. 130] Many others have read *scenographia* in place of *sciographia*, and have understood the same thing, that is, the description of the scenes, which admirably require the use of perspective. Thus the high palaces and handsome loggias, the magnificent buildings, the sumptuous arches and military streets that are painted in the Tragic scenes, the private dwellings, the taverns, the alleyways and lanes that are given to the Comic scenes, and the faraway towns, the retreating waters, the woods and hovels and sheds of the shepherds that are found in the Satiric scenes, all require the suitable point of view in the horizon to regulate that which is seen on those faces. From these things are born the projections, the narrowings, the striking of light and shadows, the entrances and exits of the sides and the members of the buildings, both near and far, and the crossing of the rays and the ratio of the angles under which they are seen, according to the appropriate variety of the aspects. Thus considering this part, they would have it that here it is understood that perspective is an idea, or species, of disposition, to which they accommodate the definition of *sciographia* set out by Vitruvius. This understanding is not satisfying to me, because it is necessary that the species of disposition placed within its genus have among themselves a certain similarity, in which as a species they concur within its genus. The plan, called *ichnographia*, and the elevation, called *orthographia*, which are species of disposition, concur in the nature of disposition, such that what is born and what grows are the same thing, and they participate in the nature of its genus, as all the species of animals have the nature of their genus, which is animal. But we would wish to adduce perspective within the genus of disposition, which it doesn't have anything to do with. I am speaking now of disposition as Vitruvius means it, which must be found in every work of the architect, be it in buildings, or machines, or any other object. So I understand the profile to be one of the ideas of disposition dictated by Vitruvius, as something that concurs with other species within disposition, leads to a great amount of cognition of the quality and measures of the buildings, and aids the accounting of costs, and the ordering of the thicknesses of the walls. If Vitruvius had not placed the profile there, he would have erred greatly, both because he would have omitted a necessary species, and because he would have included one that did not participate in the nature of its genus.

Be that matter as it may, Vitruvius sets out three manners of scenes in the fifth book: Tragic, Comic and Satiric. On the other hand, it is true that perspective is necessary to make the scenes, and it is a work of fine judgement to place the point appropriately so that all that is seen painted will represent a site and such a naturalness of things that nothing is forced, or precipitous, or deformed and unseemly, as is seen in the scenes of many: the houses way too small, the buildings that spill over, the foreshortenings drawn to such a low point that neither from close up nor far away can they be seen with delight, in addition to the falsity of many parts that the eye can't bear to see. Many ingenious designers dealing with this place the point behind the scene, that is, behind the panel or canvas or plane that is painted. So it might well be that the thing seen is placed now in front of the plane or termination where it is painted, and now behind, and now on the same plane, as Commandino has learnedly demonstrated.[3] So resorting to the precepts given in Part I, one can with reason remedy every defect. Nor do I wish to reprise that which is demonstrated in Vitruvius regarding the compartition and disposition of the theatres of the Greeks and the Latins, both to avoid annoying those who are reading, and because it is not my intention to describe theatres. I will only speak about the scenes, and about the refined findings of the ancients, such as how, without losing any time, they passed from a tragedy to a comedy, and after to comedy to a satire, and finally from any kind of fable pass to another with its appropriate scene. So, in the niches and holes and openings of the fixed scenes there turn some wooden machines that have three faces, on each of which were the perspectives of the fables that had to be represented.[4] So, on one of the three faces was the Tragic adornment, on another the Comic and on the third the Satiric. From this it came about that, wanting to recite a tragic tale, they turned towards the pulpit and the spectators those faces on which were painted the buildings of the Tragic scene. I say 'those faces', because there were three niches or holes, one in the middle and one per side, the one in the middle being the larger, and in each of them was placed one of these triangular machines. If they wanted a comedy, with little effort they turned towards the spectators the face which had the Comic ornaments, and **[p. 131]** they did likewise when they wanted to recite the satire and rustic things. Here it is necessary to point out that all of the faces of the machines placed in the three openings, or niches, corresponded to one point; that is, that the three faces of the Tragic scene were regulated by a single point, and the three of the Comic scene by a single point, and the three of the Satiric scene by a single point. In this way the three faces composed a single aspect and perspective, as the rationale demands, so that there is nothing false. Now I will describe the parts of the buildings one by one, such as the bases, capitals, columns, architraves, and then I will show how to put them together and to form all three sorts of scenes.

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[3] Federico Commandino, *Ptolemaei Planisphaerium* (1558); see for example the figure on p. 15r.

[4] The placement of the 'triangular machines' in the Roman theatre is shown in Barbaro (1567, p. 249), Williams (2019, p. 411).

Chap. II. Description and shading of the Tuscan base

Nothing (as I have said) can be placed in a degraded plane unless it is first had in perfect form, so when we wish to degrade the members and parts of buildings, it is necessary to know their measures. To begin, therefore, with the Tuscan base,[5] I say that it is as high as half of the thickness of the column at its foot, although some say its head. As can be seen in figure A (Fig. 4.19), the space from point *e* to point *x* is half of the diameter, that is, the thickness of the column. That measure will thus be the height of the base, *da*. Let space *da* be divided into 8 equal parts marked with points 1, 2, 3, 4, 5, 6, and 7, and 4 of these are given to the *orlo*, called the plinth, which in the Tuscan base is circular, but in the other bases is square. Three of those spaces are given to the *bastone*, called the torus, and one is given to the cimbia, called the apophyge, or *listello*, which in the Tuscan base is part of the base but elsewhere part of the column. These are the terms of the height of the base and its parts.

The lengths are these. The diameter of the plinth is two parts more of the six into which the diameter of the column is divided, as from point 6 to point *c*. The semicircle of the torus projects out to the outer edge of the plinth; its centre is shown in the figure as a point, which is the limit of the projection of the cimbia or *listello* above it. The curve that is made at the foot of the column [the apophyge] must be made gracefully. The shaft of the column thus comes on top of the apophyge at point *g*, and dividing the space from point *g* at the end of the apophyge into two equal parts, and adding one of those towards point *h*, and carrying space *gh* over the line of the shaft of the column in point *f*, there you shall place the compass and draw an arc. Then placing the compass at point *h*, you will cross that arc at point *f*. Then placing the compass at point *f*, with the same opening you will make the curve at the foot of the column, as shown in figure A (Fig. 4.1). In that same figure is half of the plan of that base. The perfect form is shown in figure B (Fig. 4.2), which is very easily understood since, being similar to the description above, it does not require an explanation. The degraded and shaded form of this base are shown in figure γ (Fig. 4.3).

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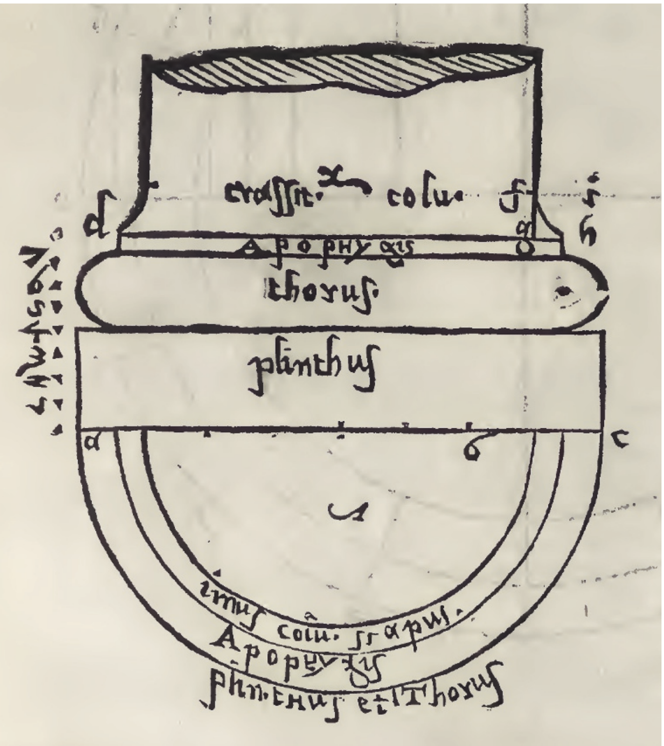
[5] Cfr. Barbaro (1567, p. 142 and figure p. 143), Williams (2019, pp. 244-246).


crassit. colu.
d
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g
h
Apophygis
thorus
plinthus
7 6 5 4 3 2 1
a
b
c
imus colu. scapus
Apophygis
plinthus et Thorus

Fig. 4.1 **[p. 131]**

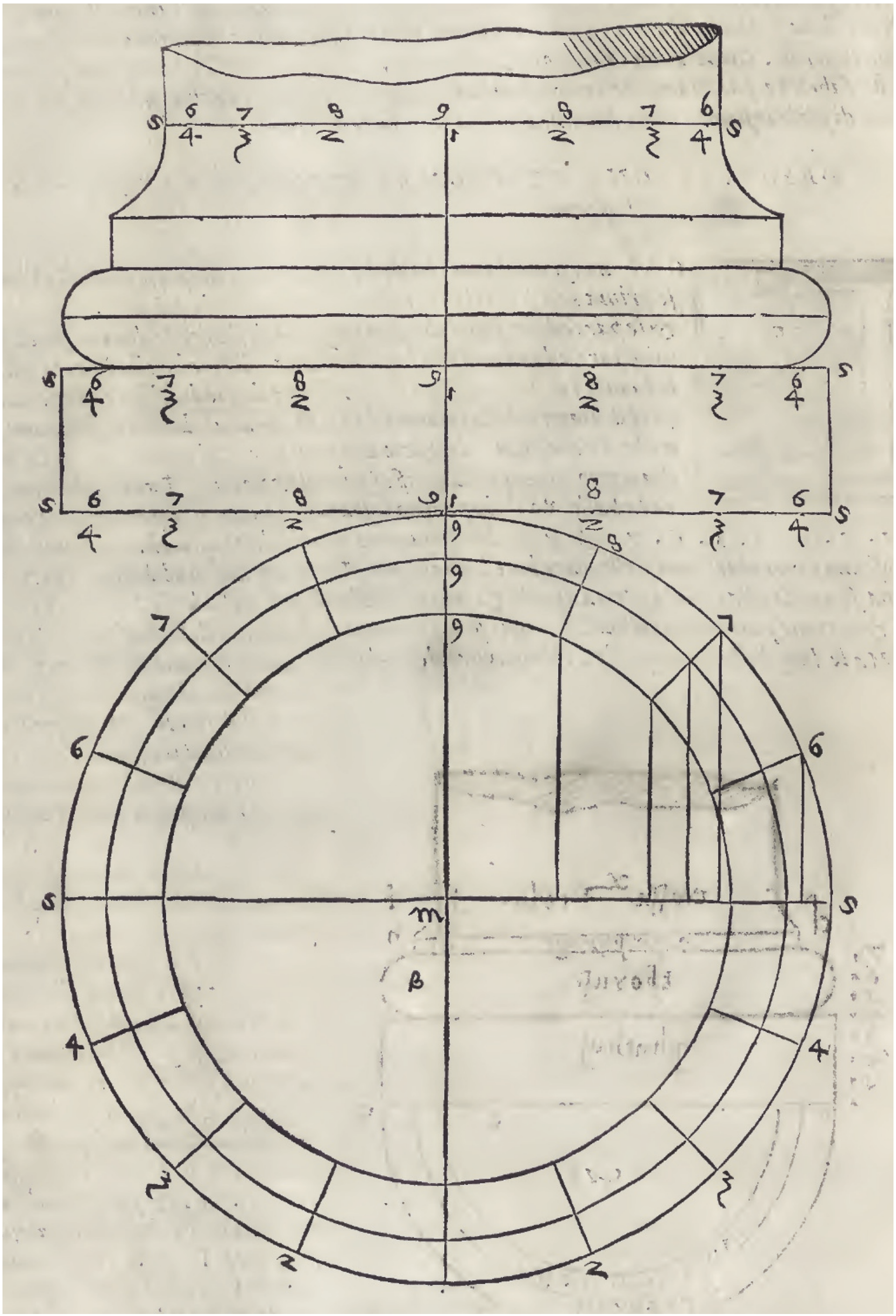
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Fig. 4.2 [p. 132]

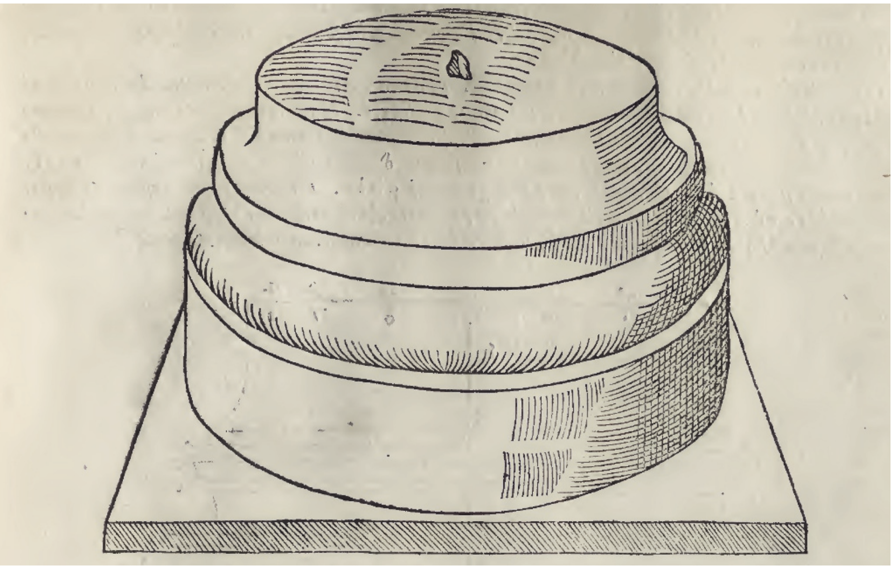

Fig. 4.3 **[p. 133]**

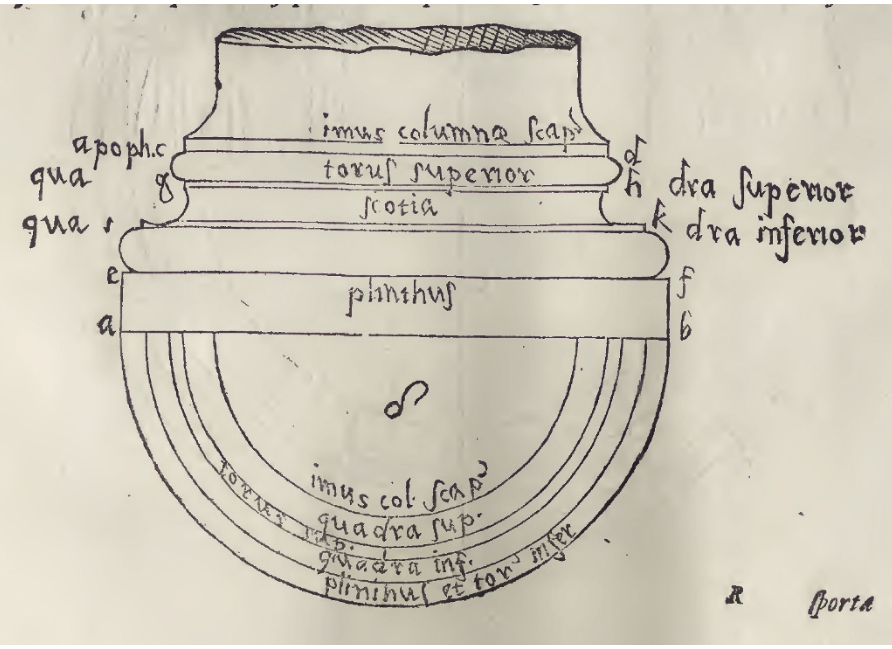
imus columnæ ſcap.
apoph.c
qua
g
torus ſuperior
d
h
dra ſuperior
ſcotia
qua
k
dra inferior
e
f
plinthus
a
b
imus col. ſcap.
torus ſup.
quadra ſup.
quadra inf.
plinthus et tor. infer.
R
ſporta

Fig. 4.4 **[p. 133]**

Chap. III. Description and shading of the Attic base

To make the Attic base[6] you will draw line *ab*, which will be one and a half times as long as the thickness of the column at its foot. This you will divide into 18 equal parts with invisible marks, and this is the length of the base. The height must be 6 parts of the 18, a characteristic that it shares with all the bases that are as high as half of the thickness of the column at its foot.[7] To the height of the plinth you will give two parts. To the height of the torus you will give one, and a half shall be given to the *cavetto*, called the scotia. To its little fillets, called *gradetti*, one and a half, and to the upper torus one. The space given to the scotia is divided into 6 parts, one of which is given to the fillet or *gradetto* below, one to the one above, leaving 4 for the scotia itself. The upper torus **[p. 134]** projects as far as the plinth. The lower fillet does not go beyond the diameter of the curve of the lower torus. The diameter of the upper torus is beyond the shaft of the column by a third part of the distance by which the plinth projects beyond the shaft of the column.

The fillet of the cimbia, which here is part of the column, does not go past the diameter of the upper torus, and its height is half of the height of the upper torus. The scotia is made with the compass, placing one of its feet at the midpoint of a line that is drawn from the end of the upper fillet to the end of the lower fillet, as shown in figure δ (Fig. 4.4) of the Attic base. Its plinth is *abef*; the lower torus *efik*; the scotia with its fillets *ghik*; the rest is of the upper torus and the cimbia is *cd*. The curve is made in the way that was described above. The perfect set out for the degradation is shown in figure C (Fig. 4.6), below which is the degraded and shaded form of this base (Fig. 4.5).

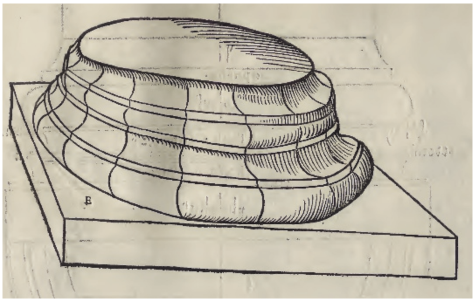

Fig. 4.6 **[p. 135]**

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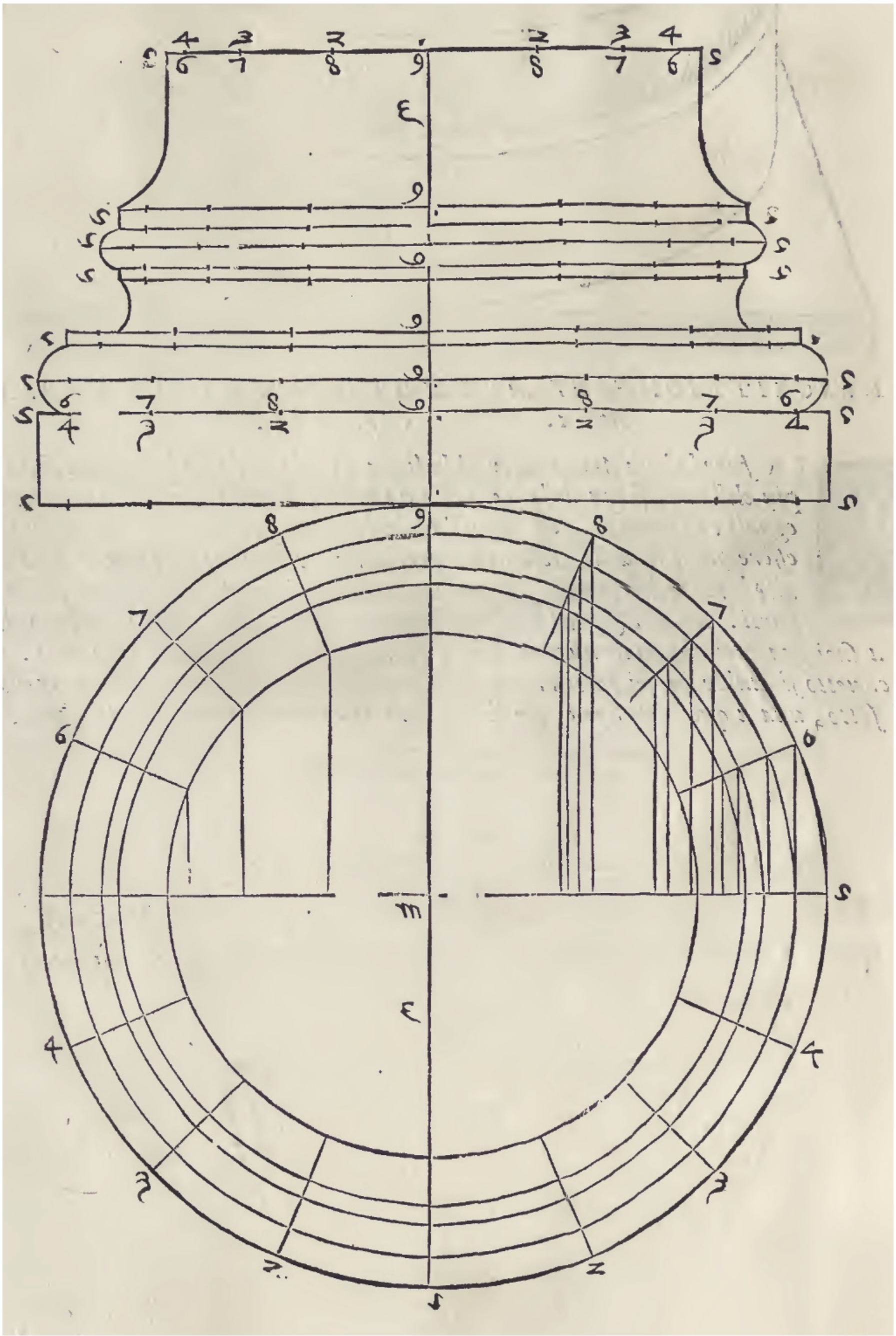
[6] Cfr. Barbaro (1567, p. 144 and figure p. 147), Williams (2019: 247-249).

[7] The thickness of the column at its foot is 12; the length of the base is 18; the height of the base is 6 (1/2 of the thickness). The 1:2 ratio of base height to column thickness is common to the Tuscan, Attic and Ionic bases that Barbaro discusses in Part IV.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6 7 8 9 8 7 6
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3 3
2 2
1

Fig. 4.5 **[p. 134]**

Chap. IV. Description and shading of the Ionic base

The Ionic base is made in this way.[8] Draw line *ab*, whose length will be equal to the diameter of the column at its foot plus a quarter and an eighth part of that. Let us say that the diameter is 16 parts, to which we add a quarter, which is 4 parts, and an eighth, which are 2 parts of the 16, and thus line *ab* will be divided into 22 invisible parts; this will be the length of the Ionic base. The height will be of 8 parts, that is, half of the thickness of the column [at its foot]. The plinth is a third of the height of the base, that is, a third of 8 parts; the remainder is divided into 7 parts, 3 of which are given to the upper torus, two to the upper scotia, called *trochilus*, with its astragal and to its supercilium, and two to the lower scotia with its astragal and supercilium. The astragals are made the eighth part of the scotia, but it will look good when the lower scotia is larger, so that it projects to the end of the plinth. The projection beyond the thickness of the column is made in this way. Take three parts of the division of line *ab*, which are the eighth and sixteenth part of the height of the base, and divide that in half; that will be the projection over the base beyond the thickness of the column, placing half to the right and the other half to the left. The projection of the base where the cimbia is, is made in the same way.

The height of the cimbia is one third of that of the torus, the centre of which is on the line that drops down from the projection of the cimbia.

The astragals must be touched by a line that drops down from the end of the upper supercilium to the end of the fillet that is above the plinth and below the lower scotia. The scotias are made as described above with the compass, and come out nice and refined.

The perfect and degraded forms of the Ionic base are shown in figure ς (Fig. 4.7).

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[8] Cfr. Barbaro (1567, p. 149 and figure and legend pp. 152-153), Williams (2019, pp. 253-254).

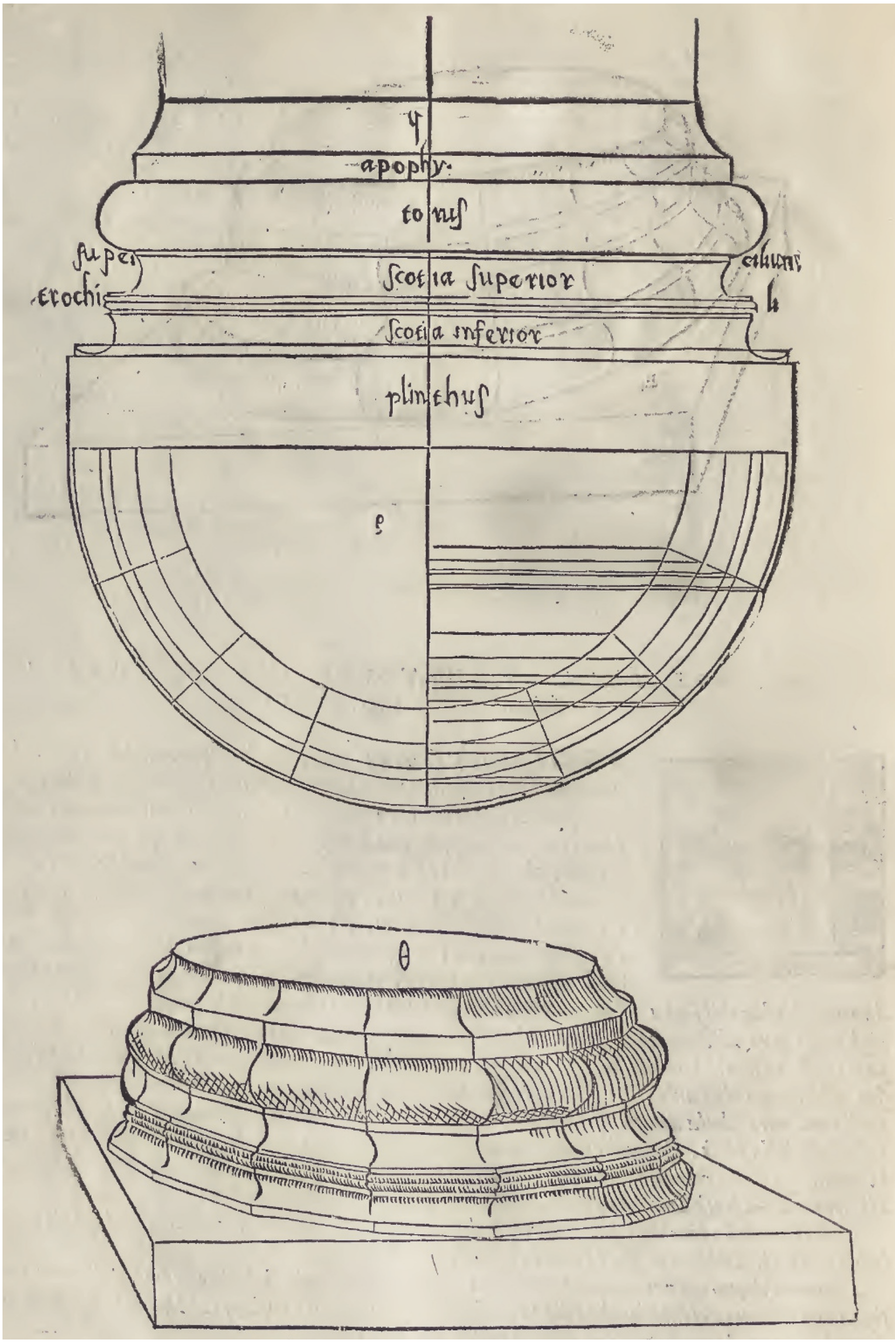
ψ
apophy.
torus
ſuper
trochi
cilium
h
Scotia ſuperior
Scotia inferior
plinthus
ρ
θ

Fig. 4.7 **[p. 136]**

[p. 137]

Chap. V. Description and degradation of columns

Having formed the bases, order leads us to reason about the columns. These are either round or square, or of many angles. All are easy to draw; it is true that some are fluted, the perfect forms of which are made as will be shown below. In all columns the measures must be noted. So, the Tuscan is six heads high and it narrows on top by the fourth part of the thickness at its feet;[9] the Ionic is eight heads and the Corinthian nine. They taper or contract according to the following rule.[10] The columns that are up to fifteen feet high are tapered by the fifth part of the thickness at their feet; if they are up to twenty, the thickness of the shaft at the feet is divided into six and a half parts, five and a half of which are given to the upper part; if from twenty to thirty, the diameter at the feet is divided into 7 parts, and 6 are given to the upper part; if from thirty to forty, then of seven and a half parts of the thickness at the feet, six and a half are given to the upper part; if from forty to fifty, of 8 parts of the thickness, 7 are given to the upper part. With these rationales the columns are narrowed and tapered. The swelling in the middle of the column, called *entasis* in Greek, is made in the way given in Vitruvius, or as Andrea Palladio says in his book on private houses.[11]

The perfect form of the fluting is shown below, with the degradation of a column with 8 sides lying on a plane,[12] as shown in figures Λ and μ (Figs. 4.8 and 4.9). The fluting is shown in the illustrations i and k (in Fig. 4.9). If you want other ways of doing fluting read Labbacco[13] and others who have described the antiquities of Rome.

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[9] Cfr. Barbaro (1567, p. 143), Williams (2019, p. 246).

[10] Cfr. Barbaro (1567, p. 133), Williams (2019, pp. 230-231).

[11] In truth, Vitruvius only promises to describe entasis, but does not actually do so; see Vitruvius, *The Ten Books of Architecture*, III.III.13. In his commentary on Vitruvius, Barbaro defers to 'Serlio and some others' (Williams 2019: 231 and n. 59). For Palladio's explanation, see Andrea Palladio, *The Four Books of Architecture* (1997, p. 18).

[12] This exercise comes from Piero dell Francesca, *De prospectiva pingendi* (Mss. Regg. A 41/2, fol. 29r).

[13] Antonio Labacco (or dell'Abacco), *Libro d'Antonio Labacco appartenente a l'architettura nel qual si figurano alcune notabili antiquita di Roma* (1552).

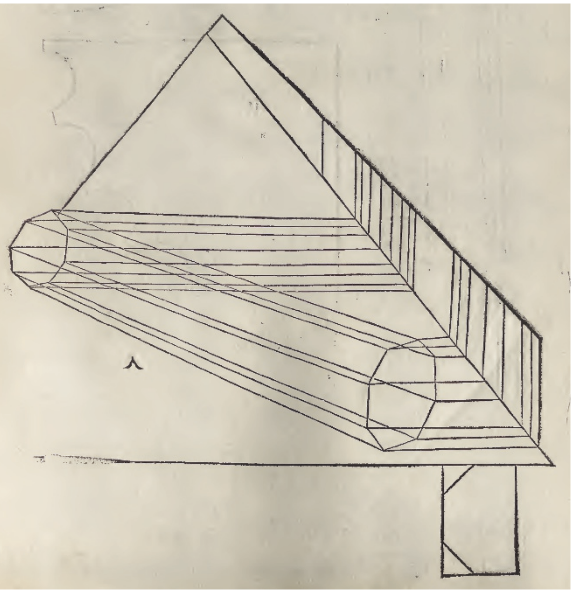

Fig. 4.8 **[p. 137]**

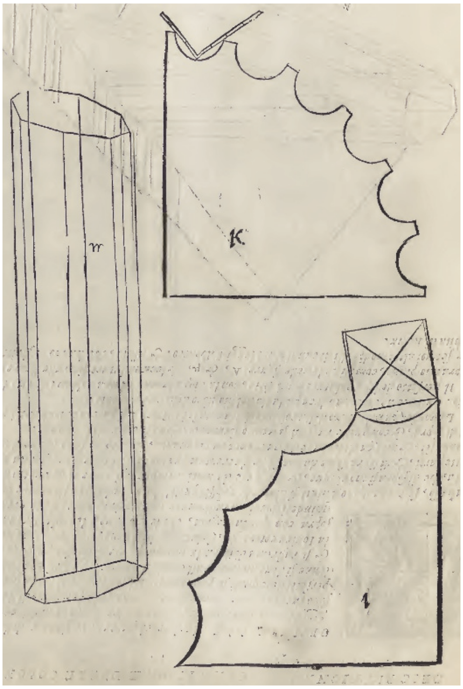

Fig. 4.9 **[p. 138]**

[p. 139]

Chap. VI. Description and degradation of the Tuscan capital

The Tuscan capital is very simple, as are its bases, columns and other parts, and its buildings, and imitates the ancient roughness of the simple Etruscans. It is as high as its base,[14] that is, half of the shaft of the column at its foot. Its height is divided into three parts. One is given to the upper *zocco* or dado, called the abacus; that in the middle to the *ovolo*, called the echinus; the third to the necking, called the hypotrachelion by the Greeks, which narrows to the upper part of the shaft of the column, where there is a curving that is called the apophyge. The abacus falls straight down plumb with the shaft of the column, while the echinus is made with the compass, as I will describe below. You will first divide the part that is given to the necking into six parts, one of which you will give to the fillet or *listello*, called the *annulus*, which is below the echinus. It projects by as much as it is high, that is, the sixth part of the necking. The other 5 parts remain for the necking. You will draw an occult line from the profile of the column upwards to the abacus. This line will leave a space that is the projection of the abacus beyond the upper shaft of the column. You will divide this space into two equal parts, which is **[p. 140]** from point *b* to the edge of the abacus, and come back from point *b*, which is the upper shaft of the column, one of those two points towards point *C*, and there place the foot of the compass. Opening it to the end of the fillet draw the circular line from that to the edge of the abacus, and you will have the echinus, below which is the fillet and the necking. Below the necking is the curve called the apophyge, or apothesis,[15] which is formed in the following way. You will make its curve, which is called *torulus*, twice the height of the fillet that is below the echinus. The centre of the astragal will be on a line that falls vertically from the end of the fillet. The cimbia has that same projection ending on the line just mentioned, and is as high as the fillet under the echinus. The curve is made with the compass, with the same rationale that is used in the base at the foot of the column, as shown here, with the perfect forms shown in figure y [*recte* N] (Fig. 4.10) and the degraded in figure ξ (Fig. 4.11).

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[14] Cfr. Barbaro (1567, p. 142 and figure p. 142), Williams (2019, p. 246).

[15] Although the term 'apophyge' is sometimes used for both of the curves, a distinction can be made between the apophyge at the foot of the column and the apothesis at its head.

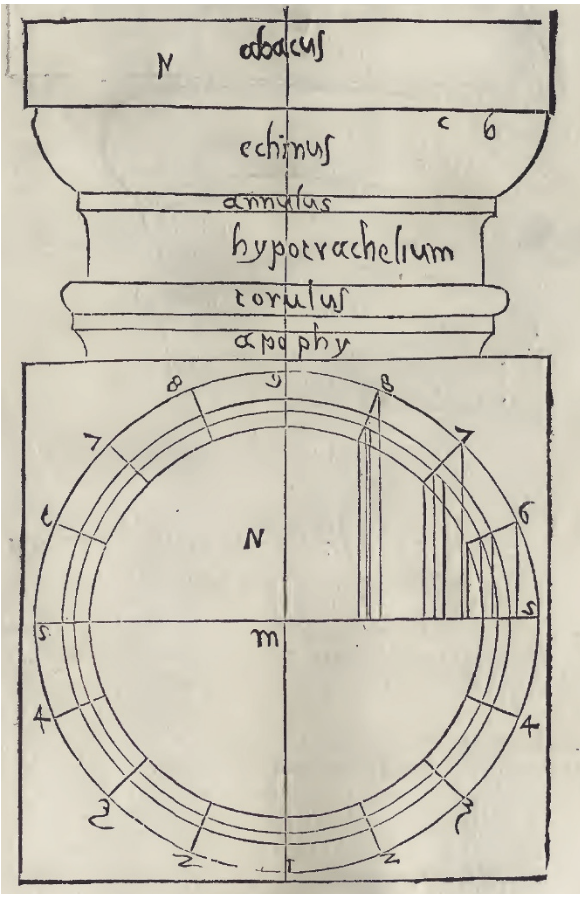
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Fig. 4.10 **[p. 139]**

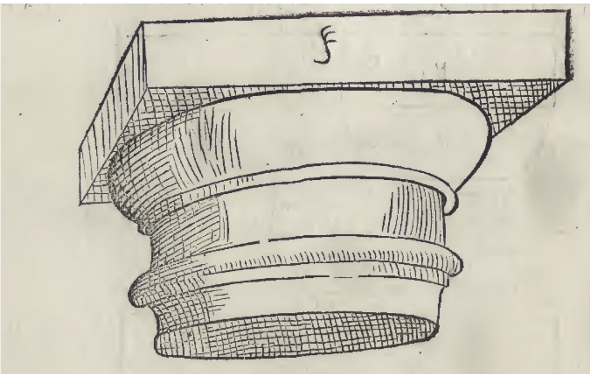

Fig. 4.11 **[p. 140]**

Chap. VII. Description and degradation of the Doric capital

The thickness, that is, the height, of the Doric capital is half of the thickness of the column at its foot.[16] The width is equal to the thickness of the column at its foot plus a fifth on each side. Although Vitruvius says a sixth, the fifth turns out handsomer. You will divide the thickness of the column into three parts, one of which will be given to the abacus, or dado, with its cyma, another to the echinus with its fillets, and the third contracts to the necking of the column. The height of the abacus with its cyma is divided into 5 parts, 3 of which go to the abacus. The two remaining parts are divided into three, two of which are given to the cyma, and one to the fillet or *listello* above. Below the abacus is the echinus with its annulets; its height is divided into three parts, of which two are given to the echinus and one to the three annulets, all three of which are of the same height; each projects by half of its height. The echinus has a projection that is two thirds of its height, and is made with the compass placed at the end of the upper fillet. Making a small arc below the abacus, and then with the compass at the same opening, one foot of it is placed below the edge of the abacus and the other crosses the arc; at the point of crossing is made the centre, and the curve of the echinus is drawn with a compass. This is made with its fillets; the one that is below, which is the third part of the height of the capital, contracts to the necking, called the *sottogola*, or *freggio* by some. This, with its curve, comes gently to the astragal and fillet **[p. 141]** straight down over the upper part of the shaft of the column. The height of the astragal is equal to all three fillets plus the half of one, and projects outward by as much as the echinus above it. The fillet is as high as half of the astragal, and its projection is equal to the diameter of the astragal. Above the capital the ancients used to place an addition which was not very high and which rested on the abacus over the upper part of the shaft of the column; they did this so that the architrave would rest squarely (*sul vivo*) on the columns and would not break the projections. The plan and the perfect of this capital are shown in figure O (Fig. 4.12) and the degradation is shown in figure π (Fig. 4.13). This is all well understood from what has been said.

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[16] Cfr. Barbaro (1567, pp. 144-145 and figure p. 147), Williams 2019, p. 249).

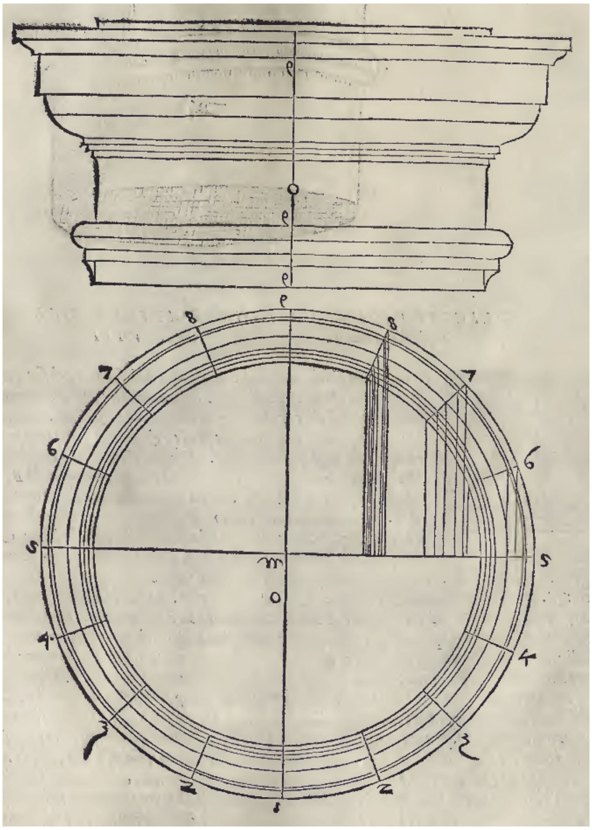
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Fig. 4.12 **[p. 141]**

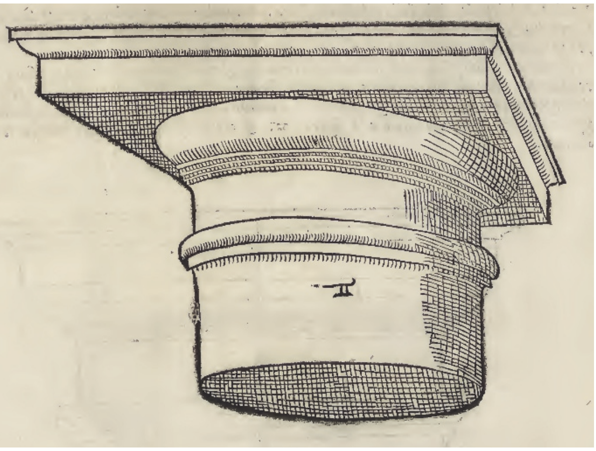

Fig. 4.13 **[p. 142]**

[p. 142]

Chap. VIII. Description and degradation of the Ionic capital

With the Ionic capital there are many difficulties in forming its perfect and its elevation, as well as its degradation and shading, so it requires the effort of reasoning clearly. So, first you will take the thickness of the column at its foot and divide that into 18 parts, to which you will add one.[17] This will be the length and width of the dado, called the abacus. Then let there be line *AB* such that from *A* and *B* fall lines vertically by a distance that is half of line *AB*, that is, as much from point *A* to point *I*, which contains nine and a half parts. Then there falls from one of the ends line *AD*, whose length will be equal to *AB*, and let it be divided into nine and a half parts. Of these you will give one and a half to the thickness of the abacus. One is given to the cyma of the abacus, which is made to resemble a gently curved letter S, and half is given to its fillet. Having finished the abacus, below it there remain 8 parts of line *AD*, which are given to the volute as follows. Retract back from the end of the fillet, or cyma, one part of the nineteen of line *AB*, and there place point *C*, from which will fall line *CE*, parallel and equal to line *AD*. On this line is to be placed the eye of the volute, which is in that part that divides the four and a half parts above from the three and a half below. The eye has for its diameter one of the 8 parts of line *CE*, below the abacus. Draw through the centre a line across that will divide the eye into 4 parts, which Vitruvius calls *tetranti*.[18] Then is formed in the eye a square, each side of which is equal to the half-diameter of the eye, and its centre the same as the centre of the eye. Its corners are in the middle of the *tetranti*, and from the corners are drawn the diagonals, each of which is divided into six parts. I say that those parts are the twelve centres of the volute. The first centre is in the upper right corner of the square; the second is in the upper left corner; the third is in the lower left corner; the fourth in the lower right corner. In this way will be furnished the first turn of the volute, which in each fourth **[p. 143]** will have reduced the eye by half. The first centre of the second turn is the first point close to the upper right corner; the second is the first point close to the upper left corner; the third is on the left in the lower part, close to the lower left corner. And with this method the volute is drawn. Its thickness, or fascia, is a half of one of the nine parts into which line *AB* was divided, and its centres are the same and with the same order as those used to draw the first volute. The volute has its channel, which is a hollowing within the fascia of the volute. This channel occupies a diameter and a half of the eye, and its depth is equal to twelfth part of the height of the volute, that is, a twelfth of the 8 parts that remain below the abacus. Because the volute is not part of the Ionic capital, but is an ornament and a thing placed on top of it, so it is necessary to note how to form that capital, which has its echinus, which the moderns call *ovolo*, the Greeks cymatium, and the Latins echinus. The height of this is two parts of the 8 of line *AD*. The fourth part and its projection beyond line *AD*, which is the flat edge of the abacus, is the same size as the diameter of the eye. For this reason the lines are made to fall from the edge of the abacus. Its curve is made with the compass. Its projection under the channel is equal to the diameter of the eye beyond the projection of the abacus. Its height is taken with the compass, which (as I have said) is equal to two parts and a quarter of the 8 parts of the line that falls vertically from the abacus. Its lower line ends where the *tondino* called the astragal begins. Placing a foot [of the compass] on that line, draw an arc of a circle, then fix the compass at the end of the upper cyma and cross the first arc. Place the compass at the crossing and draw the curve of the cyma, above which the volute gently turns. Below the cyma is the *tondino* or astragal, as it is called, which occupies three fourths of one of the 8 parts already mentioned. The centre of the curve of the astragal is on line *AE*. The collar does not project beyond this line; its height is equal to half of that of the astragal, and it contracts with its curve to the tapering of the upper part of the column, as described above. Because we imagine the volute being wrapped around a stick and tied in the middle, it is said that that stick, called the axis, is as large as the diameter of the eye. The bands, called *baltei*, that are in the middle of the sides between the volutes do not project beyond the echinus, so that when the foot of the compass is placed in the middle of the abacus of the capital, and opened to the projection of the echinus, in turning it touches the extremity of the bands. This is the measure of the Ionic capital, whose plan is formed in perfect form on centre *V* and marked with its letters and numbers. The thickness is similarly drawn with letters and numbers corresponding to those of the perfect if the descriptions given above are well understood. The perspective of this capital is easily understood. The eye is on line *HVG*, three times as far as space *VG* [is from the point *V*]. I say the eye of the perfect, because the eye of the height is in the same distance, but in line *OP*, and the end is line *OQ*. The degradation can be understood from the correspondences of the letters and the numbers, and from the figures below (Fig. 4.14).

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[17] Cfr. Barbaro (1567, p. 149 and figures pp. 153 and 157), Williams (2019, pp. 254-255; 264).

[18] Cfr. Vitruvius III.V.5; Barbaro (1567, p. 157), Williams (2019, p. 264). Barbaro translated Vitruvius's *tetrantorum* as '*quarta*'.

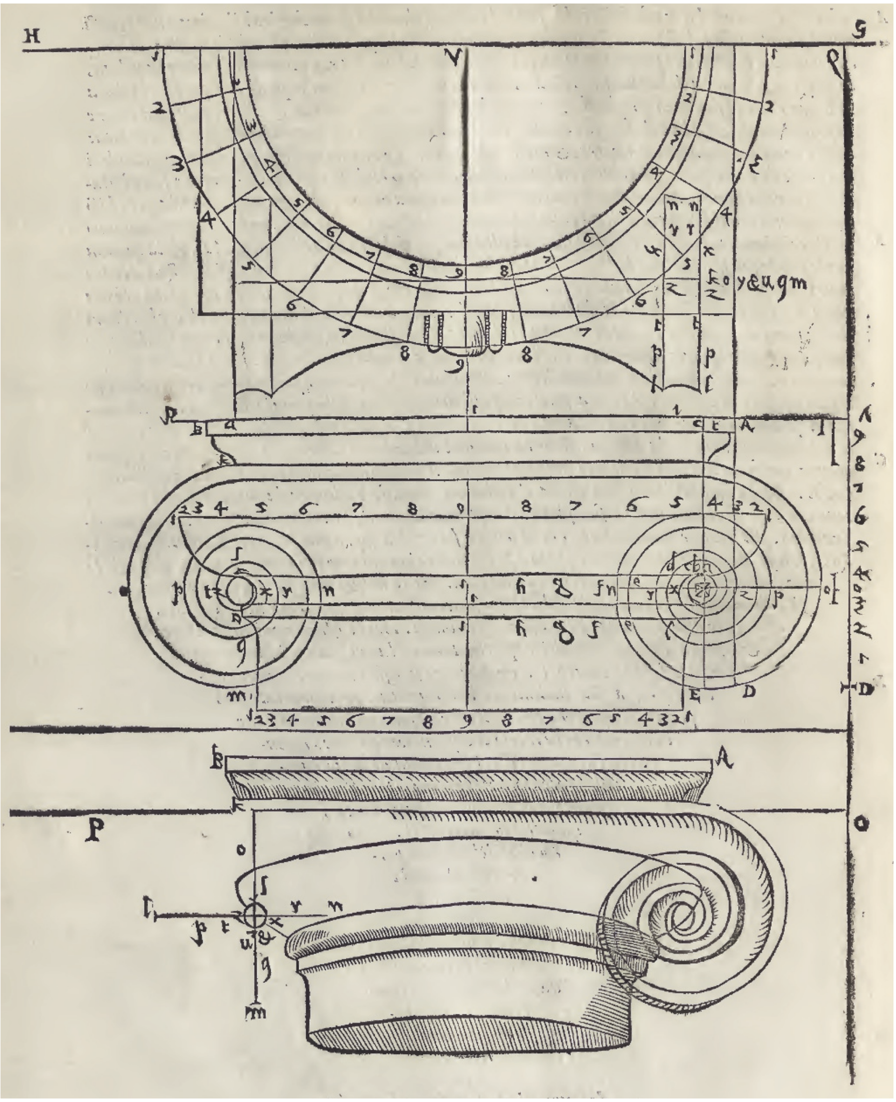
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Fig. 4.14 **[p. 144]**

[p. 145]

Chap. IX. Description and degradation of the Corinthian capital

The Corinthian capital was taken from an architect (as Vitruvius says in the fourth book[19]) who, passing through Corinth, saw at the head of a gravestone a basket with a tile on top of it, and the basket was embraced by the leaves of an acanthus, that is *branca ursina*, that was born underneath it. The basket was full of some things for the delight of the virgin who was buried there, and had been placed there by her wetnurse, who covered the basket with a tile so that it wouldn't be damaged by water. This appeared to the architect a graceful manner, seeing how the leaves and stems, and the flowers of that herbage had adorned that basket, so he transferred that form to the Corinthian capital. We call that part that is covered with leaves the 'bell', which represents the unadorned basket. Now I will tell its measures. The Corinthian capital is as high as the column is at its foot.[20] Vitruvius includes in that height the abacus or *zocco* of the capital, but it comes out better if the abacus is not included there.

The width of the abacus, that is, the dado, must be such that the diagonal lines are double the height of the capital. The faces of the abacus must be curved and arching by the ninth part of their length. The low part of the capital must sit squarely upon the upper part of the column. The thickness, or height, of the abacus is the seventh part of the height of the capital. The remainder is divided into three parts, one of which is given to the first leaf below, the second to the leaf in the middle, and the third to the stalks from which the flowers grow, which receive the abacus. Those volutes that are born from the leaves of the stems go to the outer corners of the abacus. But the smaller volutes curve inward, and will be placed below the flowers that are in the middle of the abacus on all four faces. Those flowers are as high as the abacus is, but rather longer (as is observed in Antiquity). It is thus necessary to make the bell well and dress it with leaves, and make the leaves grow out of those *caulicoli*, or stems, from which the smaller leaves grow, and from the smaller leaves the larger volutes.

The true way to curve the faces by the ninth part of their length is this. Draw line *ab* as long as the face of the abacus, and divide it into nine equal parts. One is carried to the middle over point *c*, to *d*. By means of the way to find the centre [of a circle] from three given points is found the centre of the arc.

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[19] Vitruvius IV.I.9.

[20] Cfr. Barbaro (1567, p. 155 and figure p. 156), Williams (2019, p. 261).

So where the lines made with the crossing of *a* with *d* and of *b* with *d*, there is centre *o*, as can be seen in the figure ς (Fig. 4.15). The perfect form and the degradation of the Corinthian capital are shown in figure σ (Fig. 4.15). It is necessary to well understand the plan, and the rest will then be easy.

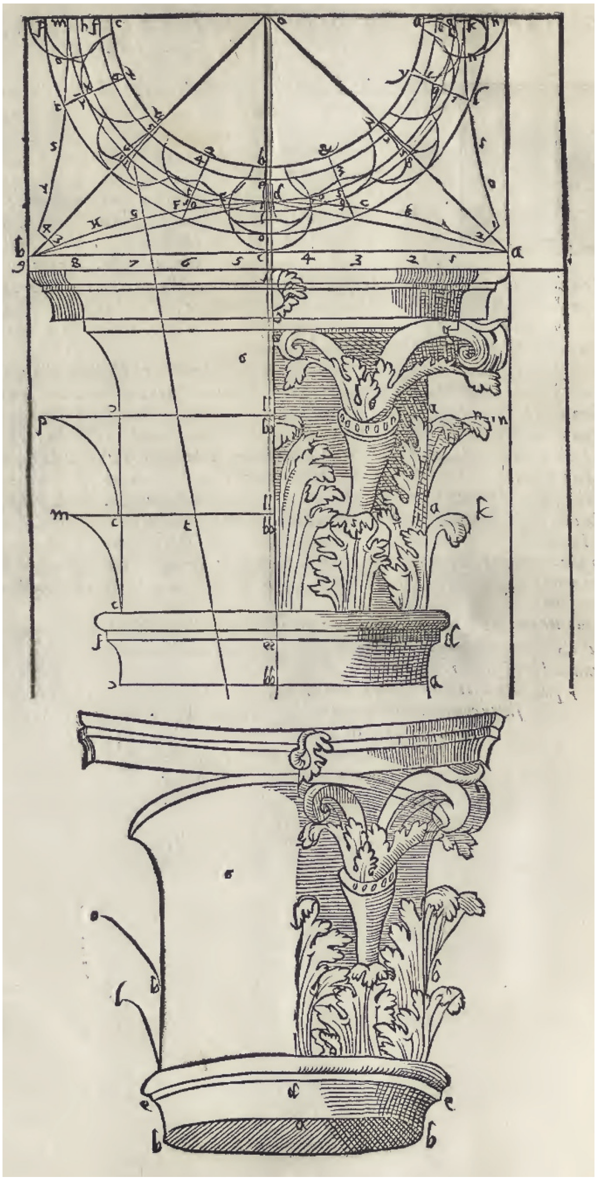

Fig. 4.15 **[p. 146]**

[p. 147]
Chap. X. Description of the Doric architrave

In treating the architraves, which are the master beams that go over the columns and tie the building all around, I would leave out their degradations, because they are easily understood, and come directly to the measures. First I will compartition the Doric architrave, leaving out the Tuscan, because it is a work in wood and has no ornamentation. The height of the Doric architrave together with the band and its guttae is half of the thickness of the column.[21] This half for now we will call the module. The band, or fascia or *taenia* as we say, is the seventh part of the module; the guttae with the little fillet is the sixth part. This little fillet goes above the guttae, and of three parts divided into sixths, it occupies one of those sixths. The width of the architrave, that is, the lower plane that is set on top of the capital, must be the same as the collar of the capital, so that in this way it comes to rest squarely, as does that addition that is set on top of the capital.

The height of the triglyphs is a module and a half, the width in the front is a module. The faces of the triglyphs are divided into six parts, and half is left on each side for the half-channels, after which one is left per side for the little flat parts that Vitruvius calls *femurs*.[22] After this there are the channels, one per side, and they are whole and as wide as the little flat parts. In the middle of the channels there is a flat part. It must be noted that the flat part in the middle corresponds directly to the centre of the diameter of the column. The metopes are as high as they are wide. The metopes that are over the corners are half-metopes, but not exactly; they are a little less than half, because in that way the compartition works out. Above the triglyphs are their capitals, whose height is the sixth part of a module. Above this is the corona, or dripstone, whose height, with its cyma, is a half module. This height is divided into 4 parts, one of which is given to the upper cyma, one to the lower cyma, and two to the space that is between the two of them. The cyma has its fillet, whose height is a third, and the other two are given to the curve of its cyma. The dripstone projects by the half plus a sixth of a module, and has its edge with small teeth below so that when water drips, it does not fall along the walls or the columns and ruin them; that is why this part is called the dripstone. Vitruvius calls that edge 'the chin of the corona', and those places the

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[21] Cfr. Barbaro (1567, pp. 145-146 and figure p. 148), Williams (2019, pp. 250-252).

[22] Vitruvius IV.III.5: *Una in medio deformetur femur, quod Graece meros dicitur.*

'scotia'.[23] The curves of the dripstone are opposite to one another, as can be seen in figure[24] (Figs. 4.16). The ancients adorned the spaces of the metopes with garlanded heads of bulls and with the patens that they used in the sacrifices. Others placed trophies there. Regarding the pediment, I will discuss that in the description of the Ionic architrave in what follows.

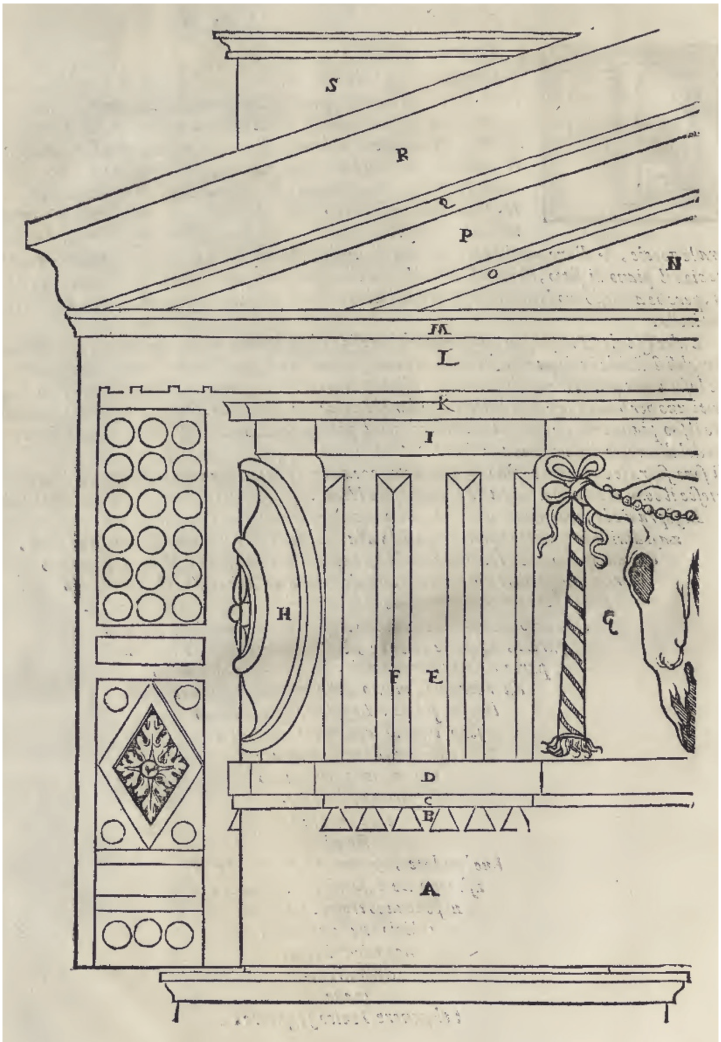

Fig. 14.16 **[p. 148]**

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[23] Vitruvius IV.III.6: *ad ipsumque mentum coronae incidatur linea quae scotia dicitur.*

[24] A blank space appears instead of a figure number.

[p. 149]

Chap. XI. Description of the Ionic architrave and the pediment

Architraves are made according to the height of the columns, as I will say in the next chapter. For now let us say that the column is fifteen feet high.[25] The height of the architrave will be half of the diameter of the column at its foot, and its width will be such that it rests squarely on the thickness of the upper part of the column. The plane on top will be as thick as the column is at its foot. The cyma, or *gola*, of the architrave is made the seventh part of the height of the architrave and must project by as much as it is high. The projection is taken from that line which would come from the tapering of the column. The remainder below the cyma is divided into twelve parts. Three of these are given to the lowest fascia, four to the middle one and five to the top one. On top of the architrave is the frieze, which is called *zophorus* by Vitruvius.[26] This measures a fourth less than the height of the architrave with its cyma. This height serves when there are no carvings, because when there are, it is made a fourth higher than the architrave so that the carvings can be better enjoyed. The height of the frieze is divided into seven parts, and one is given to the cyma that goes on top. On top of the cyma are the dentils, which are as high as the middle fascia of the architrave. The projection is equal to its height. The width, called *metochi* by the Greeks and *intersectio* by the Latins, is half of the height of the dentils. The cavity or space between one dentil and the next, also called metopa and pigeon-hole, is two thirds of the width of the dentils. The cyma of the dentils is the sixth part of its height. The cornice with the cyma is as high as middle fascia. The projection of the cornice with its dentils or cuts in the chin, must be of a height equal to the space of the frieze at the top of the cyma, that is, the cyma of the cornice. That projection is taken from the line that would come from the end of the cyma of the frieze. Up to this place the buildings are parallel to the plane.

Now it is necessary to erect the pediment, which Vitruvius calls the *fastigium*.[27] This has its cornices corresponding to the members of the cornices, and in addition it has its own raked cymae, called *simae*. The height of these is an eighth part more than the height of the cornices. Beneath these is the tympanum, that is, the plane, whose height is a ninth of the length of the cornice, measuring from the end of the cyma of the cornice. The plane of the tympanum must rest squarely, that is, if you were to drop a plumb line from the plane it would fall precisely on the collar of the capital and squarely over the column. The small piers called acroteria must be high enough so that the figures that will be set on them can be seen. Those on the corners must die into the roof; they must start directly in line with the columns, and be set back as far as the rationale of viewing requires. In some buildings that are low they are set back and their height must be equal to the top of the pediment, but the one in the middle must be an eighth higher than those on the corners. In the Corinthian it might be that the architrave and other parts are taken from the Ionic, and the frieze, which in the Ionic is bulging, can be made flat. The pediment with its ratios can be given to the Corinthian and the Doric. What I have said about the Ionic architrave is shown in the figure (Fig. 4.17).

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[25] Cfr. Barbaro (1567, p. 151 and figure p. 154), Williams (2019, pp. 258-260).

[26] Vitruvius IV.I.II: *zophoroe scalpturis ornati…* .

[27] Vitruvius III.II.2; III.II.4.

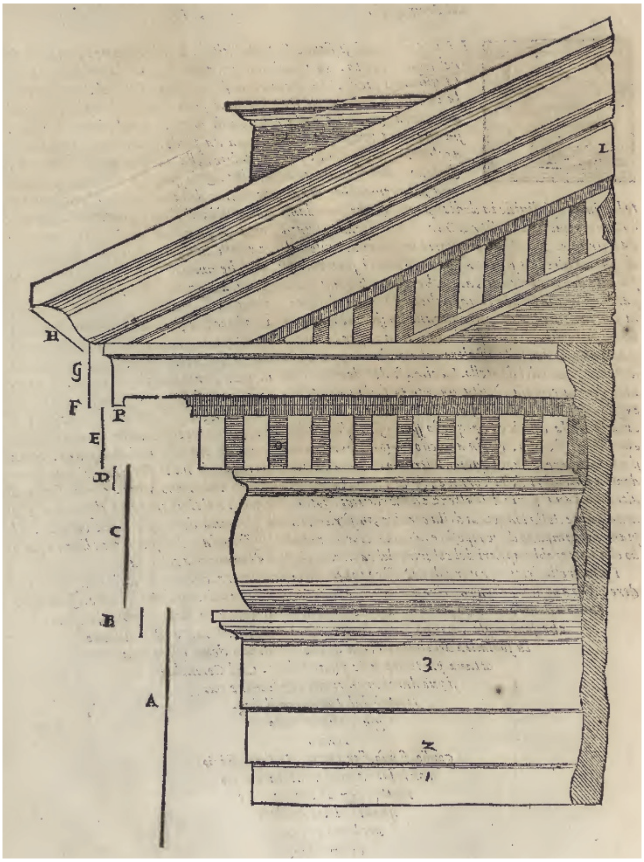

Fig. 4.17 **[p. 150]**

[p. 151]

Chap. XII. Rules for the heights of the architraves and ways to transpose them from one size to another

If the columns are from less than twelve to fifteen feet, the height of the architrave shall be half of the thickness of the column at its foot.[28] If they go from fifteen to twenty feet, the height of the column shall be divided into thirteen parts and the height of the column [*recte* architrave] shall be one of those. If from twenty to twenty-five, the height shall be divided into twelve and a half parts, and one part will be the height of the architrave. If they are from twenty-five to thirty feet, then one of twelve parts shall be the height of the architrave. Beyond this height, the height of the architrave must be determine in the same way according to proportioned parts of the height of the columns.

Now, because it often happens that it is necessary to transpose a small form into a larger one, here below I will place two figures (Fig. 4.18) that can be understood without any explanation. One is the architrave, the other the fluted column, according to how Serlio has shown them in his book on the practice of geometry.[29]

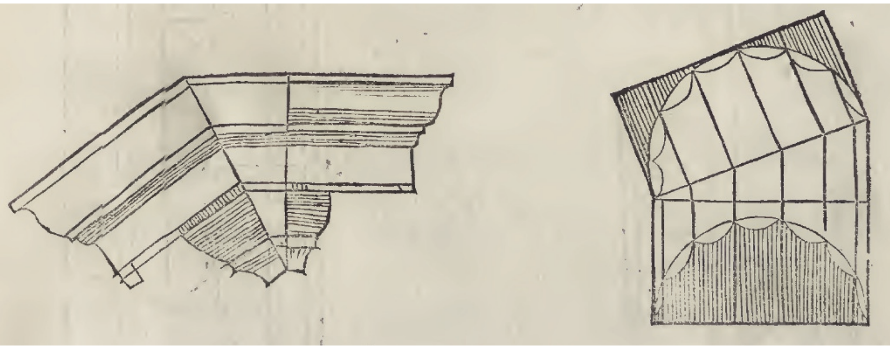

Fig. 4.18 **[p. 151]**

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[28] Cfr. Vitruvius III.V.8; cfr. Barbaro (1567, p. 158), Williams (2019, p. 266).

[29] Serlio (1566, Libro I, pp. 8r-8v).

Chap. XIII. The description and degradation of stairs

The description of stairs must not be left out, because quite often they are the beautiful parts of perspectives and scenes. These are either straight or spiral. I will speak about both manners, and first about the straight, the ratios of which are made with the set square. So, just as Pythagoras found the right angle[30] by placing together in the form of a triangle three rulers, one of which was of 3 parts, another of 4 and the third 5, as seen in the figure below (Fig. 4.19), so too the architects established that stairs shall be arranged[31] such that the straight line of 3 parts is that which comes straight down from the place of the height of the stair, that of 4 parts begins at the foot of the wall, and that of 5 parts is the shaft or slope of the stair on which the steps are to be placed. As shown in the figure, each step is formed according to the same ratio as the stair, so that the riser is 3 parts, the run is 4, and the slope is 5.

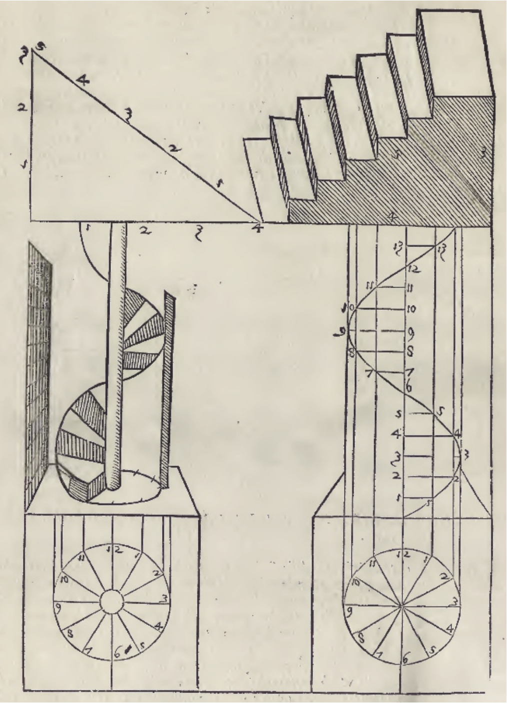

Fig. 4.19[32] **[p. 152]**

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[30] Cfr. Barbaro (1567, p. 349), Williams (2019, p. 574).

[31] Cfr. Barbaro (1567, p. 352 and figure p. 351), Williams (2019, pp. 575, 577).

[32] Cfr. the image of the spiral stair in Dürer, *Underweysung der Messung* (1525, Book I, fig. 17).

[p. 152] Spiral stairs are by way of example described with plans, and are then easily understood because we have already gone through the difficult parts (Fig. 4.19).

[p. 153]

Chap. XIV. Of arches, vaults and doors

Arches and vaults can be whole or half, or lunettes, or groins, or lanterns and domes according to desire and the place where they are made, and their descriptions are easy because of things already said. Similarly easy are doorways and the other apertures of windows or niches and other openings, when their plans are well understood. Now I will set forth the usual way of making arches even, because it can sometimes happen that it is desired to make an arch as high as another of a greater diameter. Fig. 4.20 shows an arch on the right that is a whole semi-circle, while that on the left has a greater diameter but the heights of the two are the same.[33] So, you will divide the diameters of both arches into 12 equal parts, or more if you wish, and then from the points of the division you will drop vertical lines. Where those lines cut the vault that has the whole semi-circle, there you will draw the transverse lines parallel to the diameters. In this way you will see where the ends of the second vault are: they will be where the vertical lines cross the horizontal lines, and will correspond exactly to the semi-circle, as can be seen in the next drawing (Fig. 4.20). From the figure (Fig. 4.21) can be understood the rationale of the vaults made as half-balls, called hemispheres.

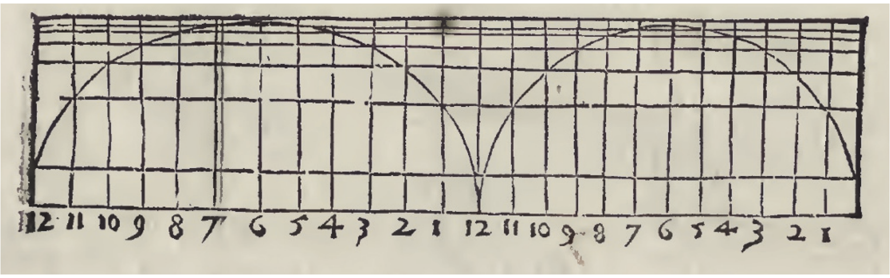

Fig. 4.20[34] **[p. 153]**

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[33] Barbaro speaks of diameter, but the term isn't exactly correct; bottom chord would be better. The plans of these would show adjoining bays of unequal lengths.

[34] Cfr. the image in Dürer, *Underweysung der Messung* (1525, Book I, fig. 33).

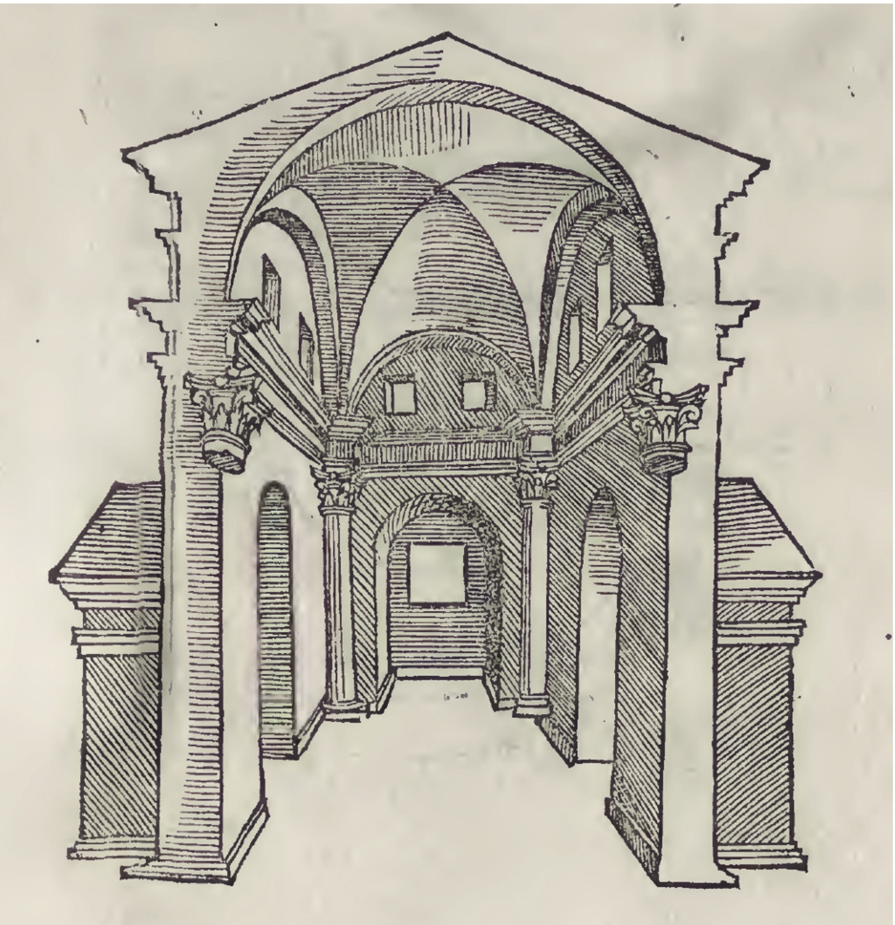

Fig. 4.21[35] **[p. 153]**

[p. 154]

Chap. XV. Plan, elevation and profile of a temple

In the plan, elevation and profile of the following temple can be seen how the architects erect their buildings, and how the vaults and tribunes are made, so I will not extend myself in describing the vaults in particular, because from the previous manners and those that follow can be derived the way to make the vaults, porticoes and curves of the chambers (Fig. 4.22).

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[35] Serlio (1566, Libro II, p. 63v).

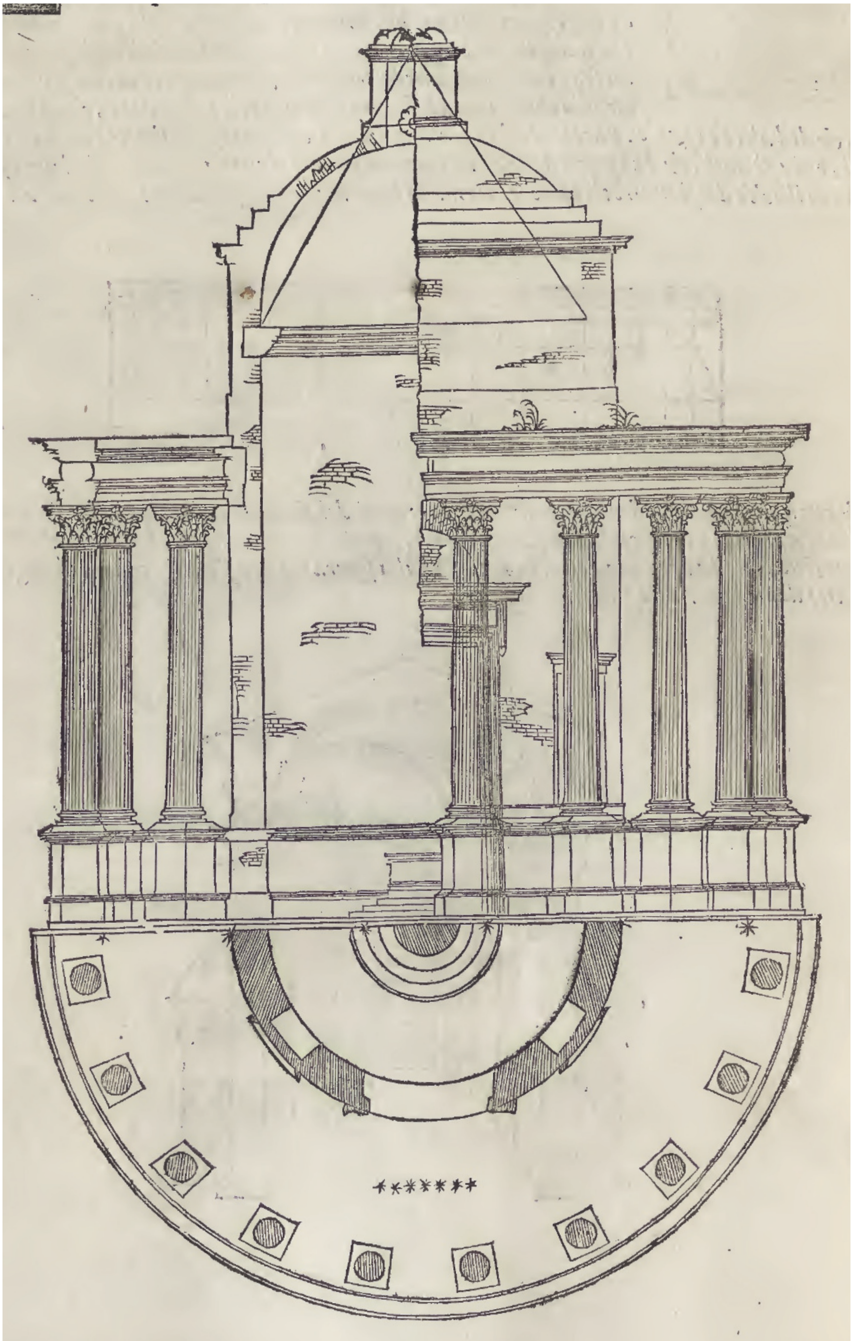

Fig. 4.22 **[p. 154]**

[p. 155]

Chap. XVI. Description of the Tragic scene

I have little writing to set forth in the description of scenes. I will only advise those who paint the scenes to place the point with great circumspection so that the things that are drawn do not appear ruined, and so that they serve the site and natural appearance. This is done conveniently by placing the eye beyond the plane of the scene considering the distance of the spectators and the size of the buildings. The distance of the spectators is that at which the listeners stay, and they cannot stay to see in places where they cannot also hear the reciters' voices clearly. For this reason I have not wearied myself with setting out figures other than those of the scenes already made by others, leaving each to be free to place them as they please. Here below is the Tragic scene (Fig. 4.23).

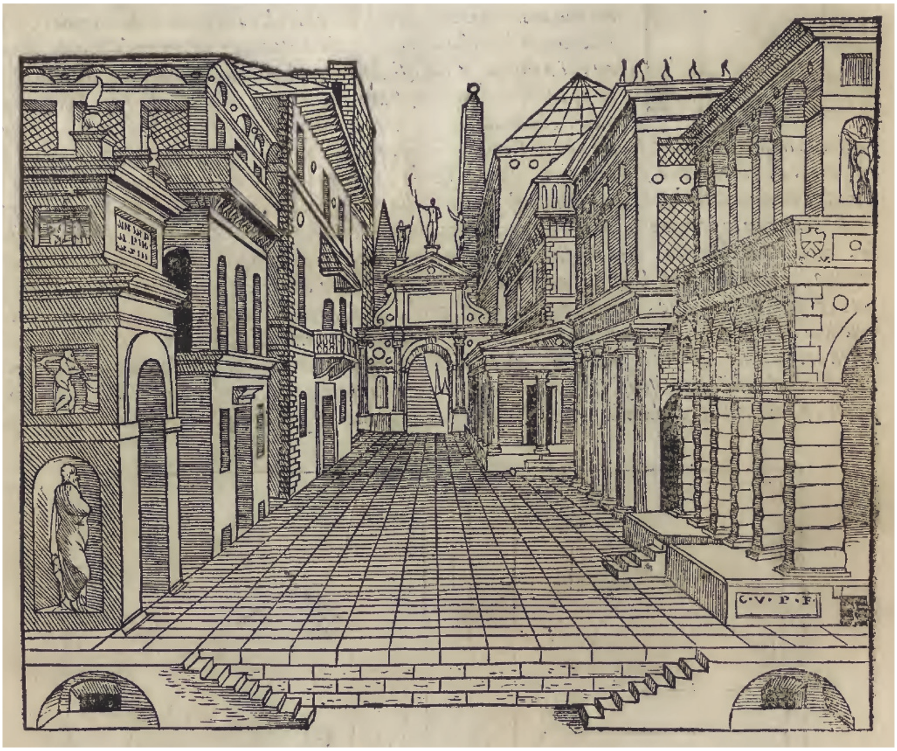

Fig. 4.23[36] **[p. 156]**

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[36] Serlio (1566, Libro II, p. 45v).

Indeed, Pompeo Pedemonte, an industrious and practical man, has conceived a way to bring the buildings of the scenes into harmony with the paintings of walls and partitions such that the paintings appear to be buildings and what you will. The way is this. Let the face of the wall be *abcd*, with a stage or *pulpitum* (as Vitruvius says[37]) contiguous with *cdfe*. On this are to be placed various houses and temples in perspective, making them low, wide and narrow, and also making streets, porticoes, balconies, squares and all that is necessary. So, let a nail be fixed into the wall at a height that is equal to the height of the horizontal line, and let that be point *e*. To this is tied a cord, of the kind used by masons, so that it doesn't deviate. The front of the stage is divided into as many equal parts as desired; for now let it be divided into twelve. The cord will be drawn to the first division of the front of the stage *fg* from point *e*, and having fixed the cord it is necessary to stretch it back to the middle of the theatre, as in point *h*, and look at the stretched cord and consider that it casts a shadow (or as the mathematicians who make clocks say, makes the line of contingency) on the stage and on the wall, say, for example, it is *emi* [*recte em*1] such that *im* [*recte* 1*m*] is on the stage and *me* on the wall, and note everything. In a similar way the cord is pulled from *e* to **[p. 156]** point 2, and looking from point *h* will make 2*V* on the stage and *Ve* in the wall. This will be done with all the other divisions, which will make lines common to both the wall and the plane. Then, wanting to build on the stage and place [things] in perspective, it is necessary to set up some rulers that are long enough vertically over the divisions of line *fg*. The cord is raised high and low, according to whether high or low buildings are desired, so that the roofs, cornices, projections, windows, doors and columns, and ultimately everything, are regulated by the lines that come from the view that is had from point *h*, looking along the cord. For example, say a house is to be built on the stage, and let it be *xyz*&. Ruler *io* [*recte* 1*o*] must be set up at point *i* [*recte* 1], and the cord stretched from point *e* and raised to point *o*, and that will be *eo*. The line formed by the cord, observed from point *h*, will indicate the top of the house, which will be *xy*, and then lowering the cord from the roof at the determined height of the windows, you will make the outlines of the windows, and with similar ways will be drawn all the other things, as shown in the figures below (Fig. 4.24).

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[37] Vitruvius V.VI.1.

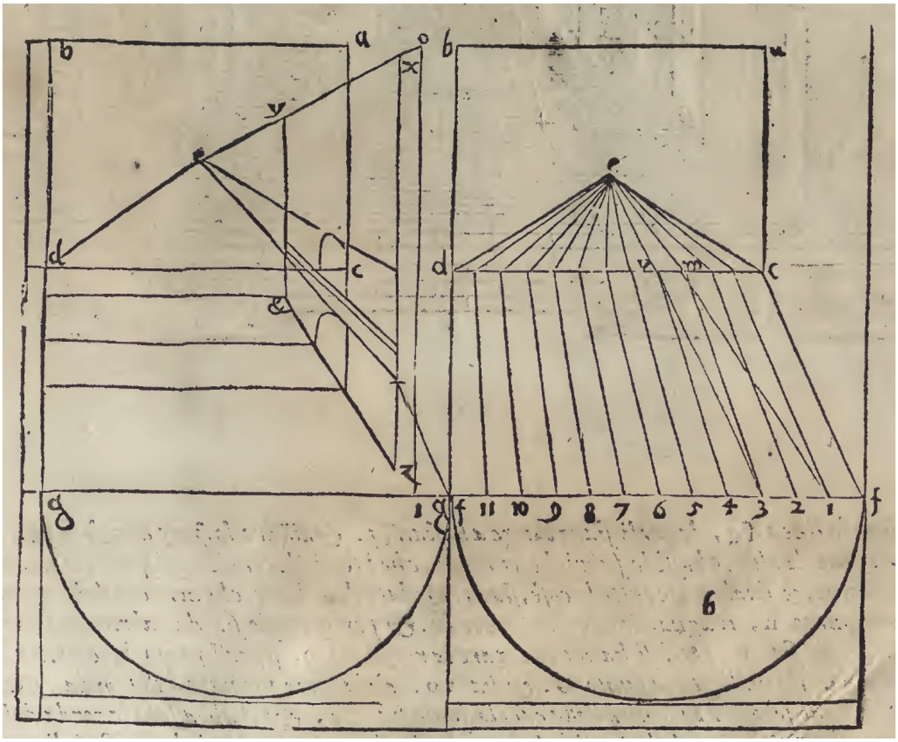

Fig. 4.24 **[p. 155]**

[p. 157]

Chap. XVII. Description of the Comic scene

The example of the Comic scene shows private buildings, since such scenes are for persons who are introduced there. So this too is free to be described by each according to the intentions of the fables that are to be recited. In this there is less knowledge of architecture than in the Tragic, because the buildings are of private persons, who accommodate themselves as best they can. Nevertheless the practice of perspective is the same with respect to the spectators and the appearance of things. The example is in the figure below (Fig. 4.25).

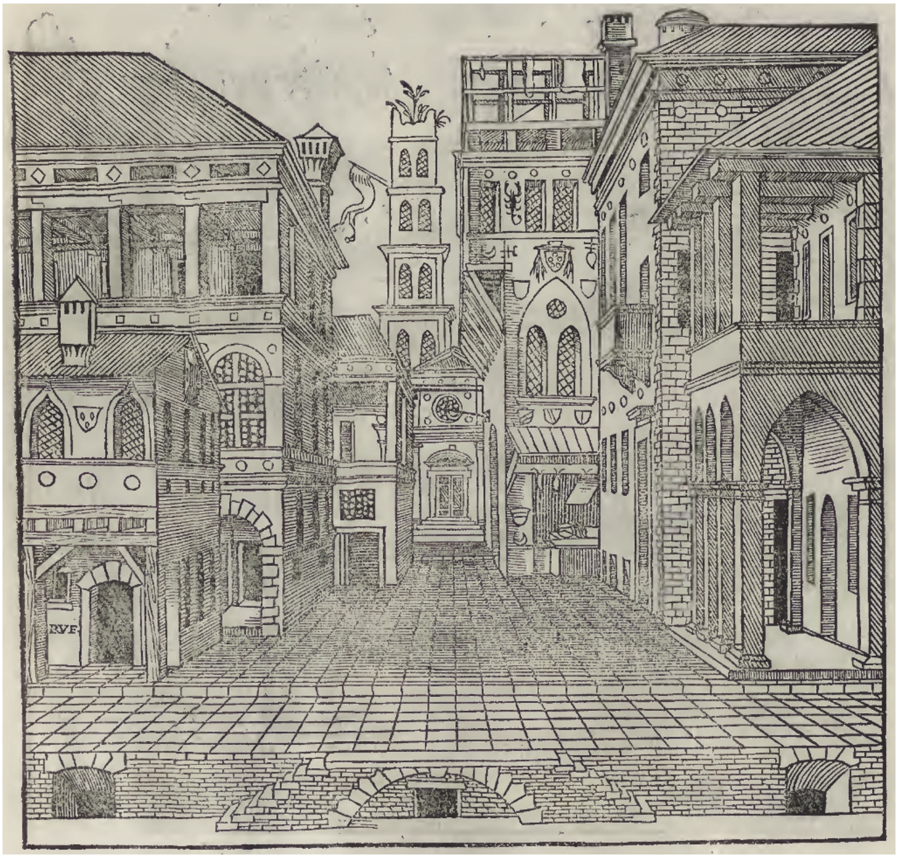

Fig. 4.25[38] **[p. 157]**

[p. 158]

Chap. XVIII. Description of the Satiric scene

The Satiric scene requires great discernment, both because it is necessary to have the consideration of the eye as in the others, and because the trees, mountains, towns, huts and shelters are by their nature things that are indeterminate. Much knowledge is needed of lights and shadows, of distances and the effects that landscapes generate, so in addition to placing the point in its place, and referring everything to that, it is necessary to well understand natural effects and imitate the true things as far as possible, with the correct colours. The example is in the figure below (Fig. 4.26).

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[38] Serlio (1566, Libro II, p. 46v).

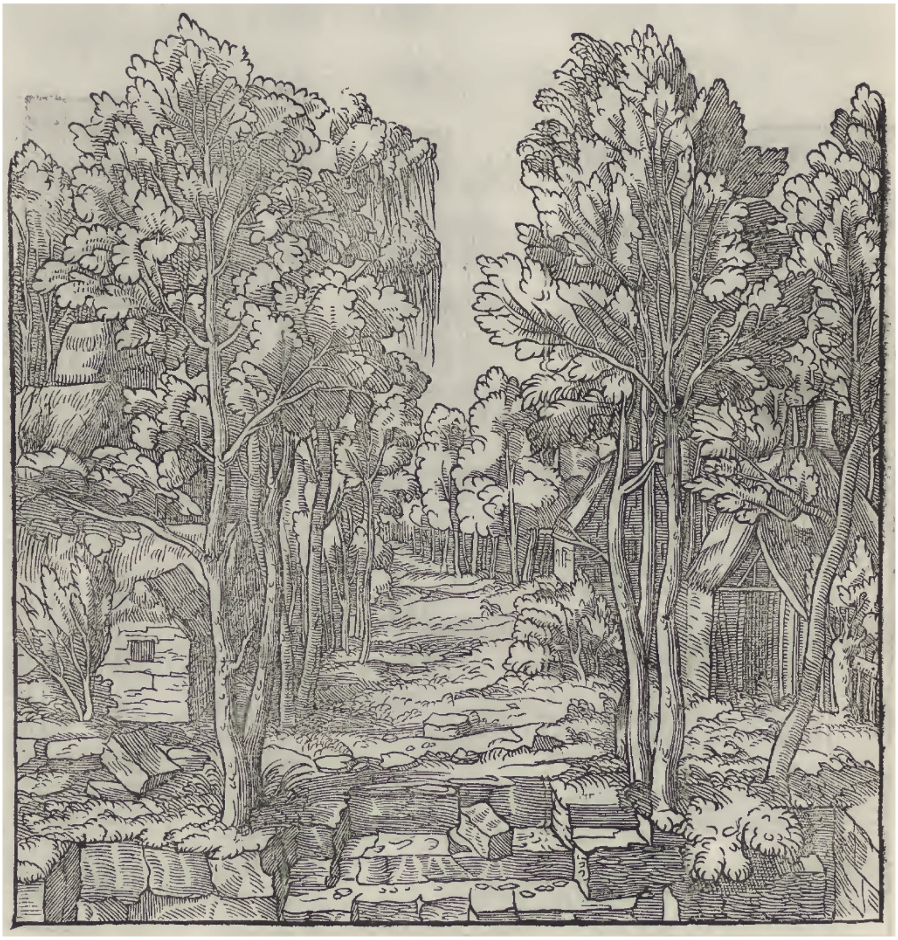

Fig. 4.26[1] [p. 158]

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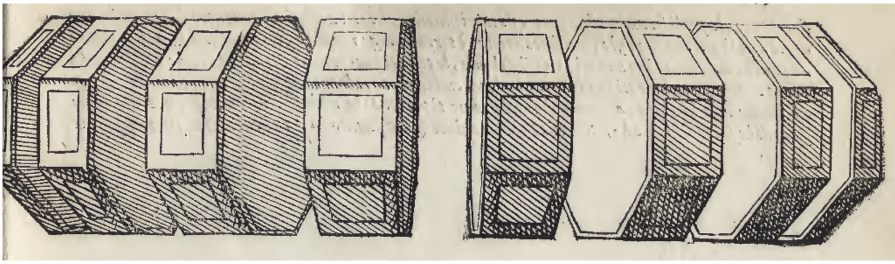
[1] Serlio (1566, Libro II, p. 37r).



Part V, in which is set forth a fine and secret part of perspective

Chap. I. [Distorting images]

[p. 159] Many times, with no less delight than marvel, are seen some panels or sheets of perspective in which, when the viewer's eye is not placed in a determined point, what appears is something other than that which is depicted, but then, when viewed from its point, there appears that which was actually made according to the intention of the painter, be it an effigy of a prince, an animal, letters, or whatever. This practice is born from those principles that I set forth in Part I. In this part it occurs to me to separate this demonstration from the other parts, in order to excite the minds to find other inventions, because with the principles given above many ingenious perspectivists have discovered beautiful things. Some have contrived to write letters on panels that cannot be read except with mirrors, and almost seem to reverberate. Some with reflections of light have drawn clocks. Some, using the medium of water for the refraction of rays, have done marvellous experiments, which they could not have done without the knowledge of nature and the properties of angles.

Chap. II. The first practice of these things

Take a piece of paper, on which you will paint one or two human heads, or anything else you want to. These you will prick holes in, as though you wished to make

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0_8

a pouncing, but with rather large holes. Then take the panel onto which you wish to transport the two heads, and make it so that it is fixed quite flat and clean. At the side of this panel you will set up the pricked paper at right angles, as though the panel was one wall and the paper another that joined up to the first and made a set square. When you have set the paper up, raise the panel with the edge to the sun, according to its height, so that, the rays passing through the holes in the paper, which are like sights (*traguardi*), you can see on the panel how the rays of the sun describe the **[p. 160]** heads. These will be stretched and pulled out in such a way that looking at them when you stand in front of the panel, they won't look like heads to you, but rather, like straight and curved lines without any rhyme or reason. But if you stand in the place where the rays of the sun come from, the heads will show themselves to you in the form they had on the paper. You will understand these things better when you practice them than when you read about them. You can also do them with a lantern, setting it as the view leads you to do. Also remember that if you look at the surface *bcde*, placing the eye in the middle at point *a*, you will be seeing under a larger angle than if the eye was at point *d*, and therefore the size of the surface will appear to you to be larger in *a* than when you are in *d*. It can also be (as has been said) that the eye being lower **[p. 161]**, in the same plane, than surface *abcd*, not only will the angle be smaller, but two lines will appear to be one and not to make an angle; if, in the same surface as before, the eye were to be at point *I*, line *ab* would appear to be one with line *bi* (Fig. 5.1)

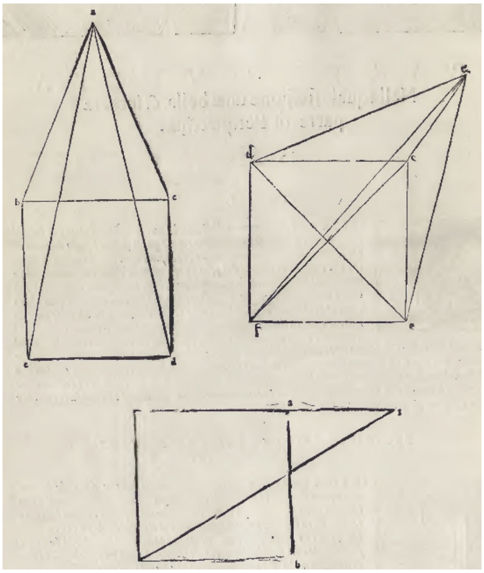


Fig. 5.1 **[p. 160]**

Chap. III. The second practice of these things

It is possible, without the sun and without the lantern, and without the pricked paper, to do the same thing. The first way is with the rules set out in Part II regarding around the description of the planes and the perfect figures. The second way is with instruments, about which I will reason in the Part IX. So, among all else, it must be noted that it is necessary, in order to better hide what is painted with these practices, that the perspective painter who has to paint the two heads or whatever else, know how to do the shading, and to cover the painting with different strokes of the brush so that it gives one appearance from far away of things painted, showing towns, rivers, mountains, stones and other things different from what is painted. He can and even must deceive, cutting and separating the lines that should be straight and continuous, so that away from the proper point of view is not shown that which appears from the proper point. Here is the example. The line *ab* is separated from line *cd*, and it will appear separated when the eye is anywhere except in the plane where they are. But if the eye is placed in the same plane, the two will conjoin, because the space between them is not seen. Likewise, if you make the forehead of a figure in one place, and the nose in another, and the chin likewise in another, such that the end of the forehead is *b*, the beginning of the nose is *a*, the end of the nose *c*, and the beginning of the chin *d*, then when looking at the figure from the proper point it will appear that *b* is conjoined to *a*, and *d* to *c*. But when looking at the figure from anywhere else, the distance and space between one part and the next would be seen, and it wouldn't be recognised that the painting represented a head; instead the nose would appear to be one thing and the forehead another, especially if the painter knew how to hide the shape of the nose with some other similar shape, making it or pretending that the thing that is supposed to be a nose is a stone, and the forehead a clod of earth, as he sees fit (Fig. 5.2).

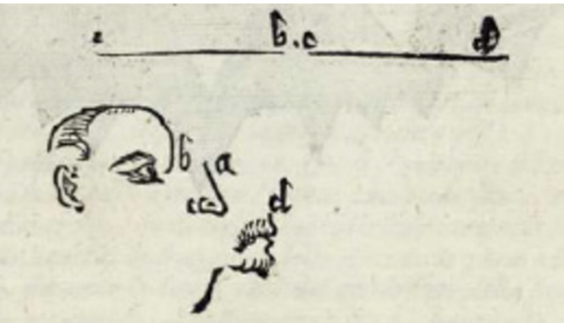
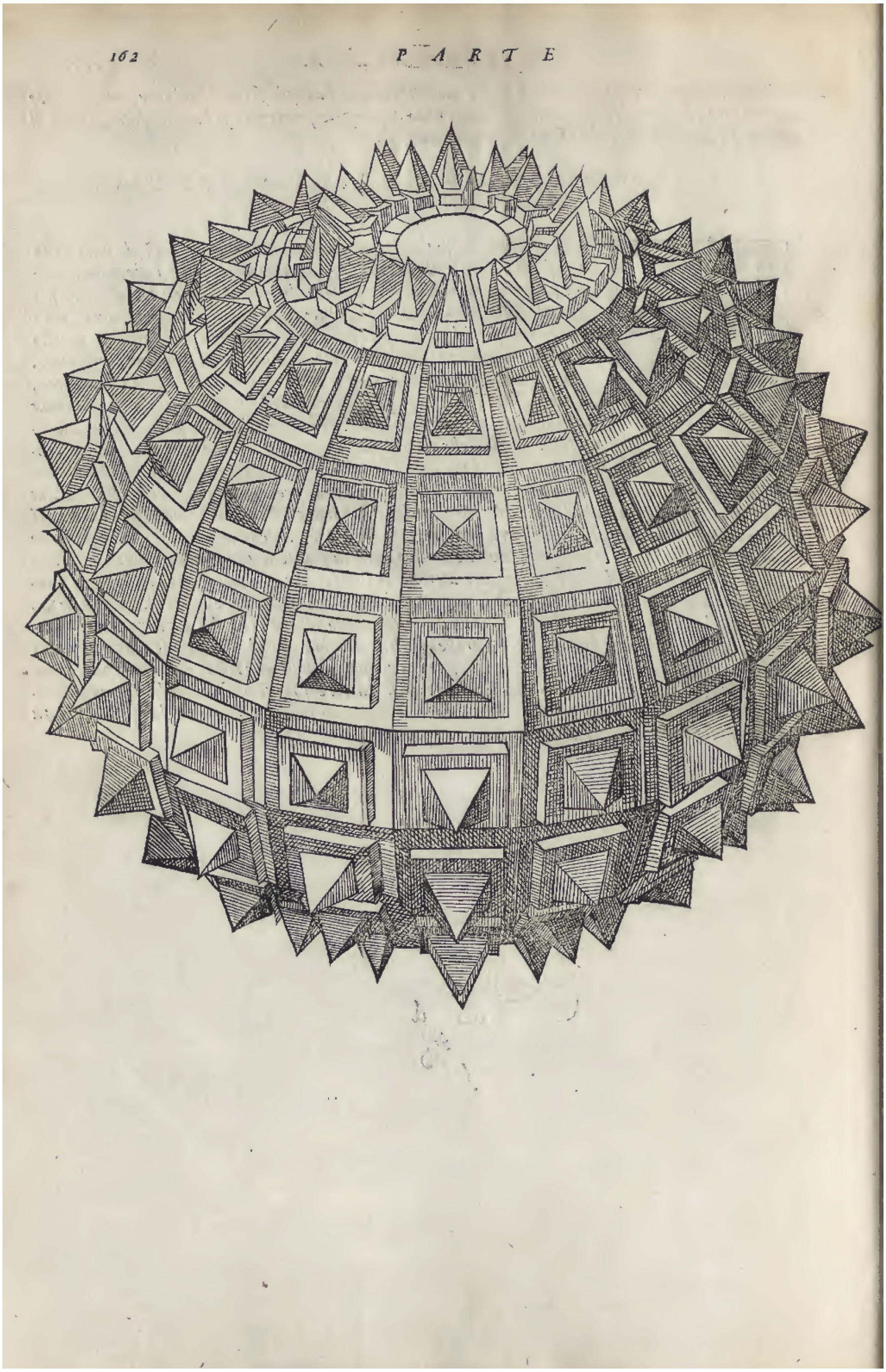


Fig. 5.2 **[p. 161]**



162 PARTE

Part VI, which is called *Planispherio*

Chap. I. Unfolding, description and degradation of the sphere

[p. 163] A fine, ingenious and useful invention was that of the ancients for casting the points and circles of the sphere onto planes with proportion and correspondence of ratios, because its demonstrations were of the greatest benefit to the investigators of celestial things. And because they were founded on perspective, it seems to me that it is reasonable to devote a part of my efforts to the practice of such a fine invention. So selecting the way that is most convenient among many, I will make understood by examples what is desired of this material. First I will come to the unfolding of the sphere, as I have done in the description of the bodies in Part III, and if those first five bodies are regular, so much more so must be the sphere because of its uniformity, and also because it circumscribes the other bodies. Then following the shading, with the rules previously set out, I will come finally to casting the sphere onto the plane, as taught by Ptolemy in his treatise dedicated to that.[1] So, the unfolding of the sphere is done by some in the following way. Divide a quadrant of a circle into three parts, with points *a*, *d*, *e*, *c* (Fig. 6.1).

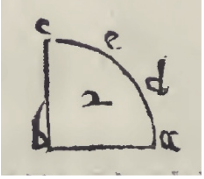


Fig. 6.1 **[p. 164]**

Let there be drawn a line such that *ad* of the quadrant goes into it thirty times, and according to that space, let said line be divided into thirty parts marked with the numbers 1, 2, 3, 4, up to 30. Take then with a compass the space of ten of those parts, and place the foot of the compass on the number 1. Draw an arc of a circle that passes through the number 11. Then with the same opening place the compass over the number 2, and draw the arc that will pass through the number 12. Going on like this, 12 arcs will be made. Then, the compass remaining with the same opening, place one foot over the

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[1] Commandino, *Ptolomaei Planisphaerium* (1558).

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0_9

number 29 and make one by one 12 arcs that will cut the first ones, and with those form certain lunette-like figures (Fig. 6.2).

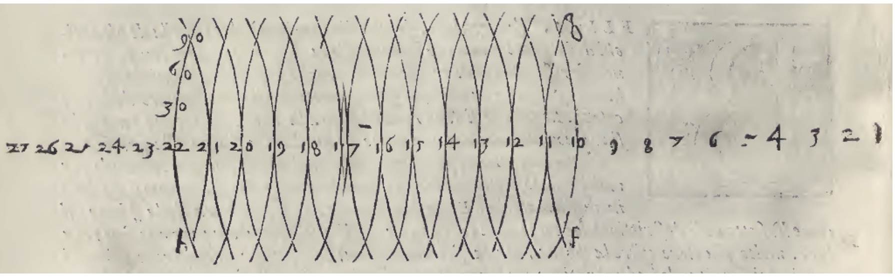

Fig. 6.2[2] **[p. 164]**

In this way you will have the unfolding of the sphere. If you then have someone lathe a ball whose half-diameter is like line *ab* of the quadrant in figure 2 (Fig. 6.1), cutting away the excess paper, and arranging the unfolding of the sphere over it with those arcs closed, you will have the perfect sphere, as was said (Fig. 6.3).

In this way are cut the papers of maps of the world in order to arrange them over the balls, and they come out just right. When it would please us to locate the meridians, straight lines can be drawn from the crossing of one arc with another in each lunette, such as from point *f* to point *g*. When the ball is closed, all the points come to be united at the poles, and **[p. 164]** and each great or small circle of the sphere [the parallels], whether straight or oblique, can be marked, dividing one of those circumferences of arcs into degrees, as is seen in the last arc marked with the numbers 30, 60, 90 (Fig. 6.2), according to the declination of the sun, or according to other respects noting the signs of the heavens or the parts of the earth. The perfects of the spheres [plan and elevation] are in the figures marked 3 (Fig. 6.3), of which one has the two poles with one on top and the other below (Fig. 6.3, right), and the other has a pole in the middle as you can see (Fig. 6.3, left). Since in a larger form things can be seen better, I will draw said sphere larger.

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[2] Cfr. the image in Dürer, *Underweysung der Messung* (1525, Book IV, fig. 34).

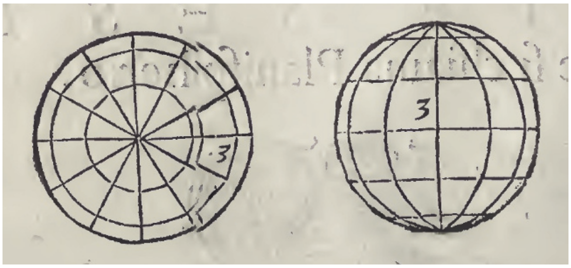

Fig. 6.3[3] **[p. 164]**

So, let on centre *m* be made circle *A*, in the square 1, 2, 3, 4. Let it be divided for now into 16 parts, with the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 above and below point *A*. Then having drawn the diameters, lines fall from each point of the semicircle above to the similar numbers and points in the semicircle below: 8 onto 8, 7 onto 7, and so on with the rest. Where those lines cut the diameter *A*9 place the letters *B*, *C* and *D*. Placing the foot of the compass in centre *m*, and opening it to point *B*, draw the circle *B*. Closing it then to point *C*, draw the circle *C*, and finally with the space from *m* to *D*, draw the circle *D*. If from the points of circle *A* you draw the lines to centre *m*, you will divide all 4 circles into 16 equal parts, as shown in figure 65 (Fig. 6.4), which is the perfect of the ball with the pole in the middle.

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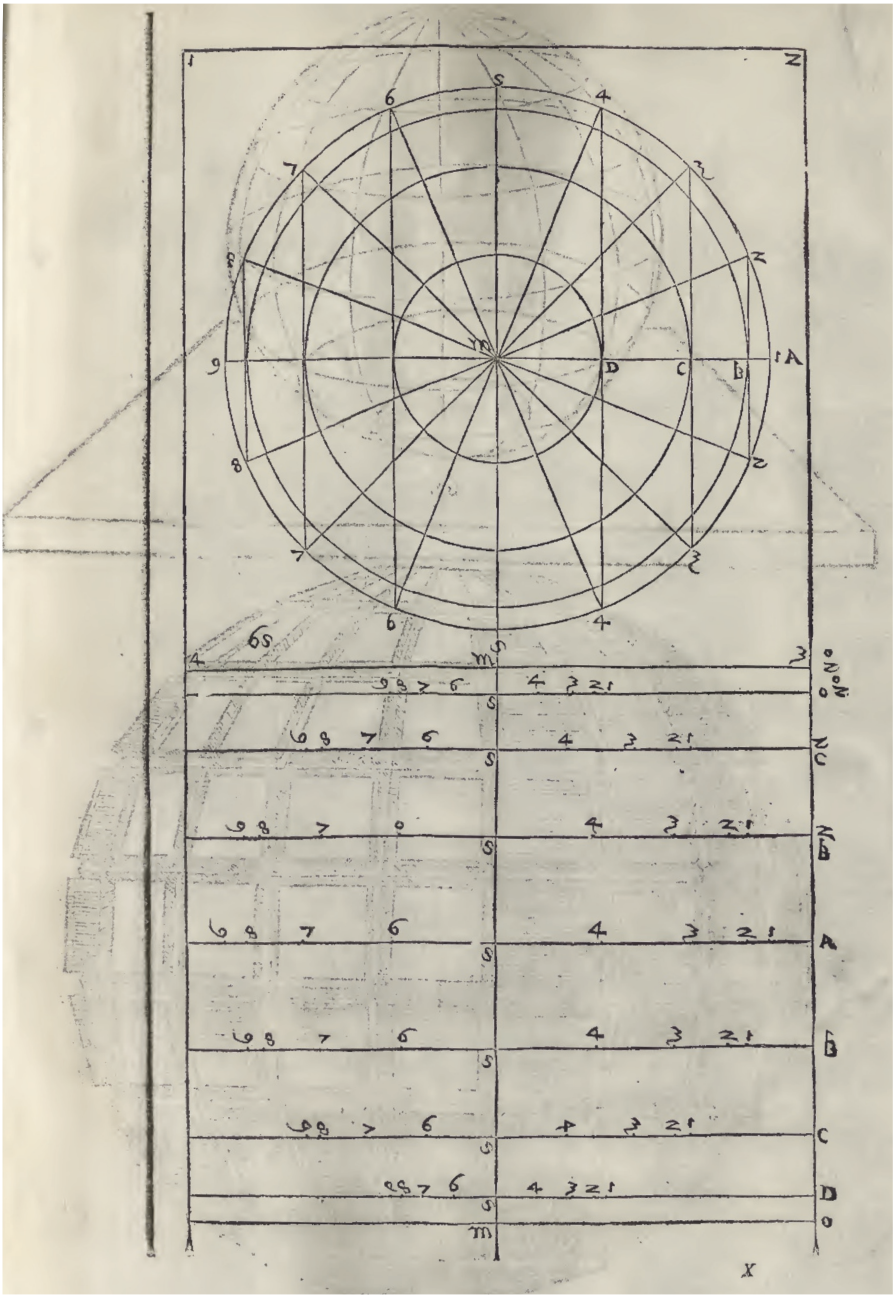
[3] Cfr. the image in Dürer, *Underweysung der Messung* (1525, Book IV, on the page following fig. 34).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C
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m
X

Fig. 6.4 [p. 165]

From this are taken the widths with a thread and the compass, as described above [in Part III] in the formation of the mazzocchio. The heights are actually taken from the same ball that has two poles, and with the transportation of the points onto the diameters, and with the same practice of the two compasses [of height and width[4]] the sphere can be degraded with its circles, as shown in the degradation in figure 65 (Fig. 6.5).

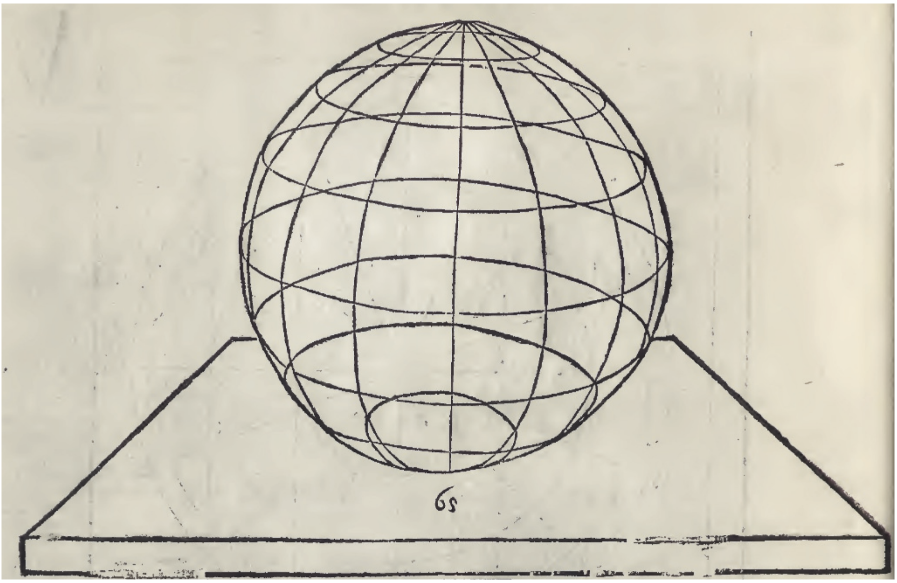

Fig. 6.5 **[p. 166]**

The same rationale used to make the mazzocchio with its pyramids can be used to make the ball that thrusts out its pyramids, or that has square coffers on its surface, of other things that the painter wants (Fig. 6.6).[5]

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[4] See Part III, Chap. XXXVII **[p. 120]** for the description of the 'two good, well-made compasses'.

[5] Barbaro does not limit himself to suggesting variations of the sphere to the reader. He personally carried these experiments out in the extended Italian manuscript It. IV, 39=5446, fols. 119v-138r and 168r-215r'.

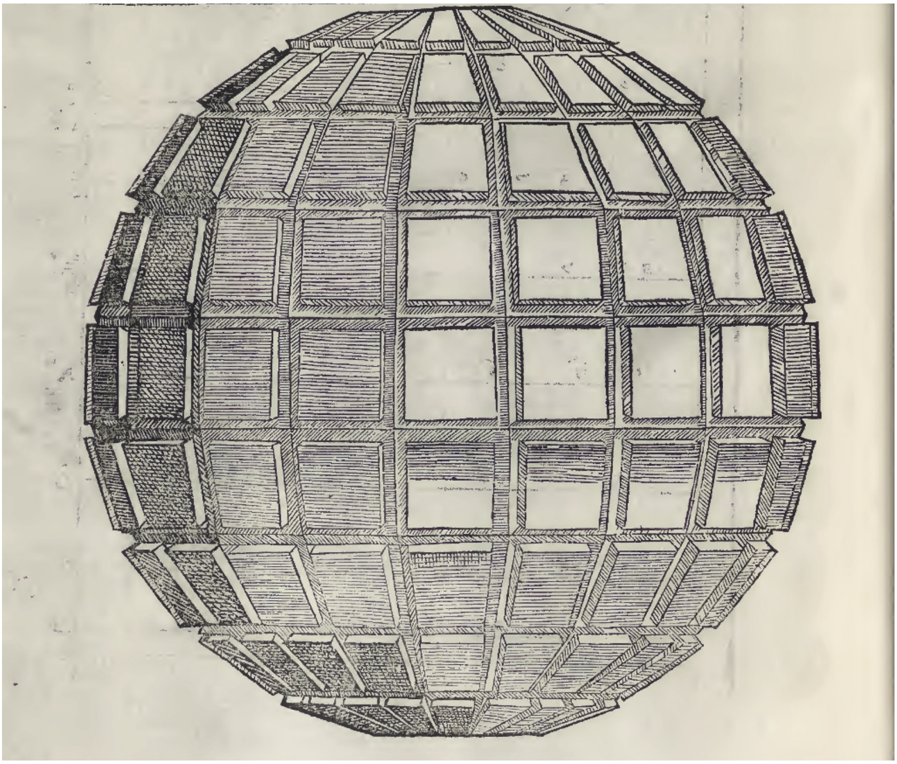

Fig. 6.6 **[p. 166]**

[p. 167]

Chap. II. Projection of the sphere onto the plane according to the ancients

Let us imagine that the eye is in one of the poles of the sphere, and let that be the Antarctic pole, since for the contemplators of the heavens it is convenient to look from the pole that rises over their horizon, and for us the Arctic pole is taken into consideration. Looking then at point *o*, where the eye is, we draw a straight line to point *a*, where the other pole is. This line *oa* represents the hinge or axis of the world, and for now it serves us for the diameter of a meridian, which is marked *aoBC* [*recte oBaC*].[6] On centre *E* [*recte* point *a*] let there be drawn a transverse line, on which line *oa* falls at right angles, and this passes through point *a*. This line is the plane onto which the rays of vision must fall from point *o* to the points of the meridian. Transporting all the points of the sphere, except for the pole *o* where the eye is, since the eye does not see itself (as we said in Part I), the angle of contingency[7] does not fall within vision: nor is anything else described in that plane except that which is on the concavity or convexity of the sphere, there being no consideration of its depth (Fig. 6.7).

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[6] In Fig. 6.7 *B* and *C* are not shown over and below the center *E*, but to the right and left.

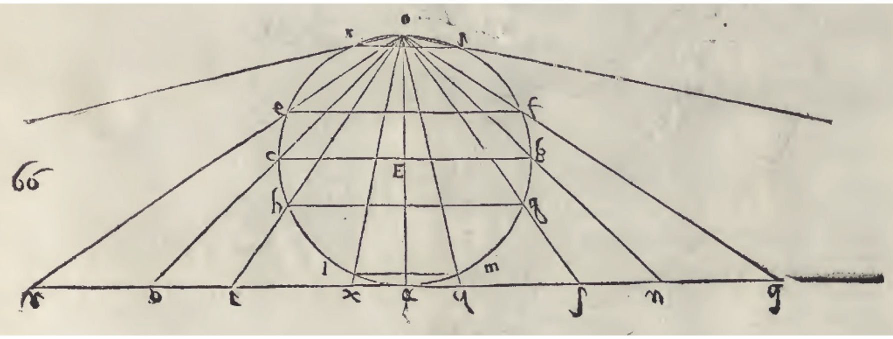

Fig. 6.7 **[p. 167]**

So, let there be drawn, in meridian *oBaC*, the parallel diameters of the circles. The equinoctial *bc* [*recte BC*], the tropic of Capricorn *ef*, the tropic of Cancer *gh*, the diameter of the Antarctic Circle *ik*, and of the Arctic Circle *lm*. It is necessary to carry the diameters of these parallel circles onto the plane *dac* [*recte* passing through *a*], so that by then forming on each diameter its corresponding circle, these circles are represented in the plane with such proportion that corresponds to the circles of the sphere imagined at the location and in their order. So, let the rays start from eye *o* and pass through the end points of the diameter of the equinoctial *bc* [*recte BC*], they arrive to the line of the plane in points *n* and *p*. I say that *np* will be the diameter of the equinoctial cast onto the plane. So if you take the space *an*, and according to that [as a radius] form one circle, you will have transported the equinoctial onto the plane. And if you want to transport the tropics, you will do it in the same way, passing the rays from the eye *o* through the end points *e* and *f*, and *h* and *g*, onto the plane to points *q* and *r*, and *s* and *t*, where the diameter of the tropic of Capricorn will be *rq*, and that of Cancer *st*. If then from the same centre from which you drew the equinoctial you draw the tropics, you will form those circles in the plane, and with the same rationale you will transport the **[p. 168]** Antarctic Circle and the Arctic Circle, which circles are called 'polar' because they are close to the poles, and because within them are the poles of the Zodiac. If in meridian *OBaC* [*recte oBaC*] you place the diameters of the other parallel circles, which are the circles of the signs of the Zodiac, according to their declinations, from those, with the rays that from the eye to the line of the plane pass through their end points, you can transport the diameters and circles as shown in figure 71 (Fig. 6.8).

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[7] For an earlier reference to the line of contingency, see Barbaro, Part IV **[p. 155]**.

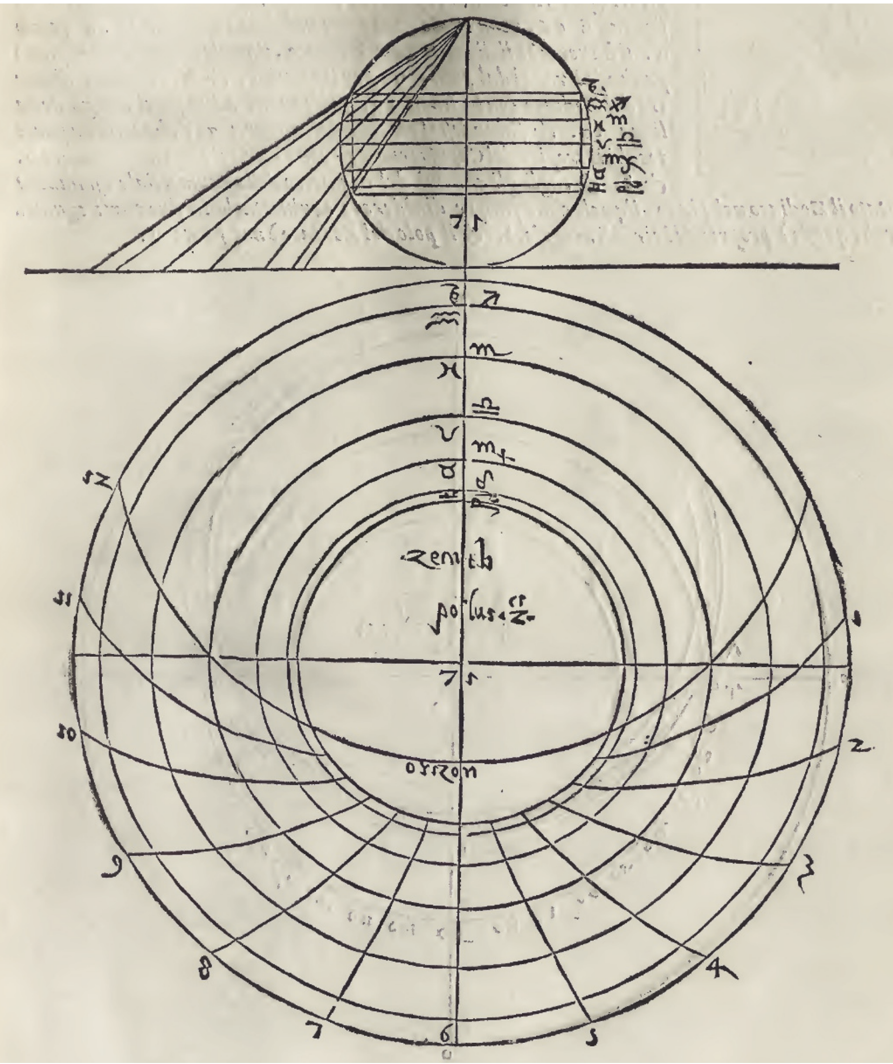

Fig. 6.8 **[p. 169]**

Having found the diameters of the parallel circles on the line of the plane, and wanting to transport said circles to the plane, you will make, at centre *a*, a circle of a size such that its diameter will be equal to the space from point *q* to point *r* in figure 66 (Fig. 6.7), and that will be the circle of Capricorn. Likewise in the same centre *a*, you will make parallel [concentric] to the circle of Capricorn a [equinoctial] circle whose diameter is equal to the space from point *x* [*recte n*] to point *p* in figure 66 (Fig. 6.7). To make the circle of Cancer, you will make on said centre *a* a circle whose diameter is line *st* in figure 66 (Fig. 6.7), and in this way you will have the circles of both tropics and the equinoctial. If you wish, from figure 71 (Fig. 6.8), to derive the other parallel circles and transpose them to centre *a* as you have done with the first three, you can do this conveniently, as seen in figure 71 (Fig. 6.8), which you will divide into 4 parts with two diameters *bc* and *de*. Note that in figure 71 (Fig. 6.8), the draughtsman has made this much larger than that which is taken from figure 66 (Fig. 6.7); I say this so that the reader will not err.
Here is seen clearly why the ratio of the tropic of Capricorn looks much larger in comparison to the equinoctial and the other tropic: because the tropic of Capricorn is closer to the eye, it is seen under a larger angle than the other circles.[8] Consequently it appears larger to us. So too the circle of the Antarctic pole: even though it is among the smaller circles, when cast onto the plane it is much larger than the others, as can be seen in figure 66 (Fig. 6.7), if the size of the paper permits it.

[p. 170]

Chap. III. How to make projections of oblique circles

The oblique circles are transported to the plane in a way similar to that of the horizons, the Zodiac and the others. For example, this would be shown in figure 66 (Fig. 6.7): if there were a line from point *f* to point *h*, that would be the diameter of the Zodiac, which, transported to the plane, the rays passing from point *o* to point *f* and *h* would make the diameter of the Zodiac with line *tq*. Then taking the half of line *tq*, and making a circle in figure 70 (Fig. 6.9), which on one side touches the tropic of Capricorn, and on the other the tropic of Cancer *b* and *f*, and cuts the ends of the diameter of the equinoctial, this will have cast onto the plane the Zodiac. This must always divide the equinoctial into two equal parts, because this is proper to greater circles [meridians of the sphere]. The pole of the Zodiac is in point *u*.

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[8] Cfr. Barbaro (1567, p. 432), Williams (2019, p. 721).

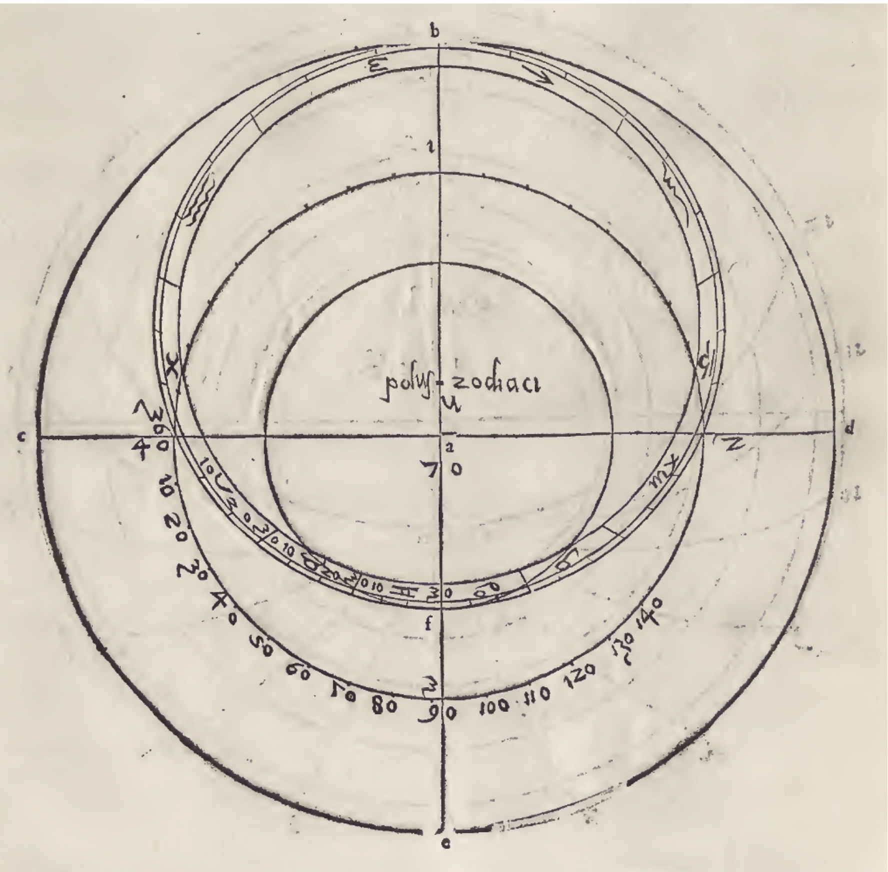

Fig. 6.9 **[p. 170]**

With the same rationale are transported the horizons and the horizontal circles, called ‘almicantarath’ by the Arabs.[9] These are circles parallel to the horizon, which are reduced degree by degree to the point that is over them, called ‘zenith’, or vertical point, as shown in the examples in figure 68 (Fig. 6.10).

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[9] Samuel Johnson, *A dictionary of the English language* (London, 1755, vol. 1): ‘ALMACA’NTAR. n.f. [An Arabic word, written variously by various authors; by D’Herbelot, *almocantar*; by others, *almucantar*.] A circle drawn parallel to the horizon. It is generally used in the plural, and means a series of parallel circles, drawn through the several degrees of the meridian’.

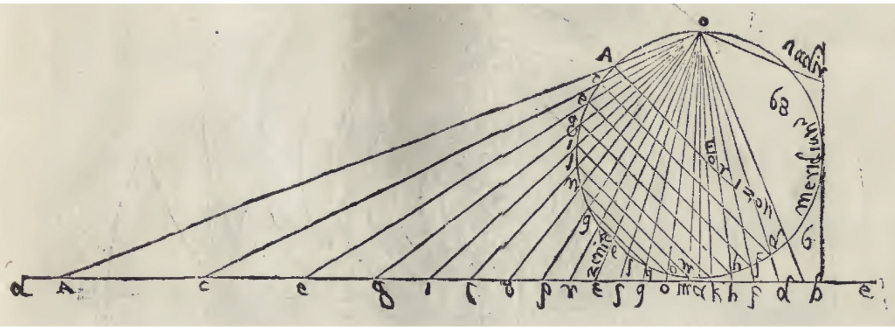

Fig. 6.10 **[p. 171]**

Let there be the same meridian *obac* [*recte oBaC*] on its plane *de*. Let the diameter of the horizon be *AEb*, according to the height of 45°, counting from point *o*, then let there be the vertical point, or Zenith *i* [*recte t*], and the diameters of the horizontal circles of 10 by 10 degrees for now. It is possible to make them of degree by degree, or of two by two, or however you wish according to the size of the drawing surface. *id* [*recte cd*], *ec* [*recte ef*], *gh*, *im* [*recte ik*], and the rest, as is seen in figure 68 (Fig. 6.10), pass then from the eye *o* through the ends of the diameters of the horizontal circles out to plane *dac* [*recte dae*], as *oad* [*recte oAA*], *oic* [*recte oce*], *oeg* [*recte oee*], *oli* [*recte oll*] and the rest, as shown in figure 68 (Fig. 6.10). Wanting then to transport the horizontal circles to the plane, you will use the same method that you used in transporting the Zodiac and the parallel circles. So, the diameter of the horizon *AEb* is on the line of the plane equal to the space *db* [*recte Ab*], and in the middle of that distance is the centre of the horizon. Opening the compass from that centre to one of the ends, that space is transported onto the meridian line, and the horizon is drawn. That horizon must pass through the points of the equinoctial, for the same reason that the Zodiac also passes through the same points, that is, because the horizon is one of the greater circles. Likewise the first horizontal circle will have its diameter in the line of the place equal to space *cd*, and the other will have its space equal to *of* [*recte ef*], and the third *gh*, and so on with the rest, as shown in figure 68 (Fig. 6.10). You will then make figure 67 (Fig. 6.11). First you will put the parallel circles, so that the circle *fz*3 [*recte ez*34] will be the tropic of Capricorn; when the Sun is in it, it makes the winter solstice, called 'bruma' by the ancients. The equinoctial is *lmno*. It is called 'equinoctial' because when the Sun is in those signs, that is, in Aries and in Libra, day is equal to night. The tropic of Cancer is *pqrs*.[10] This denotes the summer solstice, during which the day is the longest of the whole year with respect to those who live in the northern hemisphere. The Zodiac, called the 'signifer' because it carries the celestial signs, cuts the equinoctial in two equal parts at *m* and *o*; it touches the tropic of Capricorn at point *e* and the tropic of Cancer at point *r*. Likewise in that figure (Fig. 6.11) is the Arctic circle, transported, like the others, from figure 66 (Fig. 6.7), and there is also the horizon *mbo*, which passes up to the circumference of the tropic of Capricorn on both sides. Finally, there are the circles or horizontal arcs *d*, *f*, *h*, *k*, *m* and the rest of the 10 by 10 degrees, up to 90, where there is the Zenith, the vertical point.

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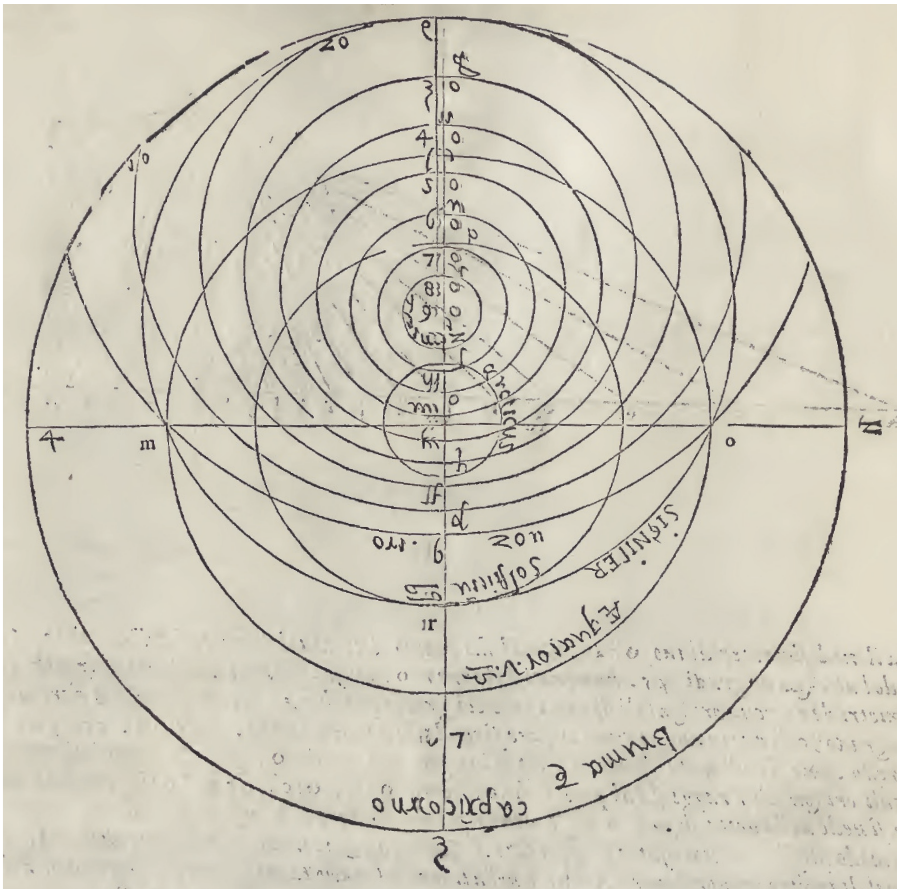
[10] Points *q* and *s* are not marked in Fig. 6.11.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capricorno

Fig. 6.11 [p. 172]

[p. 172]

Chap. IV. The projection of the circles of the heights onto the plane

Having practiced the placing of the horizontal circles, the largest of which is the horizon, easily leads us to consider how some of the circles of the sphere are transported onto the plane with circular lines and some with straight lines, and that all of those circles that pass through the poles of the world – I say all of them – are represented in the plane with straight lines. No circle that is represented in the plane by a straight line can be represented whole, because its pole is outside the plane, and all such are either greater circles or smaller circles, and because they pass through the pole, in which is the eye, which cannot see itself. But let us leave this consideration for now, and attend to casting onto the plane those circles that are called 'azimuth' by the Arabs. They are imperfect circles made to demonstrate the height of the stars which divide the horizon into 360 equal parts. All pass through the point that is over us, called 'zenith', and for this reason they are called 'vertical circles' by the Latins.

[p. 173] Let there be made figure 69 (Fig. 6.12), with the tropics, equinoctial, Zodiac and horizon, onto which must be drawn the circles of heights. So, take from figure 68 (Fig. 6.10), on the line of the plane, the space from *t*, which there represents the zenith, to point *a*, which is the point where the meridian touches the line of the plane. Carry that space to figure 69 (Fig. 6.12), from centre *a* onto line *cad*. At point 9 will be the Zenith, or vertical point through which must pass all the vertical circles called azimuth. To do this is it necessary to find the point opposite the zenith, which is called 'nadir' by the Arabs. So, you will make a circle which passes through the zenith and the point *m* and *o*, which are equinoctial points where the horizon cuts the equinoctial; this circle you will make thanks to the three points, and it will serve for the first azimuth. Its centre is in point *p*, and its nadir in point ✠ on line *cad*, extended. If you want other azimuths, you will do it in this way. Divide the first azimuth into 18 parts, starting with point ✠, and on both right and left mark 1, 2, 3, up to 9, where the zenith is. You will place the ruler over the zenith, and over point 1, marking in the azimuth, and where it passes through the diameter across the first azimuth, mark 1 on the right and 1 on the left,[11] and there will be the centre of the second azimuth. You will do this placing one foot of the compass on point 1 and opening it to point 9, where the zenith is, you will make that part of a circumference that comes from the zenith to the horizon, and where it cuts the horizon mark 10 on both sides. Then place the ruler on the zenith and on point 2 marked by the first azimuth. You will note that it cuts its diameter crosswise, and mark 2. Having made this centre, open the compass to point 9 and draw the arc of the third azimuth, which will cut the horizon on both sides, and mark 20, because that azimuth will be that many degrees away from the cut that the first azimuth makes on the horizon. With similar procedures you will construct the other azimuths, until you have divided the horizon into 360 parts, giving 90 degrees to every quarter. Since my intention is not to go farther, having already shed light on the way the ancients projected the sphere onto the plane, you will divide then all the circles below the horizon into 12 equal parts, and you will have the hours, as shown in figure 71 (Fig. 6.8).

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[11] In Fig. 6.12 point 1 is marked only on the left.

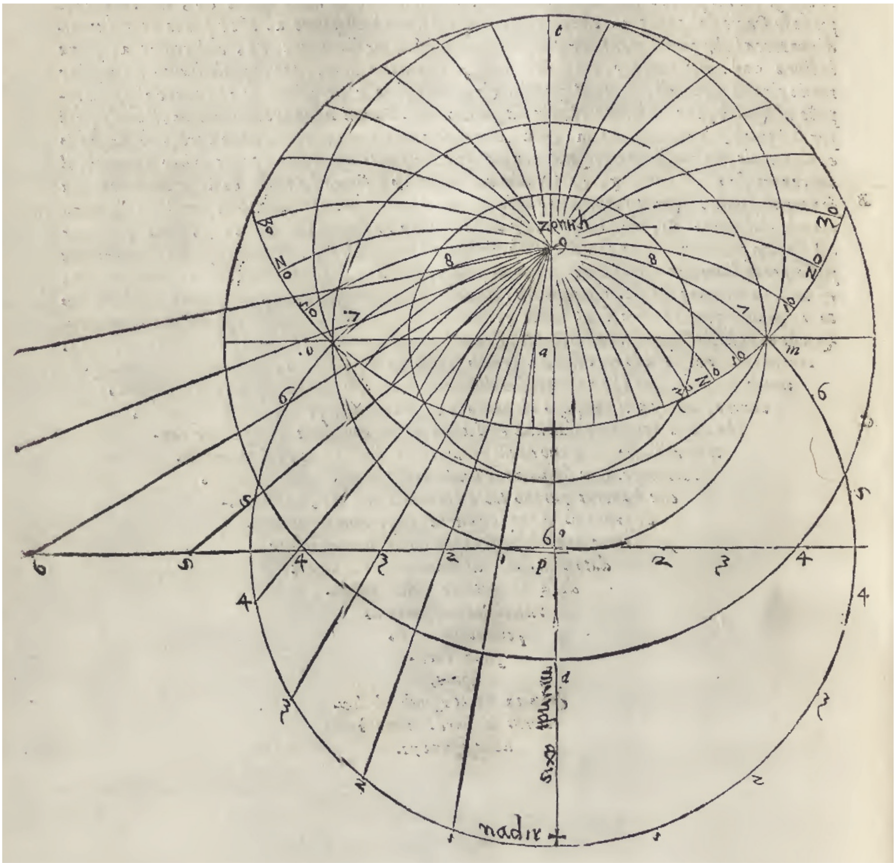

Fig. 6.12 [p. 174]

Part VII, which treats lighting, shadows and colours

Chap. I. [The importance of light, colour and chiaroscuro]

[p. 175] There is no doubt at all that the power of perspective depends on the nature of the angles, and that the appearance of things changes when the centre and the eye change, and that it is not sufficient for the erudite and expert to see and consider the bare lineaments of bodies and shapes put into perspective in order to judge things. But real pleasure and delight also come from colouring, shading, and giving the proper illumination. Imitation and expression increase to untold extent in goodness when they are considered, and every drawing comes closer to the perfect and true when it is made with the proper colours, appropriate lighting, and correct shading. If these things are done well, then on the plane the reliefs, projections and tapering of things are more gratifying because they make an effect that is more similar [to reality]. I will therefore say a few but necessary things about colours, lighting and shadows so that scholars of perspective will know how much effect will be had from one or more lights, higher or lower, brighter or dimmer, and if they come from stars, the Sun, fires or lanterns, or other sources of light, and also if they are primary or secondary, or reflections or refractions. So, primary lights are those that make secondary lights, such as the light that comes in through a window: the place in which the light falls is said to be illuminated by primary light, and all of that which is further illuminated is said to be

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0_10

touched by secondary light.[1] Of this manner are all the reflection of the rays that come from polished bodies and the refractions that are made by transparent bodies, of which there are subtle rationales in common perspective.[2] So, if we wish to imitate the true, as far as that is possible, it is necessary that there be a certain moderation in the lights and shadows; that is more readily subject to practice and observance of nature than to the laws of art. That which appertains to our purposes is the following. First, as to colour, the plane where the painting is to be made must be of a colour such that can bring out and allow to be observed all that which is painted on top of it. Thus a fine and ingenious practice is watercolour (*acquarella*), because with the water it is possible to make the same colour lighter or darker; the lighter colour is applied where the light strikes, and the darker colour is left where there are shadows. If they work in light and dark, then quite often – indeed, always – the good masters leave in place of the light the white of the paper or the panel on which they are painting. This ingenious way of light and dark demonstrates a fine example of arranging the colours when we wish to use colours. Since there is great agreement between colours and light, similar and almost the same observations can be made about them. So, there being **[p. 176]** a certain and determined part of a body that is touched by more vivid light, it is necessary that the lightest colour be placed in that part, and then deepening, little by little, from the lightest to less light, until the deepest and darkest colour is reached, so that those parts that are stricken by the light do not appear to be separated by lines from that which is in shadow. Of great reputation is the painter who knows how to unite colours and imitate nature, where in flowers and ripe fruit, and in the vermillion cheeks of youths, the colours change from the whiteness of snow to the colour of roses to deepest porphyry.[3] No one is subtle enough of eye to discern where the white finishes and the vermillion begins. This is also shown by the most delightful appearance of the celestial arc (*arco celeste*) [rainbow], in which not only is the same colour deepened or made more intense without any apparent differences, but there are also many and different colours, which appear different though there are no distinct demarcations. It is therefore necessary to diminish the lights and lightness of the colours with a certain dexterity, almost as though they die into the shadow and abandon their vividness little by little. Where sides turn and corners join there must be a certain moderation of colours and of lights so that the surfaces of the bodies appear different, as they are; if they were all touched by the same light and colour, they would not appear to be different and many, but all one. It is therefore necessary that the temperament of the lights and the colours be gently united and distinct. All of these things (as I have said) are the consequence of both observation of reality and experience combined with judgment.

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[1] Today we would say 'direct light' for primary light and 'indirect light' for secondary light.

[2] During the Renaissance Optics was called 'common perspective' (*perspectiva communis*) or 'natural perspective' (*perspectiva naturalis*) to distinguish this science from the perspective used by artists (*perspectiva artificialis*).

[3] Cfr. Alberti, *On Painting* II.47 (2004, p. 83): 'the colour also dissolves progressively like smoke into the areas next to each other'.

A marvellous dexterity must be used in round bodies or those that are rounded, since those figures requires a light that is brighter in almost a single point, while in the other parts the light fades away little by little until finally it finishes in a dark bottom of shadow. If those who engrave on copper admirably avail themselves of what I have said with a diversity of ways and number of cuts, that is, that in some places they are thinner and in other places of thicker, then shadows will appear. If they make cuts that are now simple, now double, now multiple, such that each member of the body has its realisation and its feeling, then they can realize to an even greater extent what I have said about colours and lights. Indeed, it is a marvellous thing what those engravers do, and how with the moderation of cuts they imitate cloth both thick and thin, furs, silk, velvet, brocade, without using any colour at all. And what else? The dawn, and the eastern Sun, the night and fire, storms, the reflections of water, clouds, the forms of things both animate and inanimate: they moderate their cuts so well that it is very easy to distinguish one thing from another.

Let us take care, then, in folding, and in the valleys and the peaks, to give the more brilliant light and the brighter colours at the top than anywhere else. In addition to this we must be careful to use only one light and to take it from above because light makes clearer and more sure effects in uncovering the feelings when it is taken from above. This is known by those who want to see a statue well and appreciate it, because the artfulness is better understood if the light shines down and if there is only one. There being many lights, and those shining from below, subtract the roundness and feeling of the parts of the statue from vision.

The shadows must not change the colours, but base themselves on the same colour and make it deeper. Shadow is the absence of light, and not the effect of the colour black,[4] except in the case where it is necessary to take the light away, which leaves black to be seen, that is, no colour. Light does not generate colour but makes it so that they can be seen. The colours are still truly colours in the night: green is green, and yellow is yellow, but they are not seen because they are only visible with the presence of light.[5] Thus those things that are impeded from receiving light must be darker, and those parts that are turned toward the light but are either lower and shaded by what is above, or are behind, must be more in shadow. The light that at night comes from fires is brighter where it strikes, and where it does not strike, the shadow is of a denser black.[6] That is all that needs to be said about the reception and impediment of colours, lights and shadows.

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[4] This appears to contradict Alberti, who seems to believe that black is added to colours to produce shadows. See Alberti, *On Painting* II.47 (2004, p. 83) where the painter will 'modify the colour of the surface with white or black'.

[p. 177]

Chap. II. On the projection of shadows

Now I will speak about the projection of shadows that come from bodies that are opaque and solid, which prevent the rays from passing. So, the lights that come from the eternal fires of heaven cast shadows that are almost the same as the bodies that make them. The shadows that come from earthly fires are larger than the bodies that make them. Light emits straight rays as far as they can extend, but when an opaque thing interposes, then the rays meet in that, and the shadow is cast by as much space as is occupied by the opaque body that breaks the rays of the luminous body. So the projections of shadows must be taken from the height of the light. Therefore, whoever wants to measure the length of the shadows directly must place the light in a determined place, from that make the lines fall like rays to the extremities of the body drawn out to the plane below, and there place the endpoints of the shadows, which will be circumscribed by the rays. Nor must one neglect to think about the projection of shadows, because with those one thing is made marvellously distinct from another, as the two brothers from Brescia ingeniously did in the ceiling of the Church of the Orto in Venice, a thing that merits being seen. I will end this part with an example by Albrecht Dürer.

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[5] Cfr. Alberti, *On Painting* I.9 (2004, p. 44): 'as the light disappears to do the colours, and when it returns, the colours come back with the strength of the light'.

[6] Cfr. Alberti, *On Painting* II.46 (2004, pp. 82-83): 'Therefore, to avoid condemnation and earn praise, painters should first of all study carefully the lights and shades, and observe that colour is more pronounced and brilliant on the surface on which the rays of light strike, and that this same colour turns more dim where the force of the light gradually grows less. It should also be observed how shadows always correspond on the saide away from the light, so that in no body is a surface illuminated without our finding surfaces on its other side covered in shade'.

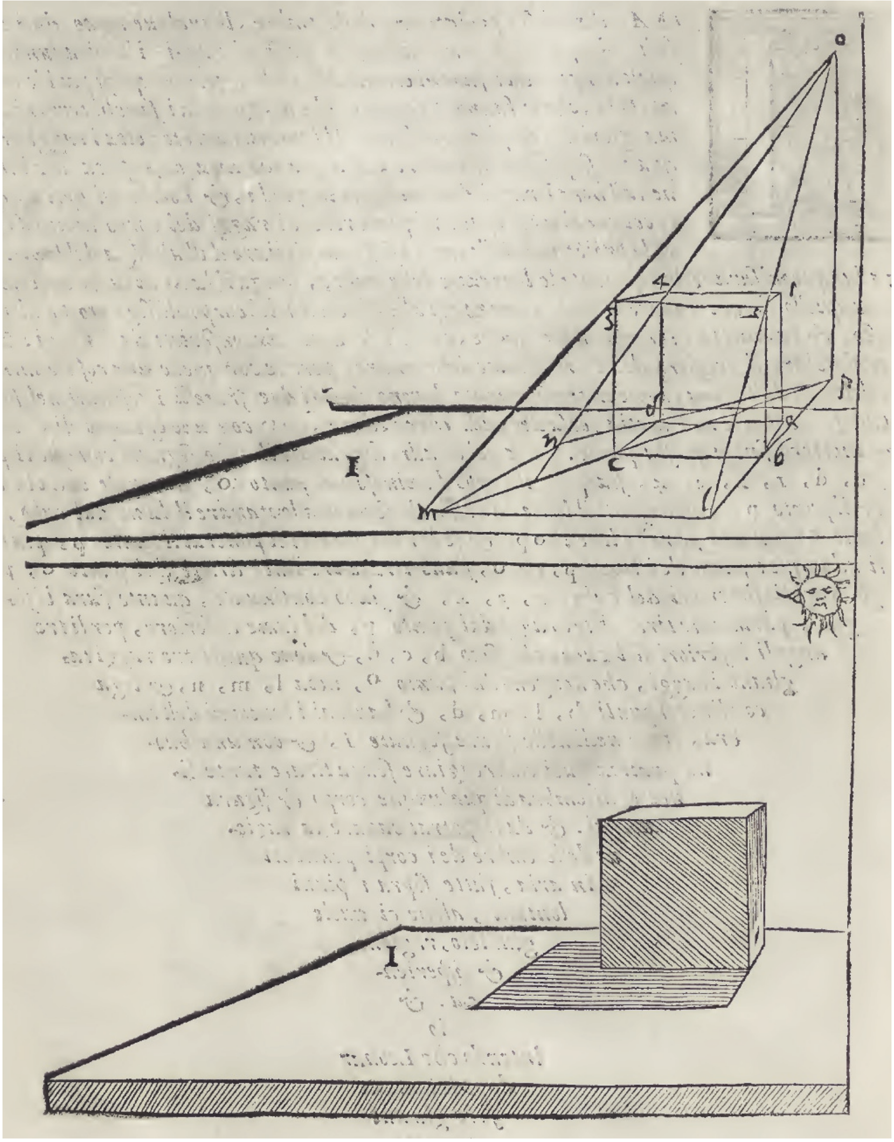

Fig. 7.1[7] **[p. 178]**

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[7] Cfr. the image in Dürer, *Underweysung der Messung* (1525, Book IV, figs. 57-58). Note that here the shadow indicated in the figure below does not match the lines in the figure above.

Over a cube in the degraded plane marked with points *a*, *b*, *c*, *d*, 1, 2, 3 and 4, let there be placed a luminous body in point *o*, from which falls the line *op*. The point *p* represents the light in a lower position. If you want to move the light away from the cube, you will place the light at *o*, higher up on the line *op*; if you wish it to be closer you will place *p* [*recte o*] lower down. Having thus placed the points of the lights *p* and *o*, draw three straight lines from point o to the upper corners of the cube, 2, 3 and 4; they should continue as far as necessary. Likewise draw three rays from point *p*, from the lower light, to the three lower corners of the cube, which are *b*, *c* and *d*. Where these three rays cut the rays that come from point *o*, mark *l*, *m*, and *n*, and connect them with lines to points *b*, *l*, *m* and *d*, and you will have the edges of the shadow, as you see in figure I (Fig. 7.1). With good practice you will be able to make the shadows without drawing many lines of whatever kinds of bodies and shapes you want. On your own you can derive the rationale of the shadows of bodies that hang in the air generate on planes that are far away, for which are needed judgment, reason, and experience. I understand that Leonardo from Arezzo [*recte* Vinci] has already compiled a treatise.

Part VIII, in which are treated the measures of the human body

Chap. I. Measurement of the human body

[p. 179] The human body has by various persons been measured variously. Some have regarded the larger parts; others the smaller parts. Vitruvius, in the first chapter of the third book, has the intention that I will set forth in what follows. Let there be line *ab* as long as you wish for the height of the body, and let that be divided into 8 equal parts with points *c*, *d*, *e*, *f*, *g*, *h*, and *i* (Fig. 8.1). I let the part marked *ac* on top be the height of the head from the chin to the top. Then I divide that same line into 10 equal parts, with the numbers 1, 2, up to 10, and I open the compass to a space equal to a tenth of the line divided into 10 parts. Placing one foot of the compass in point *c*, where the chin is, and turning the other towards *a*, I make point *o*. Therefore the space *co* is the tenth part of the whole body, and is the space from the chin to the roots of the hair, where the top of the forehead is. Then I divide line *ab* into 6 equal parts, and carry one of them from point *o* towards point *b*, and there I note *k*, where the top of the chest will be, from which to the top of the forehead, where point *o* is, will be the fifth part of the height of the body. Thus is adjusted Vitruvius's text,[1] which says it is the fourth part. In addition to this I divide the space between point *c* and point *o* into

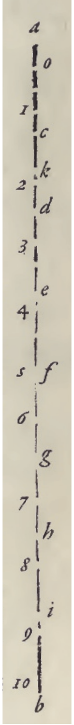

Fig. 8.1 [p. 179]

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[1] Vitruvius III.I.2; cfr. Barbaro (1567, p. 109), Williams (2019, p. 198).

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
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three equal parts, and give the upper part to the forehead, the middle to the nose, and the lower part to the space from the nose to the chin. Thus is divided the human face. The foot is the sixth part of the height; the cubit the fourth, including there the length of the hand. The chest is likewise the fourth, including the lower part of the chest, so that from the top of the chest, where point *k* is, to the top of the head, where *a* is, is the fifth part. In this way Vitruvius divides the human body into larger parts.

But Albrecht Dürer measures every particle more minutely, as can be seen in his writings.[2] Now using a middle way, which can serve the present need, I will take the measure of the human body from a part of it with which man can measure himself.

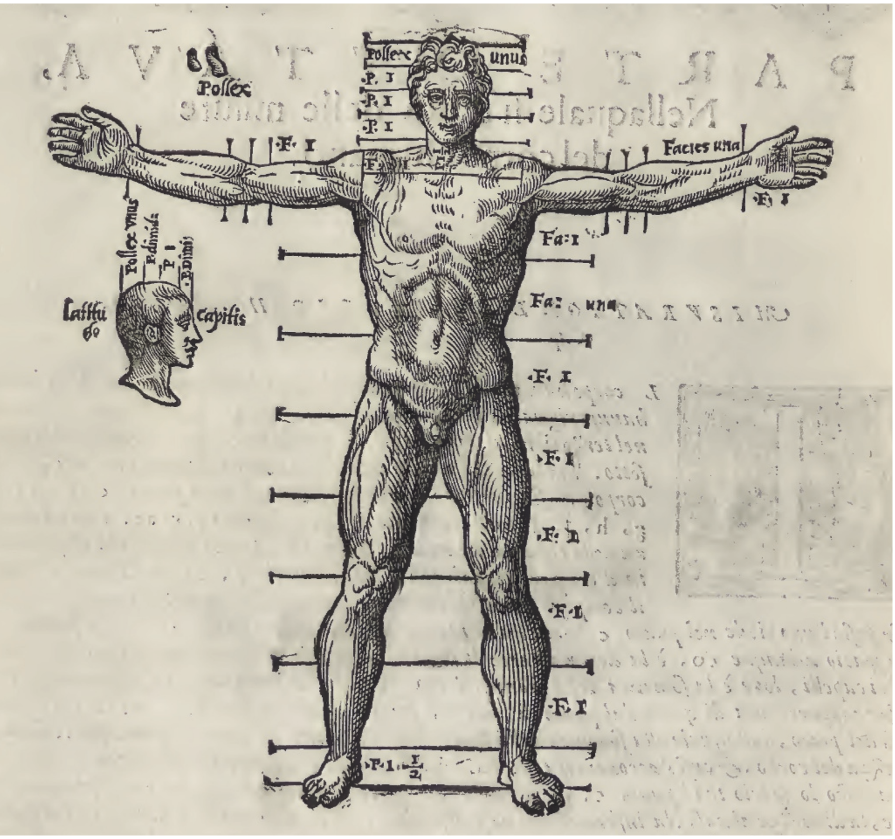
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Fig. 8.2[3] **[p. 180]**

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[2] Dürer, *Vier Bucher von menschlicher Proportion* (1528).

[3] This image was published the first time by Barbaro in his Latin translation of Vitruvius, where he presents a different way to measure the body compared to Vitruvius and Dürer: *nisi velis*

It is a marvellous work of nature and the judge of art, because art takes as a measure a modest quantity that is known, in order to come to a cognition of a quantity that is unknown, and nature has placed in the human body the part that is the measure of all the other parts in a place that it can be applied to almost every part of the body. This is the large finger of the hand, or thumb, called the 'pollex' by the Latins, with which a man can measure almost every part of himself. The example and the figure (Fig. 8.2) will demonstrate clearly what I have said, making it quite clear to the person who **[p. 180]** delights in painting that he does very well to study anatomy, and consider not only the muscles that explicate the movements of all the parts, but the bending and outstretching of the members. In addition it will make the affects and customs of men apparent, as the excellent painters have done. But here it is not our intention to reason about painting, as that is beyond our scope.

[p. 181]

Chap. II. The way to describe the plan of the human head

Having well understood the measures of the head, if you wish to make its foundation and plan easily, you will use the instrument of Dürer, called the *trasferente*, because with that the perfect of the head is transferred and carried in plan; I will exert myself to demonstrate as clearly as I am able how this is done according to Dürer's opinion, because when the way to make the plan of the head has been well understood, the plans of all the other parts will be easy and quick. So, make a square, each side of which is equal to the eighth part of the height of the entire body, and mark it with the points *abyo*, such that *ab* is the upper side, *yo* the lower, *a*[*y*] the left and *bo* the right. Draw line 1-2 whose length is equal to the side of the square, and in the middle mark 8, which signifies that said line is the eighth of the height of the entire body. Retract back from point *a* in the direction of *b* by the tenth part of line 1-2, in point *c*. Retract from the same point *a* towards *b* by the eighth part of line 1-2 and mark *d.* From the same point *a* in the direction of *cb*, retract by the seventh part of line 1-2 and make point *e*. Likewise we carry the fifth part of line 1-2 from *a* towards *b* in point *f*, and the fourth part in point *g*. Then take the twenty-second part of 80 of the

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alteram mensurandi rationem a pollice sumptam, quamadmodum superior figura ostendit; Barbaro (1567 Lat., p. 89).

entire line of the height and carry it from point *a* towards *b* in *h*. Likewise you will carry the fourteenth in *i*, the eleventh in *k*, the ninth in 1, and two seventeenths to *m*, next to point *b*. Having completed this division of line *ab*, you will let lines from points *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, *l* and *m* fall at right angles onto side *yo*; these are the end points of the width of the head. So, the line that falls from *c* will touch that part on the forehead where the roots of the hair are, and it will also touch the end of the eyebrow; the eyebrows must not be conjoined over the nose. That same line will touch the upper lip. Similarly, the line that falls from point *d* will touch the ends of the eyelids, the lower lip, and the end of the chin. The line that falls from point *e* will touch the concavity and end of the pupil, the eminent nostrils on the inside, the corner of the mouth, and the hollow beneath the lower lip. The line that comes from point *f* will touch the corners of the eyes near the temples. The line that is born from *g* will be touched by the eyebrows near the temples and by that little hollow near the jaws, and that which comes from *h* will touch the end of the neck. That which comes from *i* will touch the place where the ears are born. Line *k* will come to the end of the ear, the lower part of which will be between the lines that come from the points *i* and *k*. The line that comes from *l* will touch the conjunction of the head with the neck on the back part, and will also touch the top of the head. Finally, the line *m* will mark the ends of half the neck. In this way are had all the terminations of the parts of the head in profile with regard to its width. Now it is necessary with transverse lines to set the terminations of the height of the parts of the head. This is done in the following way. Take from point *a*, towards point *y*, on side *ay*, one part of 80 of the height of the body, and there mark *o*. From that point draw a line parallel to side *ab*. This will intersect with the line that is born from point *l*, at the top of the head. Then, again from point *a*, towards point *y*, take one of the 30 parts of the entire height in point *p*, and draw the transverse line, which will cut the line that comes from point *c* on the forehead at the hairline. Then carry half of line 8, which is the sixteenth of the entire height, into point *q*; draw the transversal and cut the line that is born from *d*, at the top of the eyelids and the top of the ear and eyebrows, which will slightly arch in the middle above that [line]. After this draw in the middle between points *q* and *t* a transverse line, which will terminate the bottom of the nose; on the same will fall the conjunction of the neck with the back part. You will then divide space *qt* into three equal parts, and from the first you will draw line *r*, which will be touched by the lower eyelids.

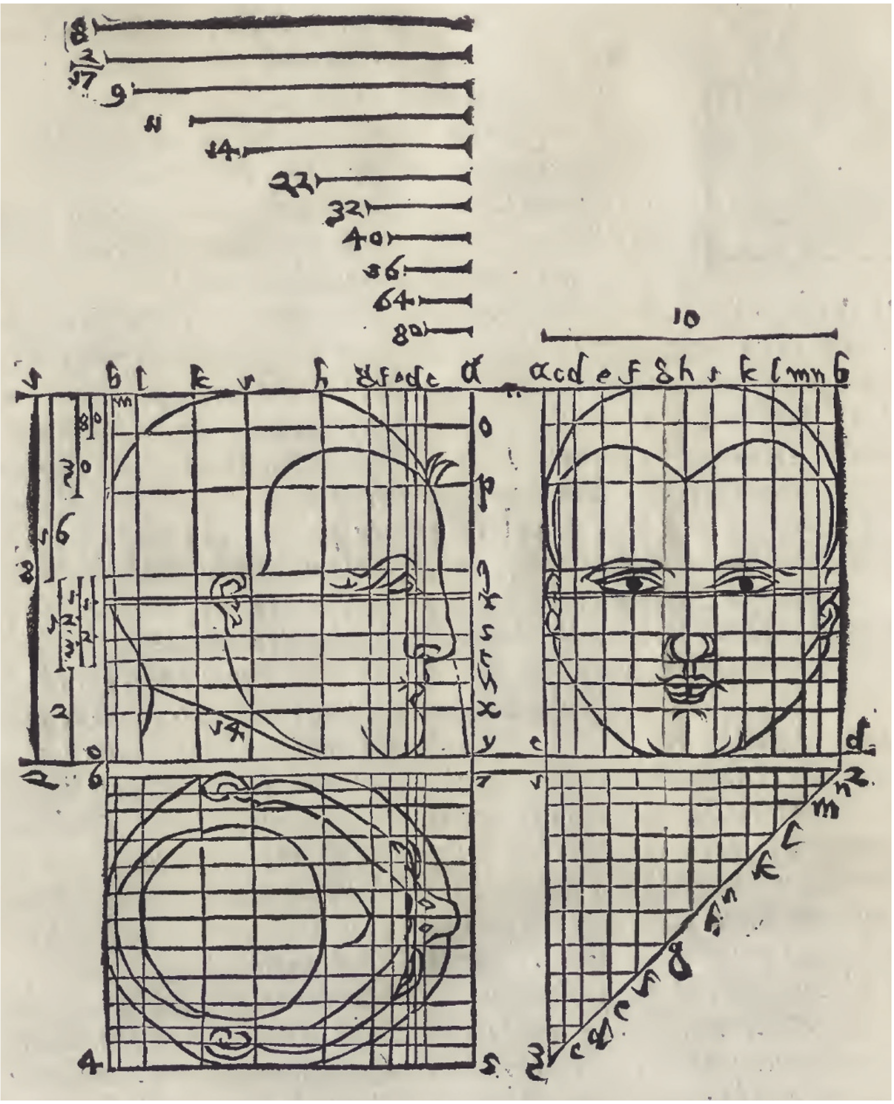
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Fig. 8.3[4] **[p. 182]**

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[4] Cfr. image in Dürer, *Vier Bucher von menschlicher Proportion* (1528, Book I, pages not numbered).

[p. 182]
You will place both corners of the eye and the pupil in the middle between *q* and *r*. Then you will divide *qt* into 4 parts, and at the third mark *f*; from that point draw a transversal line that will touch the top of the nostrils and the bottom part of the ear. Finally you will divide the space *ty* into two, and at the middle mark *x*; from that point draw the transverse line, and this will touch the top of the chin. Placing *u* in the middle between *t* and *x*, and drawing a transverse line, you will cut the mouth in the middle. Of the three parts between *t* and *u*, one you will make the upper lip, and with the two that you will make between *u* and *x*, one will make the lower lip, and the other the curve between the lip and the chin. In this way is terminated the head in profile, its outlines drawn according to the terminations established. Further, take care to draw a line from the angle made by lines *c* and *p* in the roots of the hair of the head in profile at point *y* of the square, because the prominence of the forehead will be between *p* and *q*, and that line; the nose will extend along that line. But in all of this is needed judgment and discernment in order to not make the things wooden, without any tenderness or refinement.

The measures set forth by Dürer are those of the head of a man **[p. 183]** who is rather solid, so other measures he reserves for other appearances. This I say so that it won't be believed that those measures are immutable. So I refer scholars to the reading of his things.

The head straight on and frontal is made in this way. Let there be made as in the first square of the head in profile a square of the same height *abcd*, and its length shall be the tenth part of the height of the body. Then divide side *ab*, on the top, into 10 parts at the points *d*, *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m*, and *b*. Between *m* and *b* let there be *n*, and between *d* and *a* let there be *c*, from which at right angles fall the lines to side *cd*. From the square of the head in profile from the points *o*, *p*, *q*, *r*, *s*, *t*, *u*, and *x* shall be drawn the lines parallel to side *ab*, which will divide the terminations of the head seen frontally into many small squares, into which will be put, each in its place, the chin, the mouth, the nose the forehead, the eyes and the ears, along with all the other parts according to the correspondence to the profile. Note that the lines that come from points *c* and *n*, where they cut that transverse line that come from *r*, give beginning of the face from the jaws to where they join at the chin. The ears close between the lines *a* and *c* and the lines *n* and *b*. And the auricle of the ear goes a good bit into both sides of the lines *c* and *n*. The rest is marked as you see in the head seen frontally. So the spaces *cg* and *il* give the length of the eyes and the space *gi*, the width of the nose, the distance from one eye to the other and the width of the mouth. If you want the back side, you will make a square equal to the square where you put the head seen frontally, and you will terminate, as you see the rotundity of the head, the space of the ears and the outlines of the neck, which is easy to understand solely from the example in the figure (Fig. 8.4).

It remains for us to make the plan of the head with the help of the *trasferente*. So, you will lengthen the sides *ac* and *bo* of the square of the head in one eye as much as is sufficient for you, and at some distance from the side *co*, on the lengthened lines mark points 4 and 5. Then take the width of the frontal square and carry it from the points 4 and 5 on either side of the extended lines to points 6 and 7, and close the square 4, 5, 6, 7 in which the plan is to be formed. Then extend side 6-7 parallel to the sides *co* of both squares, and on this extended line, or side, will fall line *ac* from the square of the head seen frontally, and the line *bd* of the same square, such that *ac* falls on 1 and *bd* on 2. Then from point 1, at right angles, falls line 1-3 on the extended line 4-5. From point 3 to point 2 draw the line called the hypotenuse, which closes triangle 1-2-3. This triangle is called the *trasferente*. Its compartition is as follows.

Let line 1-2 of this triangle be divided with lines that fall at right angles from the points *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m*, and *n* of the square of the head seen frontally, and arrive to line 2-3, which will be points *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m*, and *n*. From these points are drawn lines parallel to the sides *cd* and *yo*, which will divide the *transferente* into many little squares; they will also divide the square 4-5-6-7 into many small squares, cutting the lines that fall from the square of the head in profile, as you see (Fig. 8.3). For this reason it will not be difficult to make the plan in its square, considering that the lines that come from the points *i* and *g* of the *transferente* gives you the width of the nose. The line that comes from point *h* of the *trasferente* gives you the middle of the nose, and the line that comes from the tip of the nose in profile and from point *e* in its square, gives you the height of the nose. Likewise, lines *eg* and *il* that come from the hypotenuse give you the spaces of the eyes, which are comprised in the lines *d* and *f* that come from the square of the profile. With the same ways you will find the other parts, and if you set yourself to it diligently, you will correctly form the plan of the whole head, as you can see clearly in the figure (Fig. 8.4).

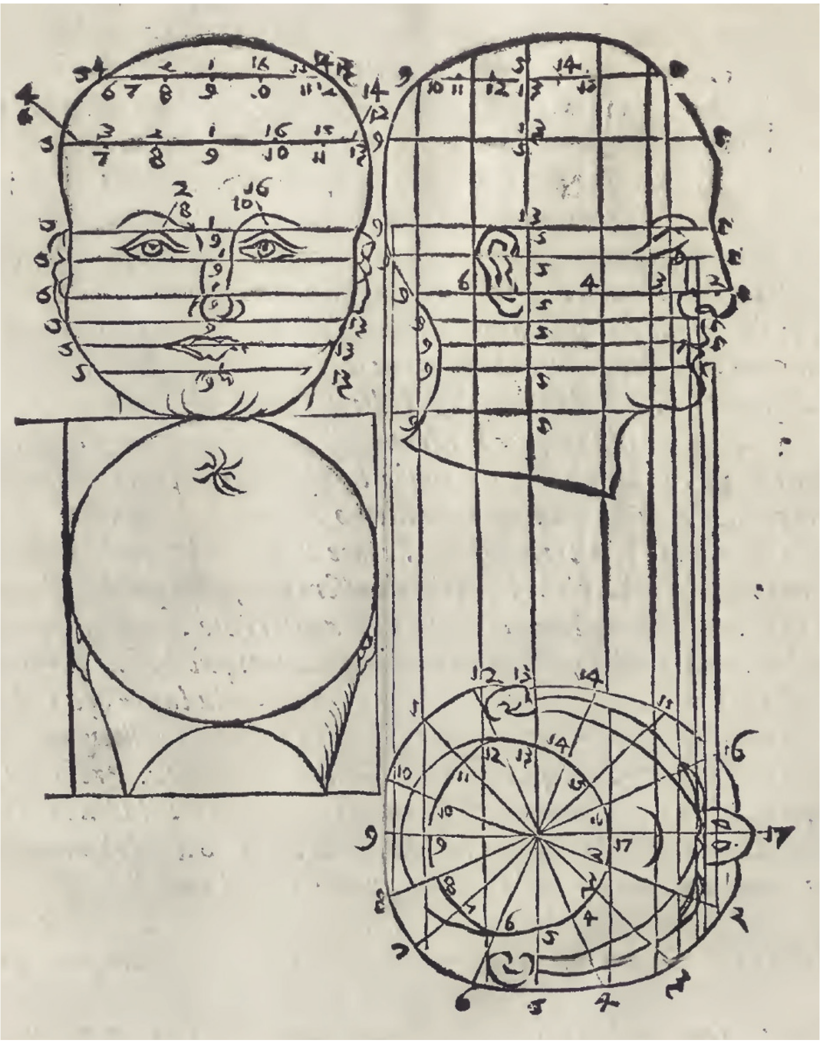
6 7 8 9 10 11 12
3 2 1 16 15
7 8 9 10 11 12
14
13
6 4 3 2
12 13 14 15
10 11
9
17
7 6 5 4 3

Fig. 8.4 [p. 184]

Chap. III. How to describe heads that are neither in profile nor straight on

From the figures alone it can be understood how heads that are neither in profile nor straight on must be made, since having set forth above with diligence the plan of the head in profile allows us to readily understand the manner of making the plan of the head that is neither straight on nor in profile.

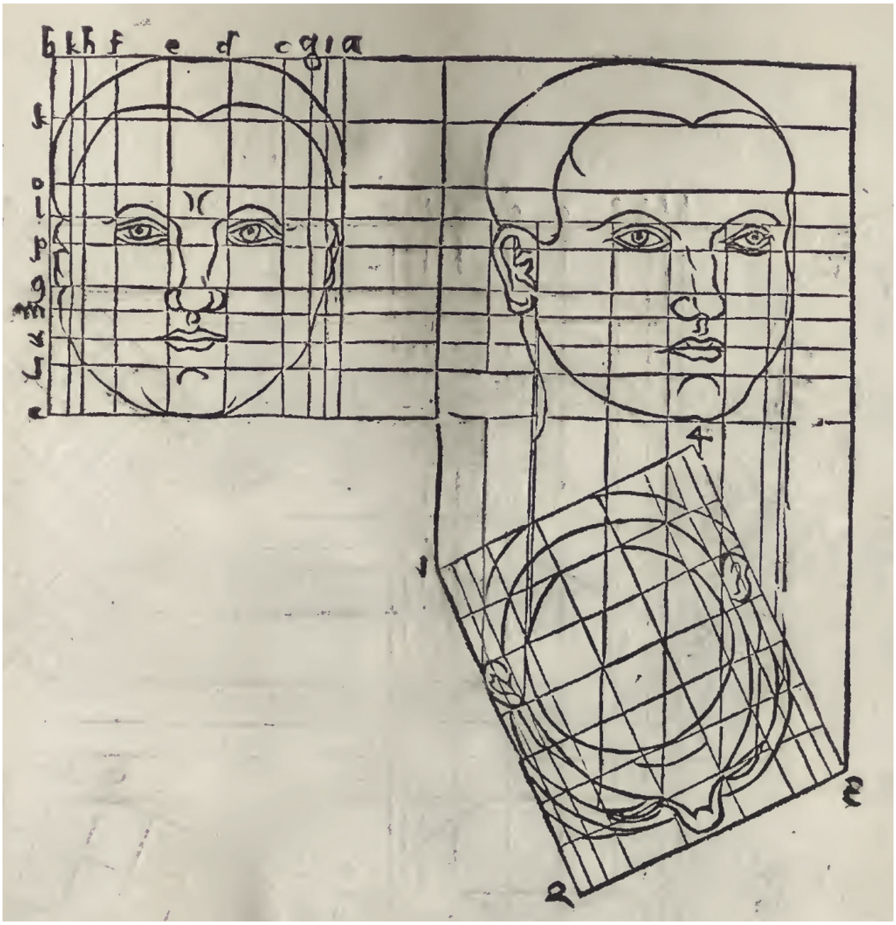

Fig. 8.5[5] **[p. 185]**

[p. 185]

Chap. IV. The way to make heads that look either up or down

If you want to transport the heads that look up – that is, those that have the chin raised – you will do it in this way. Draw the line *xy*, and on that place vertex *c* of the square *abcd* similar to the one that you made in the head in profile in Chap. II. Let point *d* be raised above line *xy* by however much you want. On line *cd* make said square, making the small squares with the rationale given above. Then make the square in which the head looking up is to be drawn, using the same rationale with which you made the square of the head seen frontally in that same Chap. II. Then from the terminations of the parts of the head in profile looking up draw [lines] to all the terminations in the frontal square, as you see [in Fig. 8.6], and mark in their places the forehead, mouth, nose, eyes and the rest. You can't go wrong if you consider what part those terminations belong to. You will do likewise when you want to transport the same head looking down, as you see in Fig. 8.6 and the others marked with their points, as they are to be put in perspective.

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[5] Cfr. image in Dürer, *Vier Bucher von menschlicher Proportion* (1528, Book IV, pages not numbered). On the left the two numbered faces are taken from Piero della Francesca's *De prospectiva pingendi* (Mss. Regg. A41/2, fol. 79r).

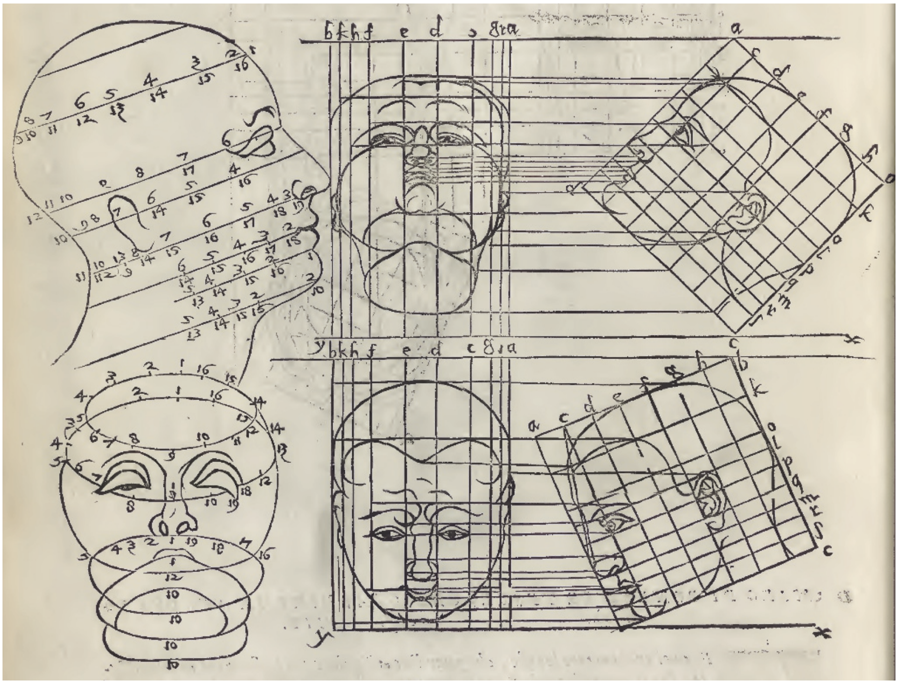
bkhf e d c 8ra
a
ybkh f e d c 8ra
a c d e f g h b k

Fig. 8.6[6] **[p. 186]**

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[6] Cfr. image in Dürer, *Vier Bucher von menschlicher Proportion* (1528, Book IV, pages not numbered).

Part IX, in which are described many instruments and ways of rendering and transporting things in perspective

Chap. I. The author's instrument for making clocks in every climate, on every plane and for every sort of hour

[p. 187] The half-sphere is used to make sun clocks in various planes, but that serves us for only one elevation. We have imagined an instrument that is not only a clock in itself, but is also an instrument for making clocks in every plane, at every elevation, and with every sort of hour. It is handsome in form, because it represents the sphere, and it is also convenient for use. It is constructed as follows. First is made a ball of copper or brass, empty inside, of a diameter that is a little less than half a foot. In this are marked the poles and the 2 polar circles, which are twenty-three and a half degrees away from the poles, and the 2 circles called 'colures',[1] and through the middle of that is marked the equinoctial, and then the tropics, which are twenty-three and a half degrees away from it, as is done in the sphere.[2] Then twelve circles are made to pass through the poles of the world; these divide the equinoctial and the tropics into 24 equal parts. These circles are called hour circles, because they denote the hours of the astrologers, which begin at noon. Of this one

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[1] In astronomy the term 'colure' refers to either of the two principal meridians of the celestial sphere.

[2] The 'sphere' is the geographical map of the world or astronomical map of the sky.

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https://doi.org/10.1007/978-3-030-76687-0_12

will serve us for the meridian, and another will pass through the points at east and west. Having marked the sphere in this way, the parts that are between these circles are removed with a rasp, leaving each with only the thickness required to hold the instrument together. Also taken away are all the arches of the hour circles that are from the tropic to the poles, except for the colures, of which (as I have said) one serves us for the meridian and the other passes through the equinoctial points. The first arc of the hour circle near the meridian serves us on one side for the first hour after noon, and on the other side for the eleventh hour after midnight; the second on one side for the second hour after noon, and on the other for the tenth hour after midnight. And so one by one up to the bottom arc of the meridian, which is to mark the twelve hours of noon and midnight. These circles are fixed and immobile, and divide (as I have said) the equinoctial and the tropics, and the other circles set there, into 24 equal parts, which are the spaces of the equinoctial hours taken by the astrologers from the meridian; these are the most certain, since they are the same distance both before and after noon, the sun being as high an hour before noon, as it is an hour after.

Now in order to set the terminations of the other hours, according to the usages of the various countries (because some begin at sundown, others at sunrise, and other set the planetary hours **[p. 188]**, always dividing the nocturnal and diurnal arcs into 12 parts, as the astrologers do), it is necessary to superimpose a moveable circle on the 2 tropics; this, however, cannot go beyond their circumference. These circles must be divided into 24 equal parts, and at the end of each be made a small hole so that it is possible to pass through them some silk threads that go from one tropic to the other. These are the terminations of those hours that we wish to set, whether from the end or the beginning of the day, or otherwise.

Now all of this ball thus divided must be placed on its horizon, and have its foot made in such a way that it does not hinder looking at something, as will be seen in the use of the instrument. Let this ball with its pins be placed at the points of east and west marked on the horizon. It can be turned and raised and lowered according to the elevation of the poles. The meridian shall be divided into 360 parts, that is, into 4 quarters of 90 degrees. Let each quarter of the horizon also be divided into 90 parts. Beginning to number the parts from the points of the east and the west, the winds can also be placed on the edge or in the thickness of the horizon. Inside this sphere is placed a stylus at one of the poles, which with its point arrives precisely at the centre of the sphere. At the point there is a small hole, so that it can be looked through, as will be seen in its use. This is the construction of the instrument, which for now I will call the 'universal clock'. To this can be added the moveable circles to make the analemma, of which [circles] there are three: the mobile horizon, the mobile meridian and the mobile vertical, but of these we have spoken at length in Book IX of Vitruvius,[3] according to Ptolomy's intention and Commandino's exposition, which is a thing worthy of the highest speculation and the most noble practice. Now follows the treatment of the use of this clock, which I will do as distinctly as I can.

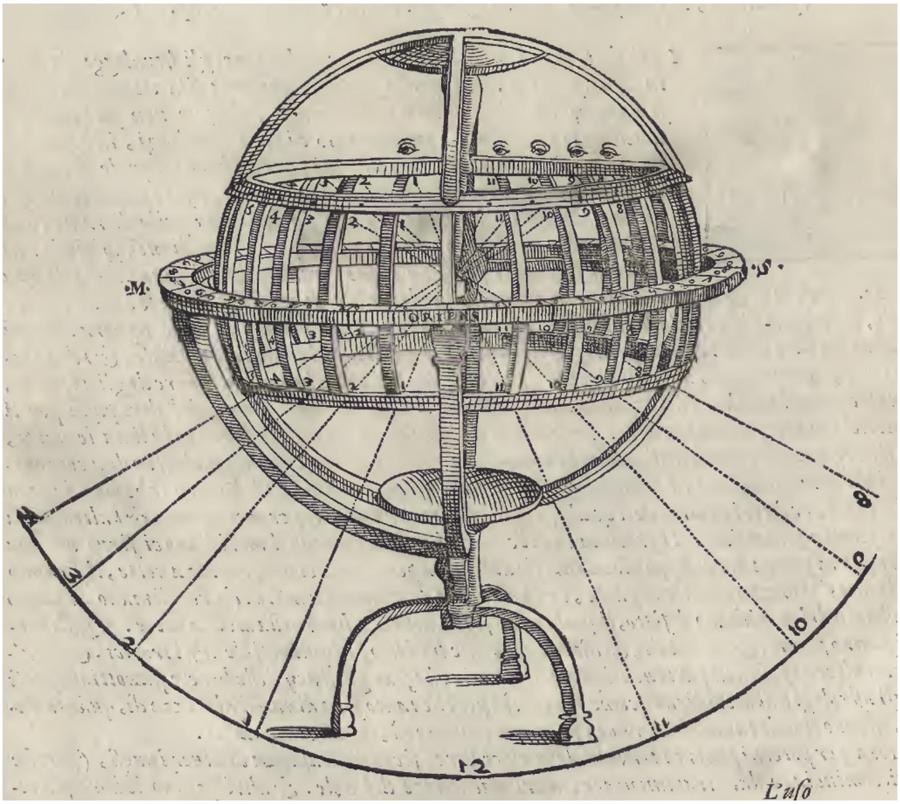

Fig. 9.1 **[p. 188]**

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[3] Cfr. Barbaro (1567, pp. 409-426), Williams (2019, pp. 683-709).

[p. 189]

Chap. II. The use of the universal clock

In wanting to describe clocks, it is first necessary to know the elevation under which the clock is to be made, because the clock is set up for that elevation. This is done by means of the degrees on the meridian, raising the pole above the horizon by as many degrees as necessary. Then, if it is desired to make the clock in the plane of the horizon, and mark the hours from noon, place the clock in such a way that its meridian is turned to noon; this will you do with the compass. Begin to look where the meridian cuts the tropic of Cancer, and direct your gaze through the hole in the tip of the stylus, and through the point opposite on the edge of the meridian and on the tropic of Cancer all the way out to the plane; where your gaze falls on the plane mark the point 12. Then, look at the cut made between the meridian and the tropic of Capricorn passing through the hole in the tip of the stylus out to the plane and where your gaze falls on the plane mark 12. With a line from one point 12 to the other, you will have both the meridian line and the twelfth hour marked for the whole year. Wishing then to mark the eleventh ahead of noon and the first after it, look along the cuts that are made by the 2 arcs of the hour circles close to the meridian in the tropic of Cancer, and through the hole in the tip of the stylus, and where your gaze falls on the plane mark 1 to the left of 12 and 11 on the right of it. Then do the same, looking along the cuts that these circles make in the tropic of Capricorn, and through the hole in the tip of the stylus all the way to the plane, mark 1 to the left of 12 and 11 to the right of it. Connect with lines the 11 and the 11, and the 1 and the 1, and you will have marked the first hour after noon, and the eleventh hour ahead of it for the whole year. With similar ways you will mark the 2 and the 10, the 3 and the 9, the 4 and the 8, the 5 and the 7, looking along the cuts of the hour arcs in the tropics and passing with your gaze, or with some very fine wire, through the tip of the stylus out to the plane, and marking and connecting the points of one tropic with those of the other. In carrying this out in practice, you will know how to mark the lines in the plane that makes the tropics and the other circles of the signs of the Zodiac, having first marked, on the arcs of the hour circles in their places, the points of the declination of the signs with tiny cuts that do not obstruct the instrument with the multitude of circles. The sixth hour on either side of the meridian is made by cutting the meridian in the middle at right angles with a line. The other hours – that is, the fifth ahead of noon and the seventh after, the fourth ahead and the eighth after noon – are made with equal spaces of the line of the 6 hours; that is, the seventh is as far from the sixth after noon as the fifth, and the eighth is as far away as the fourth, and so on for the rest.

Actually, the stylus must be placed in the plane, as if the axis of the instrument from the upper pole were to arrive to the plane, and make the hypotenuse of an equilateral triangle of which one side would be on the meridian line, and the other would go to find the tip of the stylus, so that the 2 sides would make a right angle on the plane. Here the shadow made by the hypotenuse, which represents the axis of the world, will show the hours, striking on the lines of the hours. In this way is made the clock in the horizontal plane with the hours from noon.

If it is desired to mark the hours that begin from sundown, it is necessary to know the length of the longest and shortest day of the year in that country where you want to make the clock, and then set up the threads of silk in the holes made in the tropics as necessary, and make them pass from one tropic to the other such that they cut the equinoctial of the instrument in the same cuts that the hour circles make with the equinoctial, and then look [through the hole in the tip of the stylus], as you did to make the hours of noon. So that you can more easily know where the threads go, you will first make a hole in the mobile tropic of Cancer; this should be even with the lip of the horizon, so that it is neither higher nor lower than the horizon. You will do likewise with the mobile tropic of Capricorn; and do this on the side where there are the arcs of the hours after noon, always understanding that the instrument is situated on the elevation of the pole. These 2 holes in the tropics are touched by an arc of the horizon, which serves for the 24 hours; these are not marked on the plane, since they cannot be marked because the shadows are infinitely long and equally extended to the horizon. So, the thread is placed **[p. 190]** for the 23 [hours] in the first holes above the horizon in the tropics, such that passing from one tropic to the other they cut the equinoctial in the same place as the hour circle or arc of the fifth hour after noon cuts the equinoctial. If you want to position the hour arc of 22 hours, place the thread in the second holes raised above the horizon in the tropics, which in the same way cut the equinoctial in the cut of the hour 4, and in that way step by step you will place the threads of the 21, of the 20, of the 19, and of the other hours. Working in this way you will know how many hours you can mark on the plane, which are as many as there are holes in the tropics where the threads pass above the horizon. You will know the amount by which the days increase, and in which sign the Sun is found, and that the thread of the 18 hours cuts the meridian in the same place as the hour arc of the 12 cuts the equinoctial, and what arc or line is formed in the plane by the shadow of the stylus from sign to sign. Looking along the points of the hours, and through the hole in the tip of the stylus you will transport all the hours from sunset in the plane of the horizon, as you did with the hours from noon.

With similar manners you will set the threads for the hours that begin at sunrise, but you will begin from the opposite side to place the holes in the tropics even with the lip of the horizon and the threads across from hole to hole from one tropic to the other, such that the first thread will be for the first hour of the day, the second for the second, the third for the third, and the rest one by one. Having placed the threads and situated the instrument at noon with the meridian, you will look, as you did previously, and mark on the plane the lines of the hours from sunrise.

Similarly are set the threads for the hours of the planets. So, the arc of the tropic of Cancer, which is over the horizon at the proposed elevation of the pole, is divided into 12 equal parts, and in the same manner is divided into 12 parts the tropic of Capricorn which is over the horizon, and the threads pass from one tropic to the other over the divisions of the equinoctial. This is common to all the sorts of hours, the arcs of which concur at a point on the equinoctial, that is, they don't cut the equinoctial anywhere else except in the points and cuts of the hour arcs of the hours from noon. These planetary hours are 12 on every day of the year.

The stylus or gnomon in this sort of hour – that is, from sunset, from sunrise, and in the hours of the planets – shows the hours only with its tip. That tip must be sufficiently high to touch the point of the stylus of the instrument, that is, the centre of the sphere.

There is no doubt that if you have well understood the way of making clocks in the plane of the horizon then you will know how to draw the clocks of the other planes, whether flat, concave, curved or whatever, since arranging the clock, whether hung or supported, so that it always has its meridian turned to noon and looking through the points of the hours, you can find whatever hours you wish to describe in the opposite plane, all the sorts of hours, planetary, or Italian, or Bohemian, or French, as you fancy, in every clime, and width of the country. The stylus will always be long enough so that its point touches the point of the stylus of the instrument, and that there is formed the triangle for the hours from noon according to the axis of the world.

You will have more than a little pleasure in the night setting the clock at various elevations and placing a small light in its centre, because you will see different clocks on different planes cast by the shadows that are made by the hour arcs and the threads. To make this experiment the threads must be made of copper or else they will burn. You will know that the clock on the wall made under the pole is similar to the clock in the plane made under the equinoctial, and you will see that the clock in the plane made under the pole is similar to the clock on the wall made under the equinoctial. You will find the diversity of the horizons, and you will see how the sphere is cast on the plane, something that is most delightful and pleasing, in addition to many other conveniences that can be had in measuring the heights, widths and distances, which I leave to others.

[p. 191]

Chap. III. Albrecht Dürer's instrument for making perspectives

Albrecht Dürer ingeniously discovered an instrument for putting things in perspective, which I have used and which turns out quite well. So, a frame is made of a suitable size, and along the inner sides are set some dowels, by which are made to pass in each direction a thread. The dowels are thick. On this frame is placed a door that is opened and closed as needs be. This frame is set up at right angles, on a table, and very firmly fixed. Then a thin but strong cord is tied to a ring fixed to the opposite wall; on one end it has a counterweight, and the other end is free. So, wanting to put something in perspective, such as a lute or something else, the free end of the cord is taken and passed through the frame to a point on the edge of the lute. This cord remaining fixed, a thread is made to fall from the upper side of the frame and touch the cord; the thread that touches the cord is tied to the dowel below. Then another thread is taken and made to pass across the frame and touch the cord, and cross the first thread. This done, the cord is taken away, leaving the 2 threads that cross at the right point and in correspondence to the point taken on the edge of the lute. The door is closed, and where it touches the point where the 2 threads cross, a point is marked. Then the door is opened again, and the threads are undone. Then the cord is moved to another point on the lute, the threads are crossed, the door is closed, and the [second] point is marked on it. In this way all the points on the edge of the lute are gradually marked in perspective on the plane of the door. With this instrument I have rendered many things in perspective in the chambers of the most reverend Cardinal Turnone, to his great delight. I took care to place the ring rather far away from the frame because things come out larger in this way than they do when the frame is close to the ring, which is the place of the eye, since the rays of vision are closer together when close to the eye than when they are farther away. I also set up the door carefully so that when it was closed the points of crossing of the threads touched it precisely, and things came out more certainly, especially if threads were pulled tight and the cord fixed firmly, because all such measures serve for the cleanness and correctness of the work.

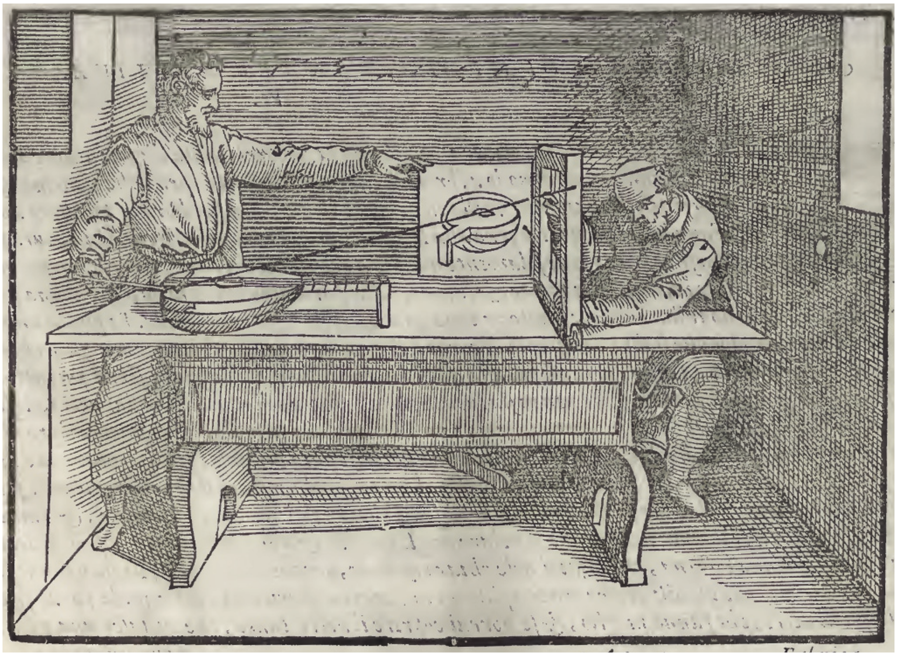

Fig. 9.2 **[p. 191]**

[p. 192]

Chap. IV. Building another instrument by Baldassarre Lanci

The ingenious engineer Baldassarre Lanci, upon my being in Siena, showed me an instrument that he discovered for putting things into perspective, which (if I recall correctly) was made in this way. In the middle of a circular brass table was fixed a dowel which could be turned around. It was 2 fingers high above the top of the table, and divided in the middle up to a certain point, such that it made 2 sort of ears, in the middle of which there was another dowel a foot high. This dowel was held by a little pin, and had enough play to allow it to be moved and slanted and straightened according to need.

At the end of the dowel was a copper tube with a small hole, which could be up to half a foot long, placed like the letter T and soldered onto the top of the dowel, in the middle of which was soldered another brass tube parallel to the first, so that when the dowel was slanted down on the little pin, both tubes were lowered and remained parallel to each other. The purpose of the upper tube was to serve to look through, while the purpose of the lower tube was to allow a thin wire to be passed through its channel in order to push it up to an elevated plane at right angles in the circumference of the instrument. This plane was no higher than the second tube – that is, it did not arrive to the height of the first – because it was necessary that the first remained free and unimpeded in order to be able to look through it. In this plane are made the things into perspective. Placing the instrument on some plane, you looked through the upper tube at the outlines of something point by point, and where the vision stopped in the upper tube, a wire was passed through the channel of the lower tube, which went to strike the plane opposite and mark a point there. With the same procedure, moving the dowel and turning to the other parts of the thing and looking through the upper tube, the points are marked on the plane with the wire.

With regard to its invention, this instrument is a fine one, but with regard to its use it needs to be better made, and made larger, and with more expedients, which I leave to the inventor, who told me that he wanted to modify it.

Chap. V. Natural manners of rendering things in perspective

With marvellous delight nature teaches us the proportioned degradation of things, and helps us in every way to form the precepts of art, so that we must be diligent observers of her on every occasion. For now I will touch on a very beautiful experiment concerning perspective. If you want to see how nature renders things in perspective not only regarding the overall outlines and parts, but also regarding colours, shadows and resemblances, make a hole in the shutter of a window of a room in which you want to see; the hole should be as large as an eyeglass. And take the eyeglass of an old man – that is, one that has a bit of thickness in the middle and is not concave like the eyeglasses of young people who are nearsighted[4] – and fit this glass into the hole. Then close all the windows and doors of the room so that there is no light whatsoever except for that which comes in through the glass. Then take a sheet of paper and place it opposite the glass and far enough away from it so that you see minutely on the paper everything that is outside the house. This happens more distinctly at a certain distance, which is found by moving the paper closer to or farther away from the glass until the appropriate place is found. Here you will see the forms on the paper as they are, with the degradations and colours, and shadows and motions, and clouds, the shimmering of water, the flight of birds, and all else that can be seen. For this experiment it is necessary that the sun be clear and bright, because the light of the sun has great power in obtaining the visible species, which you will experience with pleasure in making the experiment. For this experiment you will select those glasses that are work the best. If you wish to cover the glass so that it leaves just a little **[p. 193]** circumference in the middle that is clear and uncovered, you will see an effect that is even more vivid. So, thus seeing the lineaments of things on the paper, you can mark on it with a brush all of the perspective that appears there, and add shadows and nuanced colours according to what nature shows you, holding the paper in place until you have finished the drawing.

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[4] A convex lens corrects farsightedness, while a concave lens corrects nearsightedness.

With the help of the sun you can transport from one size to another in the proportion that is desired, and copy a clock, or fortress, a human figure, or whatever other sort of thing, as set forth by the reverend Don Gianbattista Vimercato of Milan, in his book on clocks.[5] It is done in the following way. Having drawn a clock or something else on a panel, and wanting to transport it from one size into another, you will stick into the panel a stylus that is larger or smaller than that placed first, according to the proportion that you want. You then expose that panel to the sun such that the shadow of the first stylus touches some part or beginning of an hour of the clock originally drawn, and holding the panel still, where you see that the shadow of the second stylus ends, there make a point. Then take the shadow of the first stylus made in another part or end of the hour of the clock, and note the shadow of the second stylus and make a point there. In this way you will gradually have transported every point and every line of the first clock made in whatever form and scale you want. This will work well for you if you wanted to transport a clock made on a wall into one made on the plane of the horizon. It also works from a cylinder to a plane or from the plane to a cylinder. Take care, however, to set everything up with proper thought and judgment, arranging on the panel that flat plane or cylinder that you want to transport in order for you to see the effects that straight and rotated shadows make. Through trial and error you will discover many other things that are useful and delightful. I leave it to others to set out the many other manners of compasses, rulers and instruments for making various proportional lines, or ovals and the other forms called 'parabolas', 'ellipses', and 'hyperbolas', which are sections of the cone, from which marvellous effects are made in machines and fabrications of different things.

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[5] Giovanni Battista Vimercato, *Dialogo della descrittione teorica et pratica de gli horologi solari* (1565).

Now I will also speak about a very convenient and sure way that I used to transport things. So let us say that I wish to transport one geographical table to another on a plane. I make 2 semicircles on 2 sheets of paper, that is, one on the sheet of paper from which I want to transport, and the other on the sheet to which I want to transport. I divide both circles into however many parts I want, and for the greatest convenience I divide them into 180 degrees. I then make 2 rulers, and those too I divide into as many equal parts as I want to, or into parts that are unequal between the first and the second, if I want to transport them into a larger or smaller scale. Of these rulers, one I place in the centre of the semicircle of the drawn sheet, and the other on the semicircle of the sheet that I want to draw on. Making them moveable on the centre so that they can turn, I begin to transport in the following way. I turn the ruler around the first semicircle until it touches the point, or city, that I want to transport, and I look to see how many degrees of the semicircle is the ruler, and how many degrees of the ruler is the point or city that I wish to draw. Then on the blank sheet of paper I turn the other ruler by as the same number of degrees in the second semicircle as in the first, and there I make note on the sheet of how many degrees on the ruler was the point or city taken from the first sheet. Thus gradually, place by place, I go transposing each point. It comes out to the same scale if the second ruler is divided exactly like the first, but if the second ruler is divided into a smaller number of degrees, the transported thing will be smaller; if into a larger number of degrees, it will be larger. So you will see to setting up the 2 semicircles and the 2 rulers in a place appropriate for transposing everything, as practice will teach you.

May this all be praise and glory to Him who is the giver of the intellect, the fine inventions, and all good things.

[Appendix] How with a new instrument it is possible to know the inclination of scarps of walls, according to the invention of Giacomo Castriotto[1]

[p. 194]
To be sure, in addition to the usefulness and honour that derive from serving great kings, there is also satisfaction of mind, since thanks to the enjoyment and profession, the desire for many services awaken men's minds on occasion. The reason for this I say is found in the great King Henry [II of France] in his fortress in Picardy, for his most vigilant care, wanting to know minutely the amount of scarp certain walls had. To satisfy this desire, therefore, it came into my mind to make an instrument, and so I quickly took a piece of stiff paper in hand and cut a strip 2 fingers wide, and doubling it, I attached another, slanted, piece to a corner of it with a pin, so that the piece in the middle could move. The height of that I divided into 24 points, and I did the same crosswise, and with a mark precisely in the middle, I tied a small lead plumb bob to a string. This done, to use it I leaned it against the wall, holding the plumb bob straight, and tilting the piece in the middle, I saw according to the points that were uncovered how much scarp that wall had. Seeing 5 points uncovered, I found that it had [a scarp of] just a little below 5; when 6 points were uncovered, I have one of four, and thus gradually I found all of the scarps. Seeing thereby that the instrument was very useful, when I arrived in Paris I had one made all of metal similar to the one seen below (Fig. App.1), which, in addition to scarps, as need arises can serve as a square and a level; it also serves to transport large plans to smaller ones.

---

[1] Both the text and the image of this chapter are excerpted from Maggi and Castriotto (1564, Chap. II, 'Della Fortif. delle Città. Come con un nuovo instrumento si possino sapere le quantità delle scarpe delle muraglie', p. 42v).

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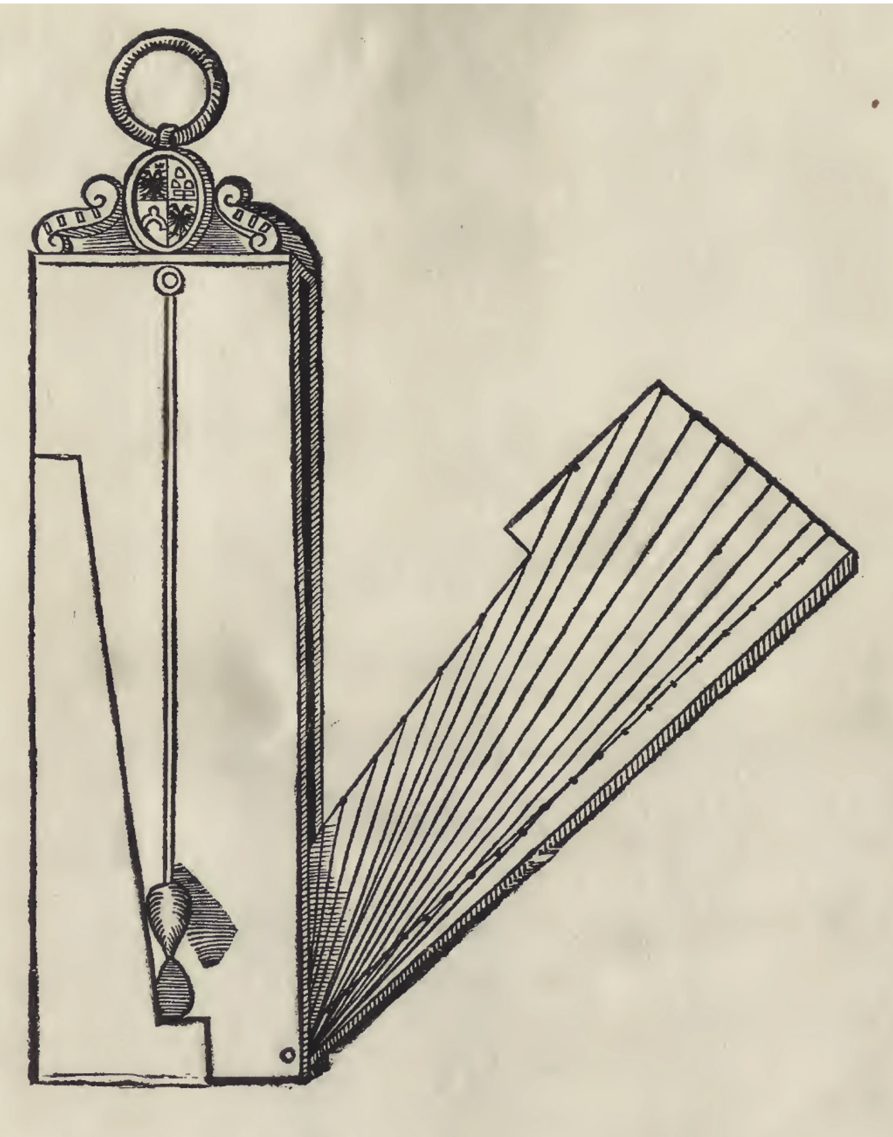

Fig. App.1 **[p. 195]**

References

Alberti (2004)

Alberti, Leon Battista. 2004. *On Painting*. Cecil Grason, trans. Introduction and notes by Martin Kemp. Repr. with revised Further Reading. London: Penguin Classics.

Barbaro (1567)

Barbaro, Daniele. 1567. *I dieci libri dell'architettura di M. Vitruvio. Tradotti & commentati da mons. Daniel Barbaro eletto Patriarca d'Aquileia, da lui riveduti & ampliati; & hora in piu commoda forma ridotti.* Venice: Francesco de' Franceschi.

Barbaro (1567 Lat.)

Barbaro, Daniele. 1567. *M. Vitruvii Pollionis de architectura libri decem, cum commentariis Danielis Barbari, electi Patriarchae Aquileiensis*. Venice: Apud Franciscum Franciscium Senensem, & Ioan. Crugher Germanum.

Barbaro (1568)

Barbaro, Daniele. 1568. *La pratica della perspettiva*. Venice: Camillo e Rutilio Borgominieri.

Barbaro (It IV, 39=5446)

Barbaro, Daniele. n.d. [La pratica della perspettiva, manuscript in Italian]. Ms. It IV, 39=5446. Biblioteca Nazionale Marciana, Venice.

Barbaro (It. IV, 40=5447)

Barbaro, Daniele. n.d. [La pratica della perspettiva, menabò]. Ms. It. IV, 40=5447. Biblioteca Nazionale Marciana, Venice.

Barbaro (Lat. VIII, 41=3069)

Barbaro, Daniele. n.d. Scenographia [manuscript in Latin]. Ms. Lat. VIII, 41=3069. Biblioteca Nazionale Marciana, Venice.

Belli (2003)

Belli, Silvio. 2003. *On Ratio and Proportion*, 2nd. ed. Kim Williams and Stephen R. Wassell, eds. Fucecchio (Florence): Kim Williams Books.

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K. Williams, C. Monteleone, *Daniele Barbaro's Perspective of 1568*,
https://doi.org/10.1007/978-3-030-76687-0

Burton (1945)

Burton, Harry Edwin H. 1945. The Optics of Euclid. *Journal of the Optical Society of America* 35(5), pp. 357-372.

Maggi and Castriotto (1564)

Maggi, Girolamo and Iacomo [Giacomo] Castriotto. 1564. *Della fortificazione delle città.* Venice: Rutilio Borgominieri.

Commandino (1558)

Federico Commandino, *Ptolemaei Planisphaerium. Jordani Planisphaerium. Federici Commandini urbinatis in Planisphaerium Commentarius, in quo universa Scenographices ratio quam brevissime traditur, ac demonstrationibus confirmatur*. Venice: Aldus (Paolo Manuzio), 1558.

Danti (1573)

Danti, Egnazio. 1573. *La prospettiva di Euclide*. Florence: nella stamperia de' Giunti.

Descartes (1860)

Descartes, René. 1860. De solidorum elementis. *Oeuvres Inédites*, vol. II, pp. 214-226. Paris: Durand.

Dürer (1525)

Dürer, Albrecht. 1525. *Underweysung der Messung, mit dem Zirckel und Richtscheyt, in Linien, Ebenen unnd gantzen corporen.* Nuremberg. 2nd ed., 1538.

Dürer (1528)

Dürer, Albrecht. 1528. *Hierinn sind begriffen vier Bucher von menschlicher Proportion durch Albrechten Durer von Nurerberg [sic.] erfunden und beschuben zu nutz allen denen so zu diser kunst lieb tragen*. Nuremberg: Hieronymus Andreae Formschneider.

Field (1997)

Field J. V. 1997. Rediscovering the Archimedean Polyhedra: Piero della Francesca, Luca Pacioli, Leonardo da Vinci, Albrecht Dürer, Daniele Barbaro, and Johannes Kepler. *Archive for History of Exact Sciences* 50(3/4), pp. 241-289.

Guidobaldo (1600)

Guidobaldo del Monte. 1600. *Perspectivae libri sex*. Pesaro: apud Hieronymus Concordiam.

Labacco (1552)

Antonio Labacco (or dell'Abacco). 1552. *Libro d'Antonio Labacco appartenente a l'architettura nel qual si figurano alcune notabili antiquita di Roma.* Rome.

Maor (2011)

Maor, Eli. 2011. *Trigonometric Delights*. Princeton, NJ: Princeton University Press.

Monteleone (2020)

Monteleone, Cosimo. 2020. *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto it. IV, 39=5446*. Canterano (Rome): Aracne editrice.

Palladio (1997)

Palladio, Andrea. 1997. *The Four Books of Architecture*. Robert Tavernor and Richard Schofield, eds. Cambridge, Mass.: MIT Press.

Piero della Francesca (Regg. A 41/2)

Piero della Francesca. n.d. *De prospectiva pingendi*, Mss. Regg. A 41/2, Biblioteca Panizzi, Reggio Emilia.

Serlio (1566)

Serlio, Sebastiano. 1566. *Libro primo [-quinto] d'architettura*. Venice: Francesco de' Franceschi.

Toomer (1984)

Toomer, G. J. 1984. *Ptolemy's Almagest*. London: Duckworth.

Vimercati (1565)

Vimercato, Giovanni Battista. 1565. *Dialogo della descrittione teorica et pratica de gli horologi solari*. Ferrara: Per Valente Panizza. (Repr. Venice: Gabriel Giolito d' Ferrara, 1566.)

Williams (2019)

Williams, Kim. 2019. *Daniele Barbaro's Vitruvius of 1567*. Basel: Birkhäuser.

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